# Dynamic Meteorology

(WAPE: General Circulation of the Atmosphere and Variability)

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## 5) Synoptic scale variability and baroclinic instability

**a) Observations (cf. cours4)**

**Levels 0 statistics (cf lecture 4)**

**b) Quasi geostrophy and related diagnostics**

**Quasi-geostrophic equations on the beta-plan, vorticity equation, Q vectors and potential vorticity.**

**c) Baroclinic instability in terms of 2 interacting Eady "edge" waves**

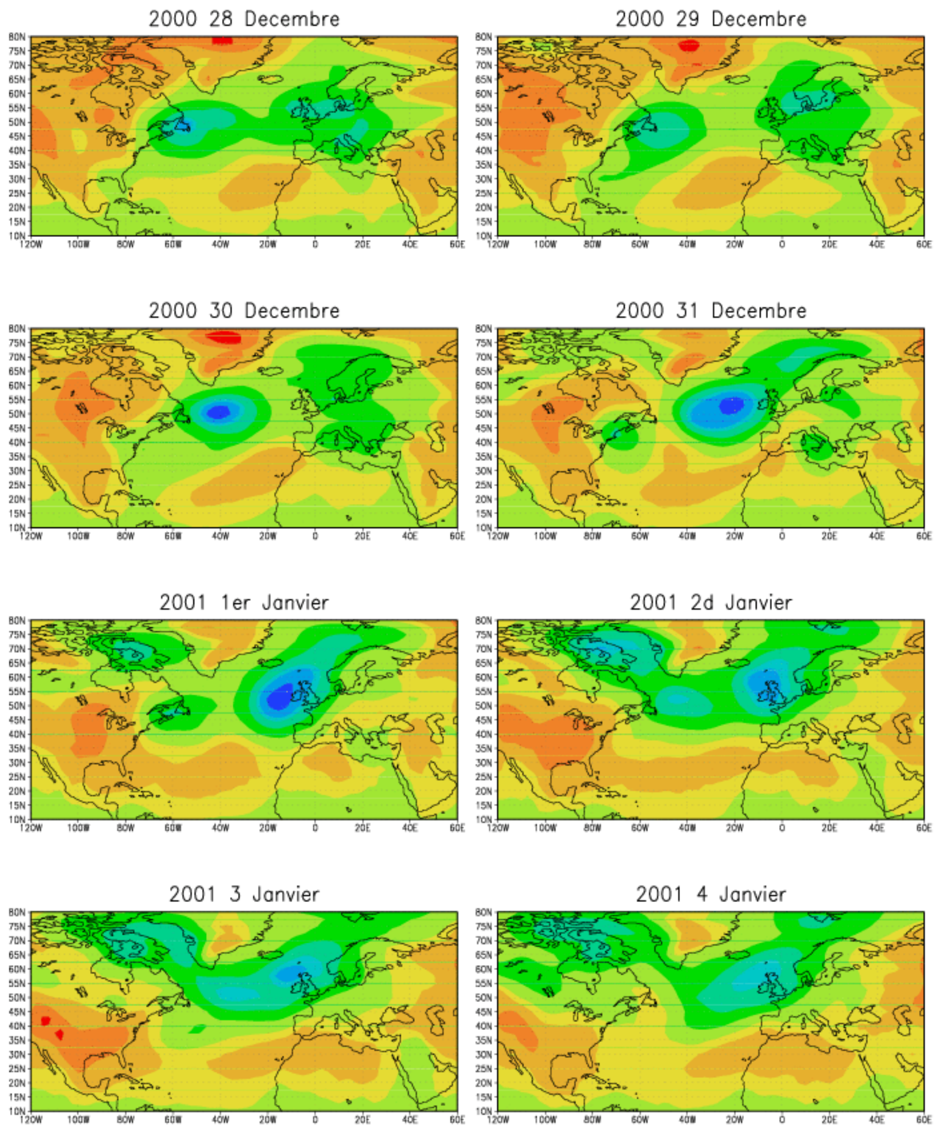
**Toy model 2**

**d) Interpretation and more realistic base state**

# a) Observations (troposphere, cf. cours 4)

Usual trajectory of a weather system developping in winter across the Atlantic

Sea-level pressure, NCEP data




Low level pressure systems usually form over the North-East Atlantic near the newfound land Island (Terre-neuve)

