

# Time and space matter: how urban transitions create inequality

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## Abstract

An increase in transportation costs impacts the welfare of households living on the outskirts of the city more than the other inhabitants and, in the short term, limited housing supply stops them from moving toward the center. Over a longer period however, urban adjustments cancel out this inequality: (1) in the center, rent level rises because of higher demand, inducing investment in additional housing and increasing city density; (2) on the outskirts, housing demand decreases until rent level decreases and compensates for higher transportation expenditures. Inertia in housing supply and household re-locations leads, therefore, to the development of spatial inequalities.

To investigate this issue, we built a dynamic model that reproduces urban transitions in monocentric cities, and enables quantifying in continuous time their spatialized consequences. Applied to the implementation of a transportation tax, the model suggests that a rapid implementation would induce (i) higher welfare losses

than can be inferred from traditional models and (ii) major redistributive effects throughout the city. Finally, the model suggests that an early and progressive implementation is to be preferred to late and aggressive action.

These results challenge current assessment methods of climate change stabilization strategies and show that it is essential to take into account urban dynamics and inequalities in the design of climate policy.

*Key words:* City, Housing, Transportation

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## 1 Introduction

As mobility needs induce a large and increasing share of greenhouse gas emissions, it is very likely that transportation systems will have to change if our societies are to respect a carbon constraint. Urban transportation systems will, therefore, have to go through significant evolutions, and ambitious urban policies are indeed a necessary step so as to achieve Kyoto-like objectives (Srinivas, 2000).

Economic evaluations of climate policies often consist in the assessment of aggregated GDP losses (*e.g.*, Tulkens and Tulkens, 2006). The analysis of urban policies, however, shall tackle impacts that are widely differentiated in space. The effects of an increase in oil prices on suburban households that are heavily dependant on private vehicles, for instance, should be distinguished from the

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effects on central city inhabitants. Cities, moreover, are slowly-evolving systems: assessing the cost of a change in transportation systems not only requires knowledge about the present and future equilibriums of the system; it also requires an analysis of the transition paths between these equilibriums, and a specific investigation of the important question of inertia (Rotmans et al., 1994) applied to urban dynamics. This paper aims at providing a framework to assess urban-system transitions, including their spatial distribution.

Beyond its general interest, the question of differentiated effects in time and space of energy policies is particularly crucial for cities. The functioning of cities relies on long-lived infrastructures in housing and transportation sectors, that can only adapt gradually to new economic conditions (Gusdorf and Hallegatte, 2007). But infrastructures are not the only source of inertia: it also takes time to households to change their locations, to modify their consumption bundles, and housing rents are also sticky to a certain extent.

We propose here a model that is fitted to address the stylized evolutions of urban systems through time and space. This Non Equilibrium Dynamic Urban Model (NEDUM) is based on the classic urban model *à la* Von Thuenen (1826), adapted to cities by Alonso (1964), Mills (1967) and Muth (1969). Dynamic analysis of cities based on the Von Thuenen framework have already been proposed before, but they only consisted in a sequence of stationary equilibriums, see *e.g.* Anas (1978) or Capozza and Helsley (1990), and a review in Brueckner (2000). Our approach is innovative in that it allows to represent non-stationary states, taking into account inertia in households relocation, in apartments' sizes, housing service production, and stickiness in housing rents.

In addition, we introduce macroeconomic feedbacks in the model by making

income endogenous: workers supply their labor force to firms that produce a composite goods, a process we represent through a neo-classical production function (Cobb and Douglas, 1928). A constant share of product is saved, and used for investments. Investments are either directed towards the productive or the housing sector, depending on their respective profitability. This interdependence between investment choices allows for the representation of crowding-out effects when housing needs make construction more profitable than productive investments.

We use NEDUM to perform two sets of numerical experiments. First, we simulate the effects of a shock on transportation costs. We show that the distribution in time of the cost of such a shock is very unequally spread between the short, medium and long run. The long term effect, namely an increase in density in response to higher transportation costs, is a classical result (e.g., Fujita, 1989), and we focus our analysis on transition phenomenons. With our calibration, roughly based on the characteristics of the Los Angeles agglomeration, a 50% increase in transportation costs leads to significant negative effects on utility levels during approximately 60 years after the shock. Households living at the outskirts of the city are most impacted during the transition period. We quantify this effect with the Gini index, which is a common economic tool designed to quantify inequalities: in our stylized city, though we assumed that all workers earn the same income, this index stays above 0.02 during approximately 55 years after the shock, and reaches a peak value close to 0.12<sup>1</sup>. This is indeed a strong effect, justifying the need for extensive analysis of the effects of urban transitions.

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<sup>1</sup> The Gini index is equal to 0 when there are no inequalities, and increases with inequality, to reach 1 when one individual earns the entire income of the society.

Second, we assume that the city government has decided to implement a transportation tax (Collier and Loefstedt, 1997), that will represent a 50% increase in transportation costs in year 2050. Before this date, the implementation path is freely chosen by the government, and we investigate the advantages of early and smooth, vs. late and aggressive action. Early implementation allows coping with the inertia of several mechanisms; however, it imposes an early constraint on economic agents, which may worsen the situation compared to late implementation. We find that there exists an equity vs. efficiency trade-off, since implementing the signal-price in less than 20 years may reduce welfare costs compared to early implementation, but entails significant anti-redistributive effects. These results show that a fraction of the population is strongly impacted by the changes in urban systems, and could be deeply opposed to these changes. The taking into account of this mechanism may be as important for policy design as the aggregate economic costs.

The remainder of this paper is as follows: Section 2 is a brief reminder of the classic urban model *a la* Von Thuenen. In Section 3, we present the NE-DUM model, that is exposed in details in the Appendices A and B. Section 4 investigates the effects of a shock on transportation costs, and Section 5 analyses various implementation paths of a given transport taxation level. Finally, Section 6 concludes and provides insights for future research.

## **2 The traditional equilibrium urban model**

In this section, the general features of the classical static equilibrium framework describing urban systems are recalled. We do it briefly because it is a traditional model, which reproduces some well established observations about

cities (Wheaton, 1974). A more detailed description can be found for instance in Fujita (1989).

In this stylized monocentric city, housing is organized around a Central Business District (CBD). A given number  $N$  of identical households inhabits the city, one worker in each household commuting every day to and from the CBD, earning the same income and sharing the same consumption level  $c$ . Transportation costs,  $T(r)$ , with respect to the distance  $r$  from the CBD are given, while housing rents  $R_H(r)$  are endogenous, and ensure that identical households reach the same utility level at the equilibrium, even though they live at different locations. Household behavior is driven by the maximization of a utility function  $U(z, q)$  describing preferences for the consumption of composite goods  $z$  and housing service  $q$ :

$$\max_{r, z, q} U(z, q) \tag{1}$$

s.t.

$$z + R_H(r)q \leq c - T(r) \tag{2}$$

Equation (2) sets the budget constraint of the household, the composite goods being chosen as the numeraire.

We use a production function of housing service  $F(L, K)$  *a la* Muth (1969): this function  $F$  takes capital  $K$  and land  $L$  as inputs, and is linear. We specify the housing service density  $h(r) = H(r)/\text{Land}(r) = f(x^*(r))$ , where  $x = K/L$ , where  $f(x) = F(1, x)$ , and the superscript “\*” denotes equilibrium values.

Generalized transportation costs are represented by the function  $T(r)$ . In the

current version of the model, the transportation system consists in a single transportation mode, used only for commuting. Also, transportation costs only take into account the monetary costs. In the future, the model could encompass several transportation modes, other trip motives, and includes the cost of the time spent in transportation, which otherwise could have been devoted to work or leisure.