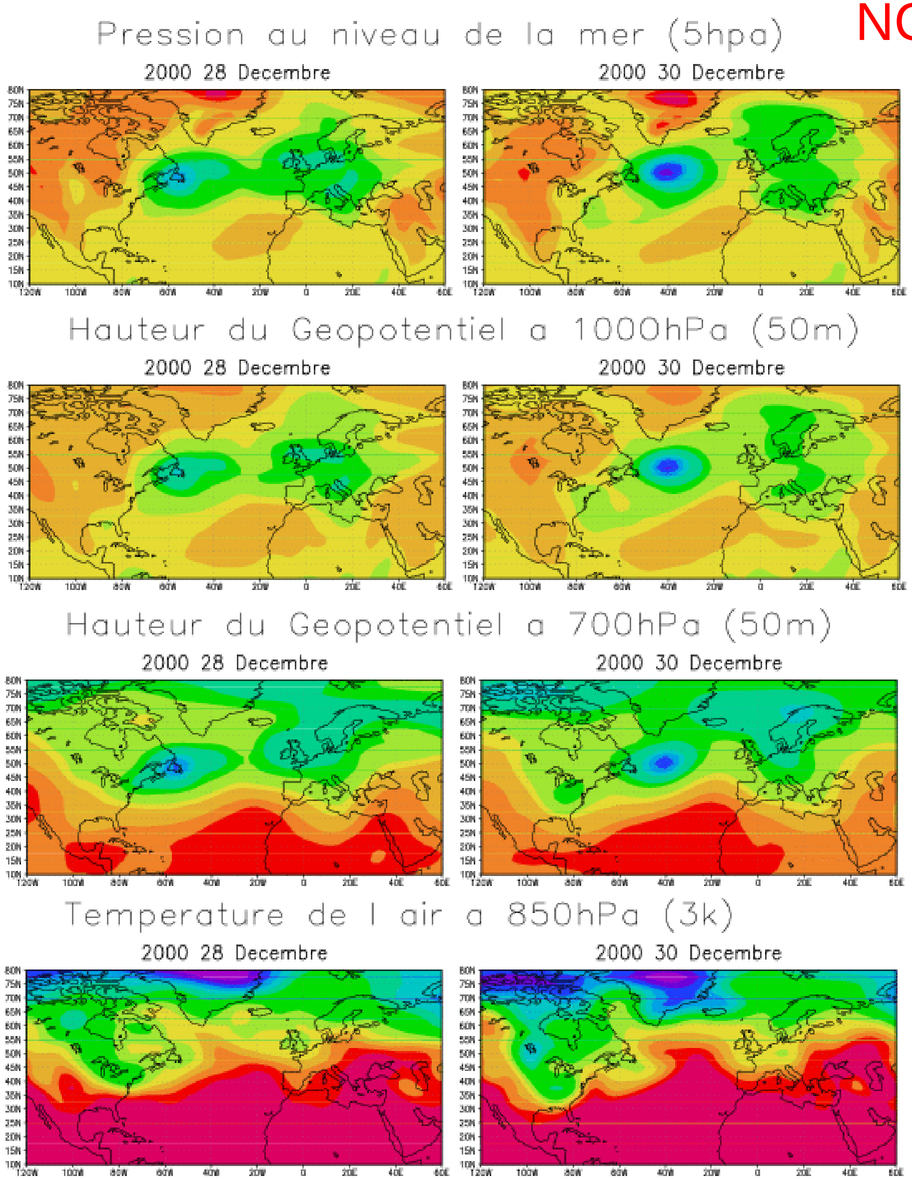
They depens as they cross the Atlantic within 3-5 days

End-up or reach a mature stage over northern west europe

# a) Observations (troposphere, cf. cours 4)

Low pressure systems impact on other conventionnal fields:
850hPa geopotential and Temperature

NCEP data




The sea-level pressure is often an interpolated quantity, we prefer to characterize the impact of the synoptic scales on meteorology fields at upper levels, for instance near above the boundary layer

For instance the geopotential height at 100hPa, and 700hPa, contains the same infos present in the SLP maps.