[ TABLE 1 ]

Table 1 presents the standard nomenclature we use, while Eqs. (1) to (7) describe the basic relationships of the classical urban modeling framework.

$$\max_{K_H} R_H(r)F(L, K_H) - \rho K_H \quad (3)$$

$$k_H(r) = \frac{K_H}{L}(r) = \operatorname{argmax} [R_H(r)F(1, x) - \rho x] \quad (4)$$

$$H(r) = \text{Land}(r) \cdot F(1, k_H(r)) \quad (5)$$

$$R_H(r) = 0 \text{ pour } r \geq r_f \quad (6)$$

$$N = \int_0^{r_f} H(r)/q(r)dr \quad (7)$$

A classic result of urban microeconomics (Fujita, 1989) is that if available land  $\text{Land}(r)$  is continuous and positive for all  $r > 0$ , and if the consumption *per capita*  $c$ , the number of inhabitants  $N$ , the transportation costs  $T(r)$  and the interest rate  $\rho$  are given, then Eqs. (1) to (7) define a unique utility equilibrium level  $u^*$ , homogenous through the whole city. This framework is adapted so as to represent realistic urban dynamics as described in the next section.

### 3 Non Equilibrium Dynamic Urban Model

The monocentric model has been mostly used to explore the characteristics of long run equilibriums. However, the existence of urban stationary equilibriums is questionable: in cities, some important economic variables vary in the short run, while other features of the city cannot adapt rapidly to changing conditions. Income and transportation costs, for example, evolve much more rapidly than housing infrastructures, which need several decades to be rebuilt. It is, therefore, very likely that the history of urban systems cannot be analyzed as a succession of stationary states, but requires a dynamical approach: assessing the effects of an urban policy requires to account for the existence and specificities of transitions.

We propose here a model that is able to capture the dynamics of urban systems, and the importance of inertia. At the microeconomic level, four key mechanisms that drive urban dynamics can be identified, as described in the following sections. In addition, we ensure that NEDUM takes the main macroeconomic feedbacks into account.

A complete description of the model, with the full set of equations, is available in Appendix A. The nomenclature is summarized in Tab. 2, and presents the new variables added to the traditional monocentric model.

#### *3.1 Households behavior*

We assume that households earn an income  $Y$ , which is partly consumed and partly invested in the productive and housing sectors (Section A.1). House-

holds also choose their housing consumption and location depending on the rents  $R_H(r)$ , as described hereafter:

- Households living at location  $r$  adjust their housing service consumption *per capita*  $q$  so as to increase their utility level  $u(r) = U(z(r), q(r))$ : taking rent level  $R_H(r)$  as given, households increase or decrease the size of their flats so as to equalize the marginal utility of housing service consumption and composite goods consumption (Section A.2.1). Adjustment in housing service consumption *per capita* is also attained through changes in the size and composition of households, for example through changes in collocation practices, or changes in the age at which children leave their parents' home.
- Households can change locations: the ones living at location  $r$  may choose to stay or move to another location (Section A.2.2). We assume they are willing to move when their local utility level  $u(r)$  is under the average utility level  $\bar{u}$  throughout the city: households living at locations where  $u(r) < \bar{u}$  are attracted to places where  $\bar{u} < u(r)$ .

Of course, the processes considered here are active in parallel: changes in flat sizes occur simultaneously with location changes, when households leave one flat to another. The changes are physically constrained by the characteristics of housing service supply: households can move only if there are unoccupied flats at their target location; they can increase their flat size only if there is a local excess of housing service supply. These two mechanisms are the basis of local changes in demand for housing service (see Section 3.2).

Most importantly, moves of households and changes in flat sizes cannot happen instantaneously, for instance because it takes time to find a new place to live. The respective inertias of these mechanisms are accounted for by specific

characteristics timescales  $\tau_q$  and  $\tau_n$  (see Eqs (A-3) and (A-11)). The intensity of these mechanisms depend in each case on the increase in utility level that households expect from these evolutions: the higher is the relative difference between  $u(r)$  and  $\bar{u}$  for instance, the more households are willing to move to location  $r$ .

### 3.2 Rent curve dynamics

Rent level  $R_H(r)$  evolves in reaction to local supply and demand of housing service  $H(r)$  (Section A.3): demand is expressed by the number of households  $n(r)$  living at this location and consuming an amount of housing service  $q(r)$ , and by the number of households willing to move to or from this location:

- rent level decreases if demand is lower than local supply, that is, if existing buildings are not fully occupied.
- If buildings at location  $r$  are fully occupied, rent levels increase if inhabitants want to increase their consumption of housing service, or if there are outside households willing to move to this location.

The orders of magnitude of these evolutions are determined by the relative difference between local demand and supply of housing service. Moreover, we assume that, for institutional reasons, housing rents do not clear the housing market instantaneously<sup>2</sup>. The inertia of rent levels evolution is characterized in the model by the timescale  $\tau_R$ .

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<sup>2</sup> In France, for instance, rents are regulated over 3-year periods.

### 3.3 *Capital and investments*

Buildings depreciate in urban systems, and are renewed or rebuilt in reaction to rental profitability. These investments have a cost and may have a crowding-out effect on other investments. This is why we added a description of capital stock evolution in the housing and productive sectors (Section A.4). Investments are directed towards either one of these two sectors (Sections A.4.1 and A.4.2). In the housing sector, investments are directed towards specific locations. The interest rate is supposed to clear instantaneously the financial capital market. This macroeconomic feedback allows for the description of crowding-out of productive investments by housing investments.

Since construction takes time (Kydland and Prescott, 1982), financial investments are transformed into productive units or into buildings with a time lag. In each sector, the timescales of this transformation of pending investments into operational investments are respectively defined as  $\tau_k$  and  $\tau_h$ .

### 3.4 *Specific functional forms and calibration*

In Appendix B, we show that, under standard conditions on general functional forms, NEDUM reaches a unique stationary state, which recovers the classic equilibrium of the Von Thunen framework. In the present section, we calibrate the model and explore the properties of NEDUM for a circular city, by adopting classic Cobb-Douglas functional forms for the utility, the housing service production and the composite goods production functions. The generalized commuting costs are assumed to increase linearly with the distance from the CBD. We reproduce in Appendix B the calculations that describe

this stationary equilibrium with our specific functional forms.

The set of parameters that determines the equilibrium state are calibrated separately from the ones that only concern dynamics.

*Equilibrium:* the parameters of our model are calibrated so that, at equilibrium, it reproduces the characteristics of Los Angeles County. Of course, such a calibration is rough, if only because the L.-A. economy is open, while we do not take into account investment coming from or going outside L.A.. In 1999, 4.3 millions workers were inhabiting the city, earning a \$20 700 yearly *per capita* income (data U.S. Census Bureau 1999). The transportation price is calibrated using 1999 gasoline prices (*i.e.* 32 cents per km on average, data American Automobile Association 1999)<sup>3</sup>. We calibrated the utility function such that, at equilibrium, housing expenditures represent 30% of households budget, while 19% of this budget is devoted to transportation expenditures (STPP, 2003). Concerning macroeconomic feedbacks, we used for calibration the aggregate American investment rates. This leads to an investment rate  $s$  of 19% (data Bureau of Economic Analysis 2006).

*Dynamics:* The parameters  $\tau_k = 15$  years and  $\tau_h = 60$  years can be approximated in an easy way, since they correspond to the construction duration of production units and buildings. Calibration of parameters  $\tau_n$ ,  $\tau_R$  and  $\tau_q$  is particularly difficult. Typical values are explored: we consider  $\tau_n = \tau_R = \tau_q = 10$  years.

*Sensitivity analysis:* Considering the uncertainty on these values, systematic

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<sup>3</sup> It appears that the calibration of this numerical value is not crucial, since our results are mostly dependant on the relative change in transportation costs.

sensitivity analyses were also carried out, varying each parameter at a time or several simultaneously. We found that the qualitative results and order of magnitude presented in the remainder of this paper are unchanged within a broad range of values (e.g., from 3 to 20 years for  $\tau_n$ ,  $\tau_R$ , and  $\tau_q$ ). In particular, the main policy conclusions remain valid for all of these values, as shown for instance by the sensitivity analysis displayed in Fig. 7.

## 4 Dynamic analysis of a shock on transportation costs

In this section, we explore the effects of a shock on transportation prices. We assume that the initial state of the city (at time  $t = 0$ ) is the stationary equilibrium described in Appendix B. Variables in the initial (resp. final) state are noted with subscript “ $i$ ” (resp. “ $f$ ”). We assume that in three years, starting at time  $T$ , transportation costs undergo a 50% increase, jumping from  $p_i$  to  $p_f = 1.5p_i$ . We use NEDUM to investigate how the city reacts to this shock.

In our numerical simulations, the state of the city converges towards the stationary equilibrium corresponding to transportation costs  $p_f$ . This convergence, as is shown below, is very slow for all values of the parameters. Most importantly, our simulations provide results that are differentiated with respect to space. At each location, we study the path followed by the economic variables in response to the shock.

### 4.1 Average utility level

The stationary equilibrium of our model verifies the classical results of comparative static analysis in urban economics (see for instance Wheaton, 1974).

Following the shock in transportation costs, hence, the city concentrates towards the CBD. Rents increase near the CBD, and decrease at the outskirts. In the long run, consumers' utility decreases: after 150 years, average utility level  $\bar{u}_{150}$  is 16% lower than initial utility  $\bar{u}_i$  (see Fig. 1). At this date, the final equilibrium has almost been reached.

[FIGURE 1]

During the transition period, housing is not adapted to the new economic conditions, and the situation of the city inhabitants is significantly worsened, compared with the final stationary equilibrium. Figure 1 shows that, 25 years after the shock, the aggregate utility level in the city reaches a level which is still 27% lower than the initial utility, and 12% lower than the final one. The importance of transition impacts and the length of this period are quite robust to changes in the timescales  $\tau_n$  and  $\tau_q$ , which are successively divided by 2 in Fig. 1: in all cases, the average utility level in the city stays under 80% of its initial level for approximately 60 years. A division by 2 of timescale  $\tau_R$  has a slightly different effect on the dynamics of the system: the average utility level goes faster above the 80% value, though the length of the transition is roughly the same.

The aggregate effects of a shock on transportation costs, indeed, stem from the interaction of microeconomic behaviors: facing new, higher transportation costs, people want to move closer to the CBD. But there is not enough space for them to move. Before the moves actually occur, therefore, rent levels and flat sizes have to change to leave room for new inhabitants close to the CBD. Over the longer term, the density of housing service supply (of which the height of the buildings is a good proxy) will adapt to the new conditions

and the higher rents in the center. Simulations with NEDUM show that the transition duration depends mostly on building inertia: it takes a long time to collect and direct great quantities of housing capital towards new locations. In the model, the reaction of the market depends on (i) the timescales  $\tau_n$ ,  $\tau_R$ , and  $\tau_q$ ; (ii) the parameters of the housing production function  $F(K, L)$  and the investment capacity of the whole economic system.

The timescale of the last mechanism is driven by the Cobb-Douglas housing production function  $F(K, L)$ , the composite goods production function  $Y(K, N)$ , and the investment rate  $s$ . This timescale is much longer than  $\tau_n$ ,  $\tau_R$ , and  $\tau_q$  and is, therefore, the mechanism responsible for the 30 years timescale of the whole system. In the sections below, we investigate the details of this ill-adaptation through time and space.

#### *4.2 Spatialized adaptation*

Initial location choices of households were determined by the initial transportation price  $p_i$  and the initial rent levels throughout the city (see Eq. (2)). As transportation costs get higher, households are willing to move closer to the CBD to spend less on transportation. As a consequence, rent levels go higher in central locations. The left panel of Fig. 2 shows that five years after the shock, rents close to the CBD have already increased, and are close to their final level, while far from the CBD, rents have not changed: the population density has not changed, even though households are willing to leave these locations. This population has indeed to stay there since there is no unoccupied flat yet in the central buildings to allow for their moving in (see the right panel of Fig. 2).

[FIGURE 2]

Of course, rent levels feedback to the rest of the system:

- Housing construction is enhanced in the city center.
- Rent levels also constrain households living close to the CBD to reduce their housing service consumption per capita, thus letting more space for households willing to move in.

[FIGURE 3]

Figure 3 shows that 15 years after the shock, many moves have occurred. At this date, since households have left the outskirts of the city, rents are almost at their final level at all locations. This is not the case for housing capital stock. Construction demands a large amount of capital, and it takes time for the housing capital stock to reach the appropriate level. As a consequence, compared to the final equilibrium, the supply of housing service is still not concentrated enough, even 15 years after the shock. This delay to reach the optimal state explains the low level of average utility attained during the transition period.

### *4.3 Redistributive consequences*

In the model, we assume that the initial state is a stationary equilibrium where all households earn the same income and reach the same utility level:  $u(r) = \bar{u}$  at all locations. After the shock, however,  $u(r)$  is not uniform throughout the city. Households located far from the CBD have to put up with very high transportation costs, but do not see their rents immediately decreasing: the

shock impacts them strongly. The left panel of Fig. 4 shows that, 10 years after the shock, those living at 50 km from the CBD can lose up to 38% of their initial utility level.

Meanwhile, households living closer to the CBD use less transportation for commuting. The initial losses are, therefore, not so high for them, amounting to merely 4% of the initial utility level 10 years after the shock. Later on, they get worse as rents increase because of the demand. Of course in the model, all utility levels converge in the long run towards a common value. This convergence is very slow, as illustrated by the left panel of Fig. 4; this slowness is partly due to the fact that, as utility levels  $u(r)$  get closer to  $\bar{u}$ , incentives to move diminish (see Eq. (A-21)).

[FIGURE 4]

The redistributive effects are significant. The right panel of Fig. 4 shows that the Gini index jumps from 0 to 0.12 right after the shock. For comparison, according to Watkins et al. (2006), at the national levels, Gini indexes of the US, the UK, and in France, are respectively 0.40, 0.36, and 0.32: those three countries have Gini indexes that are comprised in a 0.08 wide range of values, while Brazil's Gini index, for instance, reaches 0.58. In our simulation, the Gini index soars when transportation costs rise, and then decreases as the adaptation mechanisms (moves, change in flat sizes, changes in the rent levels, construction) enter into action: with our calibration, it stays above 0.02 over 50 years.

The need to invest into reconstruction enhances the negative impacts of the transition through crowding-out effects: as rents close to the CBD increase, housing service production gets more profitable, and investments are directed towards the production of housing service at those locations. As a consequence, capital is more sought after by investors, and the interest rate increases by almost 0.25 points (see the right panel of Fig. 5). On the long run, however, the interest rate asymptotically returns to its initial level. Meanwhile, there is a crowding-out effect of productive investments by housing investments, and 36 years after the shock, the production of composite goods has decreased by 1.2% (see the left panel of Fig. 5). Given its timing, and even though the aggregate product returns to its initial level in the long run, this effect is important: as transportation costs rise, household income decreases in.

It is noteworthy that, in the real world, the same mechanism that increases the cost of capital would also apply to labor. Following increased profitability of housing service production, wages go up in this sector, inducing the workers to switch from the productive to the construction sector. Comparable mechanisms occur after a natural disaster, when scarcity of qualified workers with regard to reconstruction needs causes their wage to increase. This phenomenon would enhance the cost of crowding-out effects. For simplicity's sake, we assumed that wages were fixed, and that all workers were employed in the productive sector.

[ FIGURE 5 ]

## 5 Sending a signal-price: when late is too late

We now assume that the government has decided to have a given tax level on transportation in year 2050. This situation may arise for instance in the framework of climate negotiations, if international agreements are reached on the internalization of climate change costs through a carbon tax. In this exercise, considering a 2050 target allows to analyze the importance of the implementation pace of the signal-price over long periods of time.

### 5.1 *Early versus delayed action*

We assume that the policy planning works as follows:

- with the transportation tax, workers have to pay an increased transportation cost  $T(r) = (p + \tau)r$ . The product of the tax, namely  $\pi = \tau \int_0^{r_f} n(r)rdr$ , is lump-sum redistributed to the workers and used for consumption.
- the tax level  $\tau(t)$  increases linearly from its initial level  $\tau_i = 0$  to the final level  $\tau_f = p/2$  in 2050, and remains at this level later on. The slope of the increase before 2050 depends on the starting year of implementation.