The T at 825hPa is warm ahaead the low pressure system, the warm and humid air is advected northward to bring moisture

This is the base mechanism explaining the development of the synoptic scale weather systems, and which is at the base of the Eady waves dynamics

# a) Observations (troposphere, cf. cours 4)

Statistics of the 700hPa, winter monthes (DJF) 1958-2010,
NCEP data

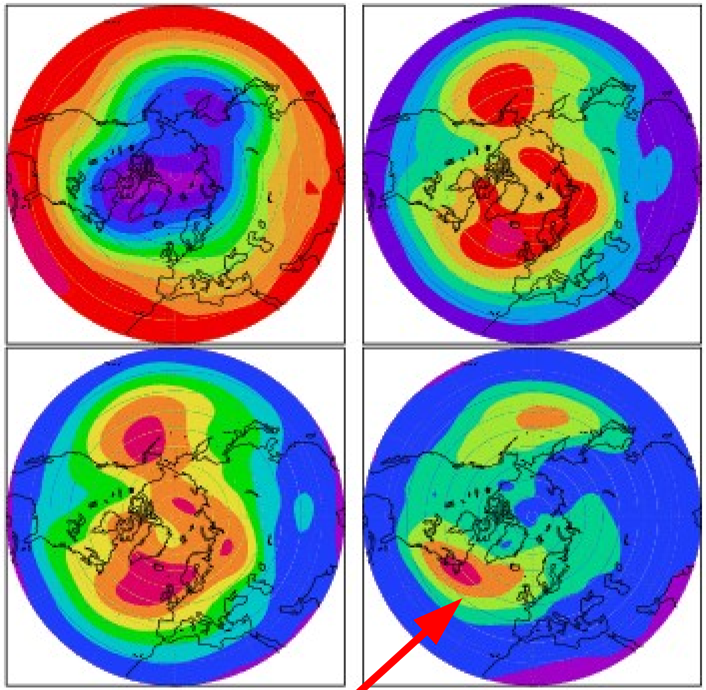
**Mean:**

Note the intensification of the westerly jet at the East of the continents (storm tracks)

And its enlargement to the East of the ocean, related to the different routes the low pressure systems can follow

**Standard deviation:**

The maxima in variability are over the North-eastern ocean.
It is more due to the changes in trajectory of the mature weather systems, than two their initial birth and development



**Low-frequency standard deviation**

It represent the largest fraction of the total standard deviation

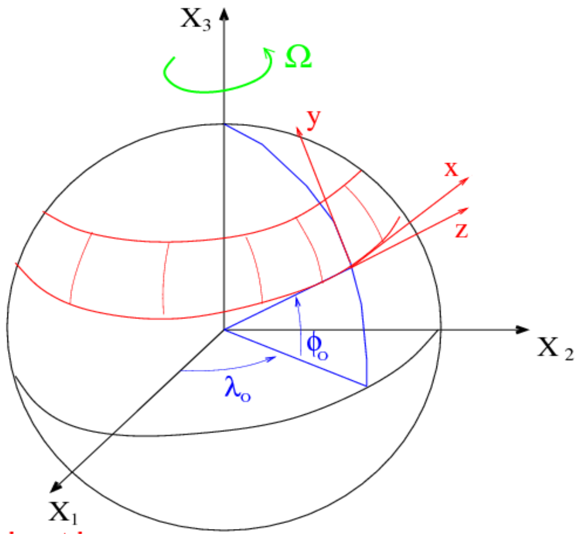
**High-frequency standard deviation:**

It directly translates where the low pressure systems form and travel during the few days after their birth

It is controlled by the developing synoptic scale disturbances

## b) Quasi-geostrophy and related diagnostics

### β plane and Boussinesq approximations in the mid-latitudes




Material derivative:

$$\frac{D}{Dt} \approx \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

With: $x = a\cos\phi_0\,(\lambda-\lambda_0)$, $y = a\,(\phi-\phi_0)$.

Coriolis term:

$$2\Omega\sin\phi \approx 2\Omega\sin\phi_0 + 2\Omega\cos\phi_0\,(\phi-\phi_0)$$
$$\approx f_0 + \beta y = f$$
$$\beta = \frac{2\Omega\cos\phi_0}{a}$$

Mass conservation: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

Sphericity terms $\tan\phi\frac{uv}{a}$ and $\tan\phi\frac{u^2}{a}$ are also neglected

$$\frac{Du}{Dt}-fv=-\frac{\partial\Phi}{\partial x}$$
$$\frac{Dv}{Dt}+fu=-\frac{\partial\Phi}{\partial y}$$
$$\frac{\partial\Phi}{\partial z}=b$$

$$b=g\frac{\theta}{\theta_s},\quad \Phi=\frac{p}{\rho_s}$$

Hydrostatic

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$
$$\frac{Db}{Dt}=0$$

Stratification at rest, $b_0(z)$

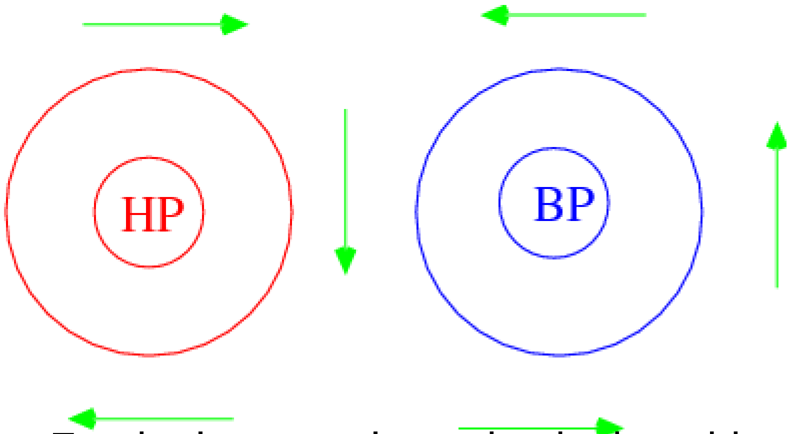
$$b=b_0(z)+b_e(t,x,y,z);\quad \Phi=\Phi_0(z)+\Phi_e(x,y,z)$$

Resulting equations :

$$\frac{Du}{Dt}-fv=-\frac{\partial\Phi_e}{\partial x}\qquad \frac{Dv}{Dt}+fv=-\frac{\partial\Phi_e}{\partial y}\qquad \frac{\partial\Phi_e}{\partial z}=b_e$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$
$$\frac{Db_e}{Dt}+N^2w=0\qquad N^2=\frac{g}{\theta_s}\frac{d\theta_0}{dz}$$

# b) Quasi-geostrophy and related diagnostics

## Quasi-geostrophic approximation




For the large-scale motion in the mid latitudes, holds the

**« geostrophic balance »**

$$u \approx u_g = -\frac{1}{f_0}\frac{\partial \Phi_e}{\partial y}, \quad v \approx v_g = \frac{1}{f_0}\frac{\partial \Phi_e}{\partial x}$$

### Quasi-Geostrophic approximation:

$$D_g u_g - \beta y v_g - f_0 v + \partial_x \Phi_e = 0$$
$$D_g v_g + \beta y u_g + f_0 u + \partial_y \Phi_e = 0$$
$$\partial_z \Phi_e = b_e$$
$$D_g b_e + N^2 w = 0$$
$$\partial_x u + \partial_y v + \partial_z w = 0$$
$$\text{where} \quad D_g = \partial_t + u_g \partial_x + v_g \partial_y$$