We study implementation periods that run from 60 years down to 0. Thus, latest implementation begins in year 2050, while earliest implementation begins in year 1990. We consider such an early anticipation, since high levels of transport taxation already existed in some countries, even though in year 1990 they were not related to climate policies.

[ FIGURE 6 ]

We estimate the impacts of these various policies over the long run, namely until year 2250. This very long period is necessary since the return to the stationary equilibrium is very slow. At this date, different paths have lead to almost identical situations, and the situation of the urban system is almost independent of the implementation policy (see Fig. 6).

## 5.2 Welfare losses

It is difficult to *a priori* predict which type of implementation (early or late) is likely to be the less costly. Smooth implementation makes inertia in the urban system less detrimental, but does not suppress completely its effects. Early implementation also induces the system to converge sooner towards a stationary state that is under-optimal, at least if the impacts of carbon emissions on the environment and the economy (*i.e.* the benefits of the policy) are not taken into account. With this limited conception of welfare, an early implementation leads to welfare losses which are accumulated over a long period of time (as soon as the policy begins). Late implementation, on the other hand, induces more abrupt welfare losses, that are more concentrated in time. As an illustration, Fig. 6 shows the average utility level for a “smooth” policy, starting in year 2000, and an “aggressive” policy, starting in year 2040.

We compare the corresponding costs to a baseline scenario where no action is implemented at all. According to our simulations (see the left panel of Fig. 7, where each curve corresponds to different values of the model parameters), welfare costs can represent a 3.8% to 2.1% loss over the entire period. These relatively high losses are due to several factors, some of which are not taken into account in published assessments of mitigation costs (Weyant et al., 2006;

Edenhofer et al., 2006): (1) consumers spend more money in transportation for a given commuting distance; (2) they live in smaller flats because of the higher burden from transportation costs; and (3) the preexisting spatial distribution of flats is ill-adapted during the transition.

[ FIGURE 7 ]

Simulations show that welfare costs decrease slightly when the action is delayed. Thus, the gains due to the neutralization of inertia effects are more than compensated by the increased losses due to the under-optimality of the final stationary state. Note, however, that this welfare assessment is not a cost-benefit analysis, since benefits are not taken into account.

### *5.3 Redistributive effects*

In assessing public policies, investigating aggregate effects is not enough, and the consequences in terms of inequality are also crucial. In our exercise, the maximum value reached by the Gini index increases when implementation is delayed; see the right panel of Fig. 6 for a comparison between a policy implemented in 2000 and a policy implemented in 2040.

Most importantly, redistributive effects are non-linear with respect to the implementation duration. On the right panel of Fig. 7, each curve corresponds to different values of the parameters we used in NEDUM. Each point in one of these curves shows the maximum value of the Gini index that will be reached during the implementation period, as a function of the year of policy implementation. We consider the maximum value of the Gini index since it is a good indicator of the potential negative redistributive impact of a policy.

Two features can be observed from the analysis of these redistributive effects:

- For all the sensitivity tests we performed, the maximum value of the Gini index remains very close to 0.02 as long as the implementation period begins before year 2015.
- If the urban policy begins after year 2020, the spectrum of Gini index values gets much larger. Late implementation, starting in year 2049, induces maximum values of the Gini index that range from 0.05 to 0.08, which is clearly a major disruption of the social situation.

As a consequence, delaying the tax implementation from year 2015 to year 2049 induces a decrease in welfare losses by 1 point, but causes the Gini index to reach significantly higher levels. This is an equity vs. efficiency trade-off that cannot be easily resolved. For instance, one may use the tax product to compensate the households living far from the center for the increase in transportation costs (instead of lump-sum distributing this product). This action, however, would distort the signal being sent to households and limit the policy. The tax is indeed meant to internalize the costs of carbon emissions and decrease mobility demand, which cannot be done without creating inequality.

## 6 Conclusion

### 6.1 Summary

This paper presents a new model, NEDUM, as a support for urban dynamics and policy analysis. Without pretending to produce precise costs estimates, the model allows for the analysis of stylized dynamical effects, and for assessing the

orders of magnitude of policy consequences. We focused on the importance of inertia in infrastructures renewal, in household moves, in changes in flat sizes, and on stickiness of housing rents.

In the long run, an increase in transportation costs translates into a decrease in average utility, since consumers spend more in transportation and live in smaller flats. In the short- and medium-run, however, the impacts on welfare are even larger: after a 50% shock in transportation costs, the loss in average utility is 70% larger over the medium term than over the long term. This is due mostly to the adaptation pace of the urban system to the new transportation costs. These consequences of inertia are worsened by crowding-out effect from productive investments to housing investments. Composite goods production is decreased by up to 1.2% during the transition, because more investment capacities are used in housing sector. Acknowledging these mechanisms may significantly change the assessment of GHG stabilization strategies, compared with published assessments, see *e.g.* Weyant et al. (2006) or Edenhofer et al. (2006).

Because of inertia, changes in urban transportation systems have also significant redistributive effects. The reason is that the demand for transportation is differentiated in space. Thus, location matters: even though all utility levels in our imaginary city are eventually equal, there exist non-trivial paths between the utility levels immediately after the shock and the final utility  $u_f$ . Following an increase in transportation costs, consumers living far from the CBD have a strong burden to cope with, and cannot immediately move to more favorable locations, because housing is not yet available close to employment centers.

The magnitude of the redistributive effects is directly related to the aggressive-

ness of the change, i.e. to the amplitude of the modifications and their pace. Considering the implementation of a carbon tax, there is a trade-off between equity and efficiency, and the redistributive effects increase non-linearly when the implementation duration is reduced.

## *6.2 Discussion*

This paper highlights that urban-policy analyses need to assess the transitory effects of policies, not only the desirability of their final results. It also shows the need for multiple metrics to measure policy consequences, in particular to take into account their influence on inequality. In this perspective, this paper makes two main contributions. From a methodological point of view, it provides a spatialized view of urban transitions over continuous time, allowing differentiated assessments of policy impacts within a city. Applied to urban-climate policies, it highlights an equity-efficiency trade-off, with a strong non-linearity in negative equity impacts when an urban policy is abruptly implemented.

In the current context of rapid urbanization — especially in developing countries — and of growing pressure to reduce energy consumption from transportation, these insights call for more research in the line of this paper, and for intensified exchanges between urban planners and decision-makers involved in energy-climate policies.

Of course, things are more complex in the real world than in the model, and transition effects depend on the specific features of each city. NEDUM is only a first step towards a dynamic assessment of urban changes, and this first version

has several limitations. Although the usual limitations involved in the classic Von Thuenen model are also present<sup>4</sup>, it seems at first view that they would only marginally interfere with our results. Main differences with the real world are the existence of several employment centers, the problem of congestion in transportation systems, and the co-existence of several transportation systems.

However, there are other limitations to our model, which may be more important and constitute a program for future research.

The first question NEDUM should be able to tackle next is the importance of anticipations: our assumption of agents' myopia is an extreme one. Agents have expectations, either false or true, and their anticipations influence the whole system behavior as well as the pace of changes. It is particularly important to include these aspects in NEDUM so as to be able to analyse commitment problems on behalf of the government, and avoid time-inconsistent taxation patterns.

Second, another important dynamic aspect is absent from our economy: population change and economic growth have not been taken into account. Clearly, growth modifies the impacts of changes in the transportation system, since economic conditions evolve continuously. Nevertheless, though the value of welfare losses would be impacted, it is likely that the sensitivity of redistributive effects to the pace of changes would not be very different than in a world without growth.

Third, we did not take into account the possibility of an accelerated rate of building turnover that could influence the vulnerability of urban systems and

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<sup>4</sup> Except for the macroeconomic feedbacks, which are present in NEDUM.

their adaptive capacity.

Finally, all households do not earn the same income. It is likely that in American cities, where low income workers usually live in the center while rich households live at the outskirts, the Gini index would reach different levels, and may even go down in response to higher transportation costs. In European cities, where city centers are mostly inhabited by rich households, the Gini index would probably get even higher. In both types of cities, anyway, important redistributive effects would occur, and their taking into account should be a priority in policy design and urban planning.