**Remark:**
The geostrophic relation used, uses $f_0$ (not f)

### Relative vorticity equation :

$$D_g\left(\frac{\partial v_g}{\partial x}-\frac{\partial u_g}{\partial y}\right)+\beta v_g+f_0\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

**Advec. Planetary vorticity**

**Vortex stretching**

### Vertical velocity equation (z>0)

$$D_g \frac{\partial}{\partial z}\frac{b_e}{N^2}+\frac{\partial w}{\partial z}=0$$

### Surface condition :

$$w(z=0)=0$$

### Surface Temperature advection (z=0):

$$D_g b = 0$$

# b) Quasi-geostrophy and related diagnostics

**Quasi-geostrophic potential vorticity conservation (prognostic equation):**

$$D_g\left(\underbrace{\frac{\partial v_g}{\partial x}-\frac{\partial u_g}{\partial y}+f_0+\beta y+f_0\frac{\partial}{\partial z}\frac{b_e}{N^2}}_{q_g}\right)=0$$

Once $q_g$ is known the entire geostrophic field is known through inversion of the elliptic equation :

$$f_0 q_g=\frac{\partial^2\Phi_e}{\partial x^2}+\frac{\partial^2\Phi_e}{\partial y^2}+f_0\left(f_0+\beta y\right)+\frac{\partial}{\partial z}\frac{f_0^2}{N^2}\frac{\partial\Phi_e}{\partial z}$$

**Surface Temperature advection (z=0):**

$$D_g b=0$$

**Omega equation (more a diagnostic tool in the following):**

A « diagnostic » equation for *w,* usefull to interpret real situations (here with **$\beta$=0**)

**Q-vector :**

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{f_0^2}{N^2}\frac{\partial^2 w}{\partial z^2}=\vec{\nabla}_H\cdot\vec{S}_q$$

where $\vec{S}_Q=\frac{2}{N^2}\left(Q_{1,}Q_2\right)$

$$Q_1=-\frac{\partial u_g}{\partial x}\frac{\partial b_e}{\partial x}-\frac{\partial v_g}{\partial x}\frac{\partial b_e}{\partial y}$$

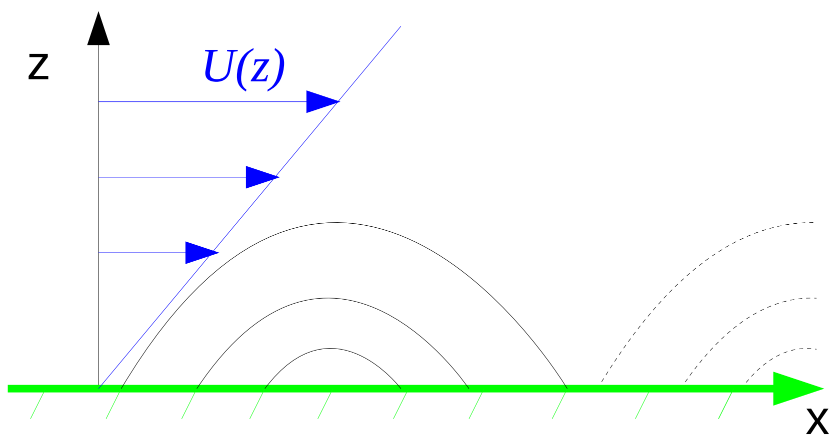
$$Q_2=-\frac{\partial u_g}{\partial y}\frac{\partial b_e}{\partial x}-\frac{\partial v_g}{\partial y}\frac{\partial b_e}{\partial y}$$

$Q_1$ plays a central rôle in « frontogenesis »

# c) Baroclinic instability in terms of 2 interacting Eady "edge" waves

## Toy model 2

The Eady "edge" wave (no beta effect, β=0):