## **7 Acknowledgements**

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## **A Appendix: The Non Equilibrium Dynamic Model**

This appendix sets up the formal representation of the mechanisms described in Section 3. Table 2 summarizes the nomenclature for the new variables added to the traditional Von Thuenen model.

[ TABLE 2 ]

### A.1 Production and consumption

While the income in Section 2 is exogenous, we specify here a production function  $Y$ , the inputs of which are labor  $N$ , and productive capital  $K$ :

$$Y = Y(K, N) \tag{A-1}$$

A constant part of the product is saved and shared between financial productive and housing investment ( $\widetilde{I}_{fK}$  and  $\widetilde{I}_{fH}$  respectively), while the other part  $c$  is used by households for consumption. For simplicity reason, we assume that land is publicly owned: land incomes  $LI$  are collected by the government, and lump-sum redistributed to consumers and used for consumption.

$$s \cdot Y = \widetilde{I}_{fH} + \widetilde{I}_{fK} \tag{A-2}$$

$$c = (1 - s) \cdot \frac{Y}{N} + \frac{LI}{N}$$

### A.2 Households behavior

#### A.2.1 Housing service per household

We assume that households permanently adapt their housing-service consumption to prices. We set the utility level  $\tilde{u}(q, r)$  reached by households living at a distance  $r$  from the CBD, as a function of housing service consumption  $q$ ; given the distance  $r$ , the amount of composite goods consumed is strictly dependant on housing choices:  $z = c - T(r) - R_H(r)q$ . We have, therefore,  $\tilde{u}(q, r) = U([c - T(r) - R_H(r)q], q)$ . Using this function, we consider that households can adjust their level of housing service consumption so as to improve their utility level.

Based on this relationship, at a given location, a change in housing service consumption *per capita*  $\delta q$  induces a change in utility:  $d\tilde{u} = \left(\frac{\partial \tilde{u}}{\partial q}\right) dq$ . If at location  $r$ ,  $\frac{\partial \tilde{u}}{\partial q} > 0$ , it is indeed rational for the inhabitants to increase their consumption of housing service. Of course, an increase in housing consumption is authorized if and only if such an increase is physically possible, *i.e.* if there is available housing at this location. The dynamics of  $q(r)$  is given by:

$$\frac{dq}{dt}(r) = \begin{cases} \frac{1}{\tau_q} g\left(\frac{\partial \tilde{u}}{\partial q}\right) & \text{if } \psi(r) > 0 \\ 0 & \text{if } \psi(r) = 0 \end{cases} \quad (\text{A-3})$$

where  $g(x)$  is an growing function of  $x$  and has the same sign as its argument  $x$ ; moreover,  $\psi(r)$  represents the number of unoccupied flats at location  $r$ , calculated as:

$$\psi(r) = \frac{H(r)}{q(r)} - n(r) \quad (\text{A-4})$$

### A.2.2 Moving throughout the city

Consumers have the possibility to move and change location across the city. They are driven by the utility levels  $u(r)$  that characterize locations. We set  $\bar{u}$  as the average utility level of consumers throughout the city. At a given location  $r$ , two cases can arise, depending on the utility level  $u(r)$ .

- If  $u(r) < \bar{u}$ , households are willing to leave towards other locations. We set  $m^-(r)$  as the number of households that are willing to move out:

$$m^-(r) = n(r)w^-(u(r), \bar{u}) \quad (\text{A-5})$$

In Eq. (A-5), the willingness to move from a location is larger if the gap

between  $u(r)$  and  $\bar{u}$  is large, and  $w^-$  is a “weight” function depending on this gap:  $w^-(u(r), \bar{u}) \in ]0, 1[$ , and  $w^-$  is a function that increases with respect to  $\left|\frac{\bar{u}-u(r)}{\bar{u}}\right|$ .

- If  $u(r) > \bar{u}$ , then households located elsewhere are willing to move towards this location. We set  $m^+(r)$  as the number of unoccupied flats at location  $r$  that are attracting households. This attractiveness is increasing with the gap between  $u(r)$  and  $\bar{u}$ :

$$m^+(r) = \psi(r)w^+(u(r), \bar{u}) \quad (\text{A-6})$$

In Eq. (A-6),  $w^+(u(r), \bar{u}) \in ]0, 1[$ , and  $w^+$  increases with respect to  $\left|\frac{u(r)-\bar{u}}{\bar{u}}\right|$ .

Based on Eqs. (A-5) and (A-6), the aggregate demand for moves and the aggregate supply of attractive, unoccupied flats, are given respectively by:

$$\widetilde{D} = \int_{u(r) < \bar{u}} m^-(r) dr = \int_{u(r) < \bar{u}} n(r) \cdot w^-(u(r), \bar{u}) dr \quad (\text{A-7})$$

$$\widetilde{S} = \int_{u(r) > \bar{u}} m^+(r) dr = \int_{u(r) > \bar{u}} \psi(r) \cdot w^+(u(r), \bar{u}) dr \quad (\text{A-8})$$

Households move when moving can increase their utility level, as long as there is available housing. However, there is *a priori* no reason why the demand for moves should equal the supply of available housing. The relationships giving the moves  $\mu(r)$  meet these physical constraints:

$$\mu(r) = \begin{cases} m^+(r) \cdot \min\left(1, \frac{\widetilde{D}}{\widetilde{S}}\right) & \text{if } u(r) > \bar{u} \\ -m^-(r) \cdot \min\left(1, \frac{\widetilde{S}}{\widetilde{D}}\right) & \text{if } u(r) < \bar{u} \end{cases} \quad (\text{A-9})$$

The variable  $d(r)$  represents the number of households that are attracted by location  $r$ . It can be greater than the intensity of moving  $\mu(r)$ , since demand

may exceed supply of unoccupied flats:

$$d(r) = m^+(r) \cdot \frac{\widetilde{D}}{\widetilde{S}} \quad (\text{A-10})$$

In Eq. (A-10), the coefficient  $\frac{\widetilde{D}}{\widetilde{S}}$  represents the number of “candidates” per unoccupied flat. If aggregate demand is smaller than aggregate supply, not all available housings will find an occupier. If aggregate demand is greater than aggregate supply, then there are more households willing to move than available housings, and not all candidates will find a new housing.

The number of households living at location  $r$  evolves according to the moves:

$$\frac{\partial n}{\partial t}(r) = \frac{1}{\tau_n} \mu(r) \quad (\text{A-11})$$

### *A.3 Rent curve dynamics*

In the classical Von Thuenen framework, housing market is at equilibrium thanks to the rent curve (cf. Section 2). It is not necessarily the case during transitions. In real life, for institutional and practical reasons, rent levels are sticky . In consequence, the dynamics for rents is directed by supply and demand for housing service at each location: if the number  $d(r)$  of households willing to move in is greater than the number of unoccupied flats, the rent level increases. If however, demand for housing is falling, the rent level at this location decreases.

Two cases need to be distinguished:

- If  $u(r) < \bar{u}$ , households are willing to move out. However, it may be the case that they cannot do it because of the absence of available housing elsewhere.

In this case, there is no reason for rent levels to decrease. For this reason, decreasing rents are function of the proportion of unoccupied flats:

$$\frac{dR_H}{dt}(r) = \frac{R_H(r)}{\tau_R} \cdot \phi\left(\frac{n(r)q(r) - H(r)}{n(r)q(r)}\right) \quad \text{if } u(r) < \bar{u} \quad (\text{A-12})$$

- If  $u(r) > \bar{u}$ , then households are willing to stay in or come by, and rent level will increase in reaction:

$$\frac{dR_H}{dt}(r) = \frac{R_H(r)}{\tau_R} \cdot \phi\left(\frac{n(r) + d(r) - \mu(r)}{n(r)} - \frac{H(r)}{n(r)q(r)}\right) \quad \text{if } u(r) > \bar{u} \quad (\text{A-13})$$

In Eq. (A-13),  $\phi$  is a growing function and has the same sign as its argument (and  $\phi(0) = 0$ ). Furthermore, we verify *a posteriori* in our numerical experiments that no housing service is provided beyond production capacity, that is:  $n(r)q(r) \leq H(r)$  is always verified at all times and at all locations.