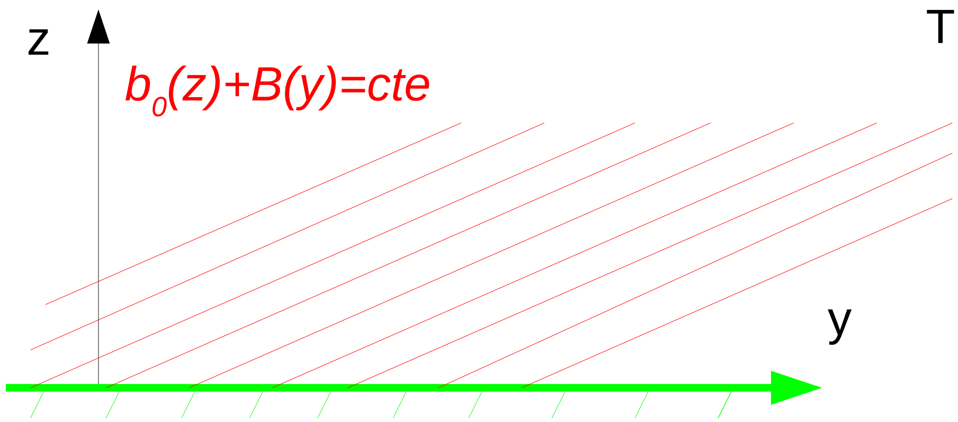



The background wind is nul at the surface (z=0) it equilibrates a uniform negative north-south negative T gradient :

$$U=\Lambda z\,,B=-f_0\Lambda y$$

The background PV is the planetary PV $f_0$ .
It is constant, so the disturbance PV is 0 :

$$0=\frac{\partial^2\Phi'}{\partial x^2}+\frac{\partial^2\Phi'}{\partial y^2}\frac{\partial}{\partial z}\frac{f_0^2}{N^2}\frac{\partial\Phi'}{\partial z}$$




The dynamics is controlled by the meridional advection of surface Temperature :

$$\frac{\partial}{\partial t}\frac{\partial\Phi'}{\partial z}-\Lambda\frac{\partial\Phi'}{\partial x}=0$$

For a monochromatic disturbance,

$$\Phi'=\hat{\Phi}(z)e^{i(kx-\omega t)}$$

0-PV condition gives : $\hat{\Phi}=\Phi_S e^{-k\frac{N}{f_0}z}$

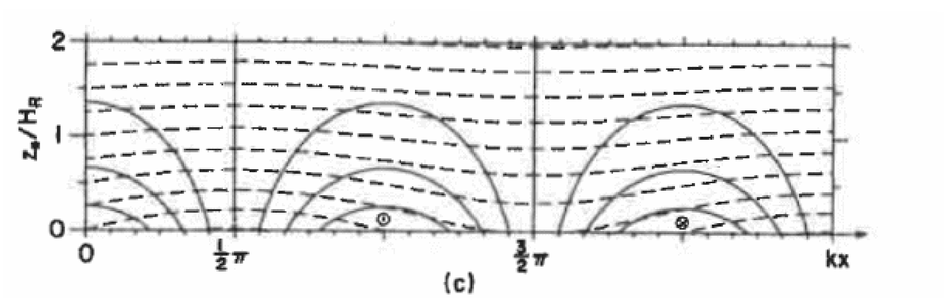
Dispersion relation : $\omega_S=\frac{\Lambda f_0}{N}$

# c) Baroclinic instability in terms of 2 interacting Eady "edge" waves

## Toy model 2

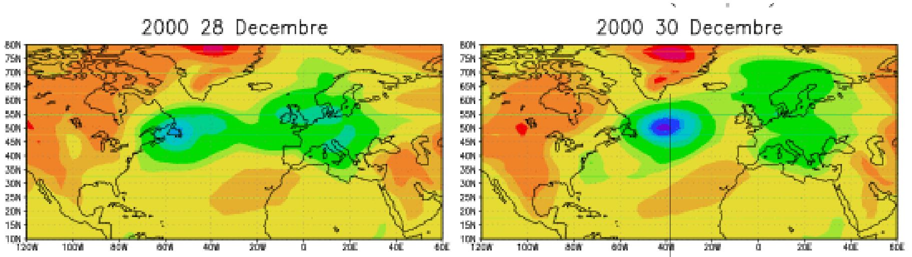
## The Eady "edge" wave (no beta effect, $\beta=0$):

The eastward displacement and the fact that hot air is advected in front of a low pressure system are realistic.