#### A.4 Capital and investment

Equilibrium in financial markets is ensured by the adjustment of the capital price through the interest rate  $\rho$ , ensuring that savings matches investment. Investments are distributed among productive and housing sectors, as explained below.

##### A.4.1 Productive investments

The variable  $K$  is the capital stock in the productive sector, and  $\delta_K$  is the depreciation rate of capital. Firms seek to maximize their profits, and have a myopic behavior: they make investment decisions as if they were at a stationary state of equilibrium. This leads to the financial investment  $\widetilde{I}_{fK}$  in the

productive sector:

$$\widetilde{I}_{fK} = \delta_K \cdot \arg \max_K [Y(N, K) - (\rho + \delta_K) \cdot K] \quad (\text{A-14})$$

Physical construction requires time (Kydland and Prescott, 1982); financial investments are hence transformed into productive capital with a time lag, corresponding to construction duration. We set  $S_K$  as the resulting stock of “pending investments” in productive capital.  $I_K$  is the real physical investment, which evolves according to the following equations:

$$\begin{aligned} \frac{dS_K}{dt} &= -I_K + \widetilde{I}_{fK} \\ I_K &= \frac{1}{\tau_k} \cdot S_K \\ \frac{dK}{dt} &= -\delta_K K + I_K \end{aligned} \quad (\text{A-15})$$

#### A.4.2 *Housing investment*

Housing is produced using land and capital<sup>5</sup>. The modeling of investments in the housing sector is based on the same principles that drive investment in the productive sector. A little complication is however added, due to the fact that the location of housing investments is driven not only by interest rate, but also by rent levels, which vary with location (see how the density of available housing service is linked to housing capital stock in Eq. (5)).

Investors owning land at location  $r$  are price-takers for rent levels and interest

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<sup>5</sup> For simplicity’s sake, labor is exclusively used for the production of the composite goods. This assumption is of course a limitation of the model: we chose to focus our analysis of crowding-out effects on capital rather than on labor.

rate. They invest  $I_{fH}(r)$  given by:

$$I_{fH}(r) = \delta_H \cdot \arg \max_{K_H(r)} [R_H(r) \cdot F(K_H(r), \text{Land}(r)) - (\rho + \delta_H) \cdot K_H(r)] \quad (\text{A-16})$$

This behavior leads to the aggregate demand for housing investment  $\widetilde{I}_{fH}$ :

$$\widetilde{I}_{fH} = \int_0^{r_f} I_{fH}(r) dr \quad (\text{A-17})$$

As for productive capital, there is a lag between financial capital  $\widetilde{I}_{fH}$ , and physically invested capital  $I_H$ , a lag given by  $\tau_h$  that corresponds to the time required to achieve the construction of buildings:

$$\begin{aligned} \frac{dS_H}{dt}(r) &= -I_H(r) + I_{fH}(r) \\ I_H(r) &= \frac{1}{\tau_h} S_H(r) \\ \frac{dK_H(r)}{dt} &= -\delta_H K_H(r) + I_H(r) \end{aligned} \quad (\text{A-18})$$

## A.5 Specific functional forms

In this section, we define the specific functional forms used in the rest of the article in order to explore the properties of the model.

### A.5.1 Basic functions

Concerning the utility function, the transportation costs, the housing service production, and the composite goods production, we use functional forms that are considered as very classical in urban microeconomics<sup>6</sup>: the utility function,

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<sup>6</sup> These functional forms are widely used in urban economics for exploratory purpose, both because they allow advanced calculations, and because they reproduce

the housing service production function, and the general production function are Cobb-Douglas, while the transportation cost function is linear with respect to the distance from CBD.

$$U(z, q) = z^\alpha q^\beta \text{ where } \alpha, \beta > 0 \text{ and } \alpha + \beta = 1$$

$$T(r) = p \cdot r \text{ where } p > 0$$

$$F(S, K) = A \cdot S^a \cdot K^b \text{ where } a, b, A > 0 \text{ and } a + b = 1 \quad (\text{A-19})$$

$$Y(N, K) = G \cdot N^x \cdot K^y \text{ where } x, y, G > 0 \text{ and } x + y = 1$$

$$\text{Land}(r) = l \cdot r \text{ where } l > 0$$

In this relationship,  $p$  is the constant marginal transportation cost<sup>7</sup>. The variable  $G$  is the General Productivity Factor, while  $A$  is the productivity associated to the production of housing service<sup>8</sup>.

### A.5.2 Dynamic evolutions

Having defined the functional forms describing preferences, production, and transportation costs, we now turn to the dynamic relationships that need to be specified.

*Housing consumption per capita:* we specify here the expression of  $\frac{\partial \tilde{u}}{\partial q}$ , used in Eq. (A-3). With the functional forms considered in Eq. (A-19), we have:

$$\frac{\delta \tilde{u}}{\tilde{u}} = \left( \frac{\beta}{q} - \frac{\alpha}{z} R_H \right) \delta q. \text{ Furthermore, we choose the simplest specification for } g$$

realistic features of agents' preferences and of goods production.

<sup>7</sup> No congestion is taken into account, even though it is an important feature of transportation systems.

<sup>8</sup> Concerning the static equilibrium and its calculation, see Appendix B

, that is  $g(x) = x$ . Hence:

$$\frac{\partial q(r)}{\partial t} = \begin{cases} \frac{1}{\tau_q} \left( \frac{\beta}{q(r)} - \frac{\alpha}{z(r)} R_H(r) \right) \cdot q(r)^2 & \text{if } \psi(r) > 0 \\ \frac{1}{\tau_q} \left( \frac{H(r)}{n(r)} - q(r) \right) & \text{if } \psi(r) \leq 0 \end{cases} \quad (\text{A-20})$$

*Moves:* we specify the weight functions we use in Eqs. (A-5) and (A-6):

$$w^-(u, \bar{u}) = w^+(u, \bar{u}) = \frac{2}{\pi} \arctan \left[ w \cdot \left| \frac{u - \bar{u}}{\bar{u}} \right| \right] \quad (\text{A-21})$$

The function  $\arctan(x)$  is a growing function of  $x$ , and converges towards  $\pi/2$  (resp.  $-\pi/2$ ) when  $x$  goes towards  $+\infty$  (resp.  $-\infty$ ), which ensures that  $w^+$  and  $w^-$  have the desired properties. The coefficient  $w$  in Eq. (A-21) modulates the strength of the force driving the moves.

*Rent evolutions:* in Eq. (A-13), we choose the simplest form for the function  $\phi$ , namely  $\phi(x) = x$ .

## B Appendix: the static equilibrium of NEDUM

### B.1 Static equilibrium: existence and unicity

In this section, we show both the existence and unicity of a static equilibrium defined by Eqs. (1) to (A-15).

*Unicity:* let us assume that such an equilibrium exists, and mark all the variables values at this equilibrium with a superscript <sup>“\*”</sup>.

Concerning productive capital, Eq. (A-15) implies:

$$I_K^* = \widetilde{I}_{fK}^* = \delta_K \cdot K^* \quad (\text{B-1})$$

Meanwhile, from Eq. (A-14) we derive that:

$$\delta_K + \rho^* = \frac{\partial Y}{\partial K}(N, K^*) \quad (\text{B-2})$$

Concerning housing capital, from Eq. (A-18), we get at any location  $r$ :

$$I_{fH}^*(r) = I_H^*(r) = \delta_H \cdot K_H^*(r) \quad (\text{B-3})$$

From Eq. (A-16), we also have:

$$\delta_H + \rho^* = R_H^*(r) \frac{\partial F}{\partial K}(K_H^*(r), \text{Land}(r)) \quad (\text{B-4})$$

The taking into account of the equilibrium utility level  $u^*$ , added to Eqs. (A-17) and (B-4), leads to a unique relationship:

$$\widetilde{I}_H = j(\rho, K) \quad (\text{B-5})$$

where  $j$  is decreasing in  $\rho$  and increasing in  $K$ .

We now consider the system of four variables  $\rho^*$ ,  $I_K^*$ ,  $\widetilde{I}_H^*$  and  $K^*$ , and four equations (A-2), (B-1), (B-2), and (B-5). First, Eqs. (A-2) and (B-2) imply that  $\widetilde{I}_H^*$  is increasing with respect to  $K^*$ . This relationship, added to Eq. (B-1), means that the derivative of LHS of Eq. (B-5) with respect to  $K^*$  is greater than  $\delta_K$ . Meanwhile, we assume that the production function  $Y(N, K)$  has decreasing marginal returns on capital, that goes towards 0 as  $K^*$  increases. As a conclusion, there is at most one possible equilibrium value for  $K^*$ .

*Existence:* if one assumes that, if  $K^* = 0$ , the derivative of LHS of Eq. (B-5) with respect to  $K^*$  is inferior to the derivative of *RHS* of Eq. (B-5), then we also have the existence of the solution.

Since there is one and only one level of  $K^*$  at equilibrium, then there is also one and only one level of the corresponding consumption level  $c$ . At its stationary equilibrium, moreover, our model reproduces the features of classic urban microeconomics models (see for instance Fujita, 1989).

## *B.2 Analytical calculations for the static equilibrium*

In this section, we characterize the static equilibrium with the functional forms defined by Eq. (A-19). We denote the equilibrium level of the variables with a superscript  $*$ . For instance Eqs. (A-16) and (B-3) give us for the equilibrium housing capital density at location  $r$ :

$$K_H^*(r) = \frac{b N p^2 (\gamma + 2)}{a c^{*\gamma+2}} \frac{1}{\rho^* + \delta_H} (c^* - p \cdot r)^{\gamma+1} \cdot r \quad (\text{B-6})$$

This relationship, added to Eq. (A-17), implies that Eq. (B-5) translates into:

$$\tilde{I}_H = \frac{\delta_H}{\delta_H + \rho^*} \frac{b N c^*}{a \gamma + 3} \quad (\text{B-7})$$

Meanwhile, Eq. (B-2), which links the interest rate and the productive capital stock, becomes:

$$\delta_K + \rho^* = yG\left(\frac{N}{K^*}\right)^x \quad (\text{B-8})$$

Using this relationship and Eq. (B-1), we derive:

$$I_K^* = y \frac{\delta_K}{\rho^* + \delta_K} \quad (\text{B-9})$$

We can now consider Eqs. (A-2), (B-7) and (B-9), which imply that the equilibrium rate of interest is the unique solution of:

$$s = \frac{\delta_H}{\rho^* + \delta_H} \frac{b}{a} \frac{1-s}{\gamma + 2} + y \frac{\delta_K}{\rho^* + \delta_K} \quad (\text{B-10})$$

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Figure 1:

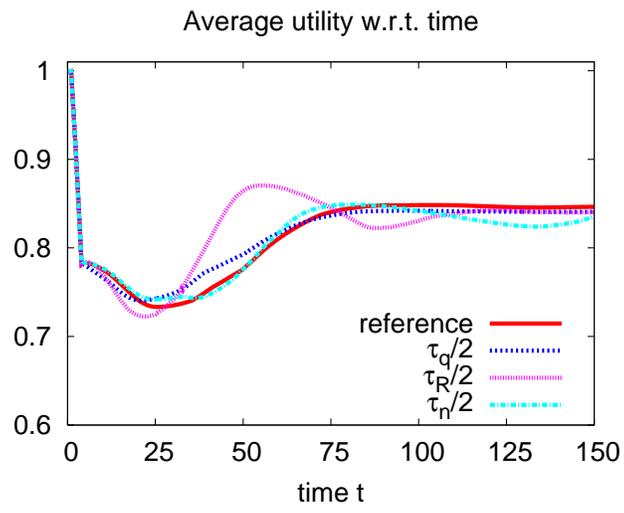


Figure 2:

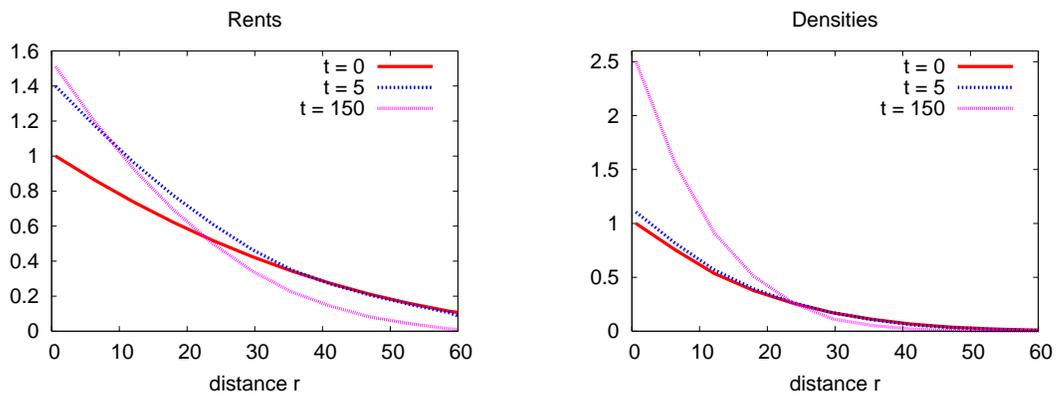


Figure 3:

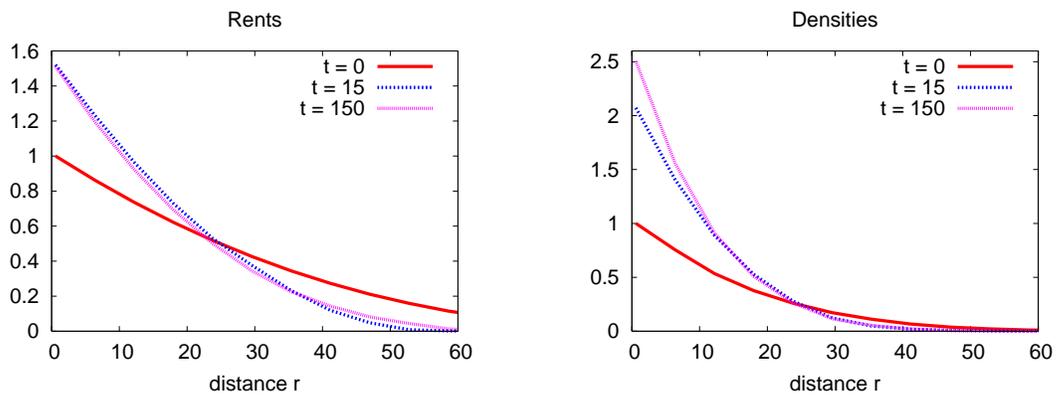


Figure 4:

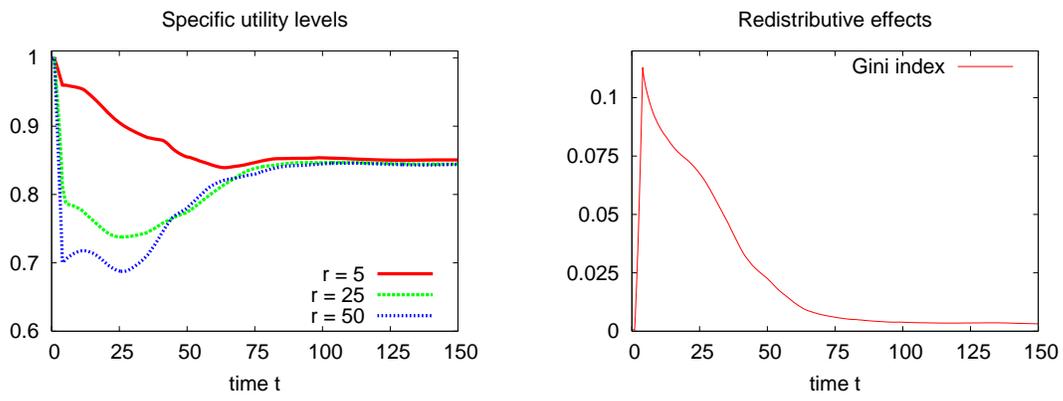


Figure 5:

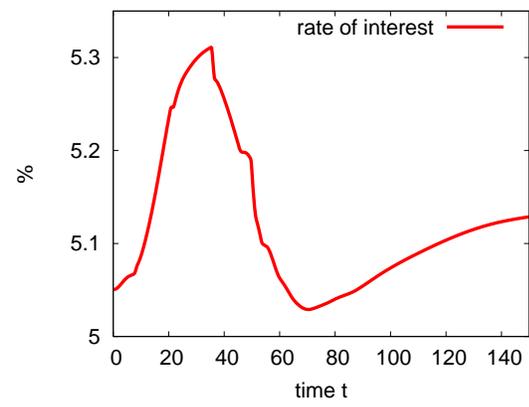
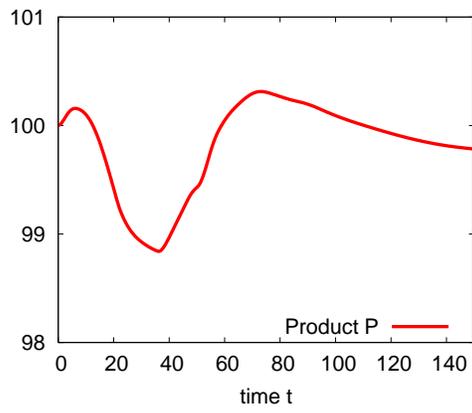


Figure 6:

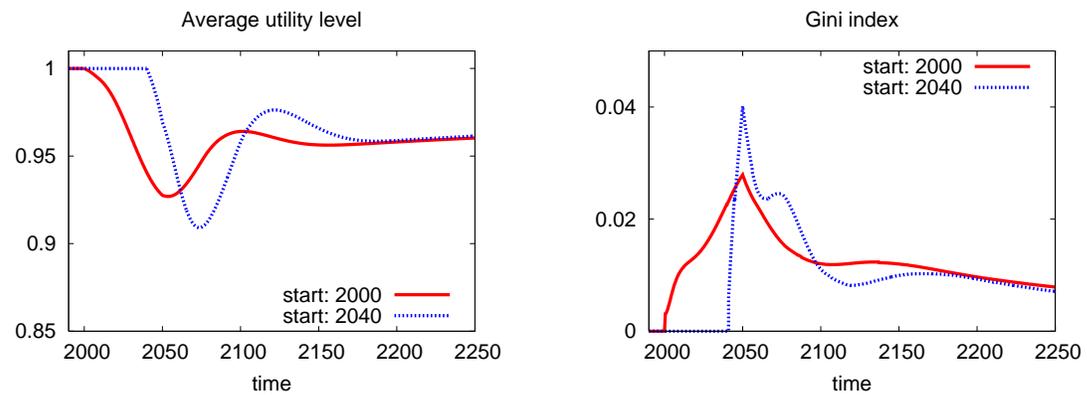
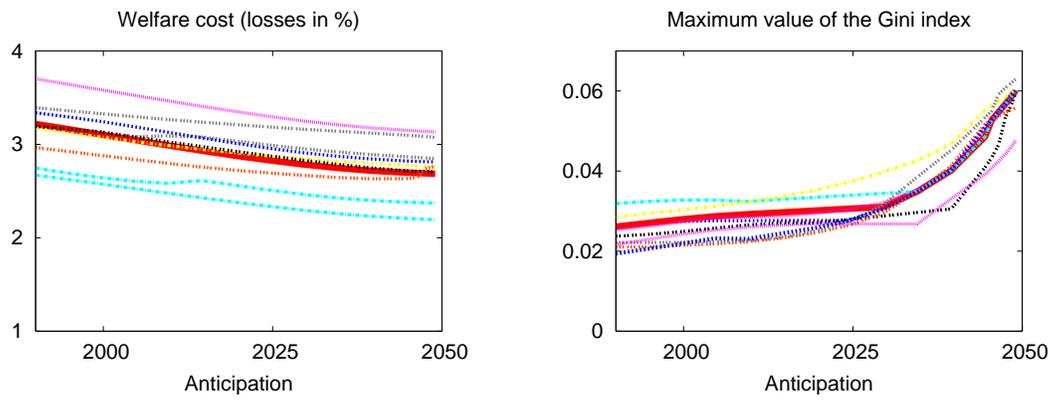


Figure 7:



caption to figure 1: The evolution of  $\bar{u}$  with respect to time. Each curve corresponds to different values of the timescales  $\tau_n$ ,  $\tau_R$  and  $\tau_q$  (index  $\bar{u} = 1$  at time  $t = 0$ ).

caption to figure 2: Response to a shock in transportation costs. Left: rent curves  $R_H(r)$  before the shock, 5 years after, and 150 years after the shock (index  $R_H(0) = 1$  before the shock). Right: density curves  $n(r)$  before the shock, 5 years after, and 150 years after the shock (index  $n(0) = 1$  before the shock).

caption to figure 2: Idem than Fig. 2 but 15 years after the shock instead of 5 years.

caption to figure 4: Left: evolution of utility levels with respect to the time, at locations 5 km, 25 km, and 50 km in the city (index  $u = 1$  before the shock) for 150 years after a shock in transportation costs. Right: evolution of the Gini index characterizing inequalities of utility levels in the city.

caption to figure 5: Evolution of the economic output  $Y$  and the interest rate  $\rho$ , with respect to time (index  $Y = 100$  before the shock).

caption to figure 6: Left: average utility level with respect to time, for policies starting in year 2000 and year 2040 (index  $u = 1$  in year 1990). Right: Gini index with respect to time for policies starting in year 2000 and year 2040.

caption to figure 7: On these two panels, each curve is related to a sensitivity test where the values of one or several parameters are changed. We show curves corresponding to a wide selection of values of  $\tau_R$ ,  $\tau_q$ ,  $\tau_n$ ,  $a$ , and  $\delta_H$ . The bold red curve corresponds to the best-guess values of these parameters. Left: the welfare costs caused by the implementation of the tax, with respect to the

year of implementation. Right: the maximum value reached by the Gini index during the transition, depending on the year of implementation.

CBD	Central Business District, where firms are located	$r$	distance from CBD
$q$	housing service per household	$h(r)$	housing service density
$z$	composite goods	$k_H$	housing capital density
$\text{Land}(r)$	land surface at distance $r$	$K_H$	housing capital stock
$n(r)$	density of households at distance $r$	$T(r)$	transportation costs
$c$	consumption per capita	$r_f$	city radius
$R_H(r)$	unit housing service rent	$R_a$	agricultural land rent
$H(r)$	housing service at distance $r$	$N$	number of households
$U(z, q)$	utility function of a household	$u$	utility level
$x^*(r)$	optimal capital to land ratio	$\rho$	interest rate
$F(K, L)$	housing service production function		

Table 1

Nomenclature for the traditional Von Thuenen model.

$Y(K, N)$	composite goods production function	$\widetilde{I}_{fH}$	financial inv. in housing capital
$\delta_K$	discount factor of the productive capital	$\widetilde{I}_{fK}$	financial inv. in prod. capital
$\delta_H$	discount factor of the housing capital	$S_H$	stock of pending investments in housing capital
$\rho$	capital price		
$\theta$	tax level	$S_K$	stock of pending investments in productive capital
$\pi$	tax product		
$LI$	Land Income	$I_H$	physical inv. in housing capital
$\tau_R$	timescale of rent evolution	$I_K$	physical inv. in prod. capital
$\tau_q$	timescale of the evolution of housing service per capita	$\tau_h$	timescale of the evolution of pending housing investments
$\tau_k$	timescale of the evolution of pending productive investments	$\tau_n$	timescale of moves

Table 2

Nomenclature: new variables introduced in NEDUM.