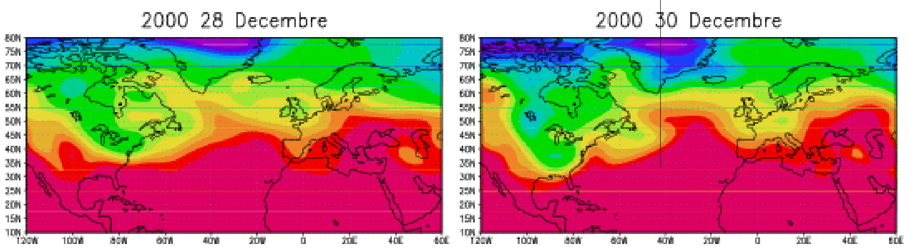



Isoline of potential temperature $b_0+b$ (dashed) and meridional wind $v_g$ (solid)

Sea level pressure




850hPa



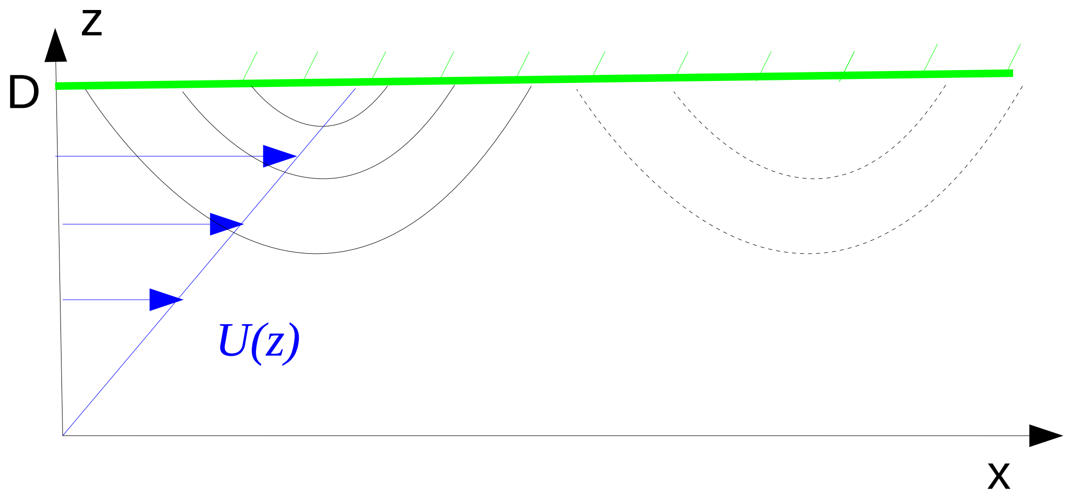

Problem : The Eady « edge wave » does not grow with time

The relation between surface pressure and Temperature is not realistic.

# c) Baroclinic instability in terms of 2 interacting Eady "edge" waves

## Toy model 2

In the Toy model 2, one also introduces an Eady "edge" wave at the tropopause!




Its dynamics is controlled by the meridional advection of tropopause Temperature :

$$\left(\frac{\partial}{\partial t}+\Lambda D\frac{\partial}{\partial x}\right)\frac{\partial \Phi'}{\partial z}-\Lambda\frac{\partial \Phi'}{\partial x}=0$$

For a monochromatic disturbance,

$$\Phi'=\hat{\Phi}(z)e^{i(kx-\omega t)}$$

0-PV condition gives : $\hat{\Phi}=\Phi_T e^{+k\frac{N}{f_0}(z-D)}$

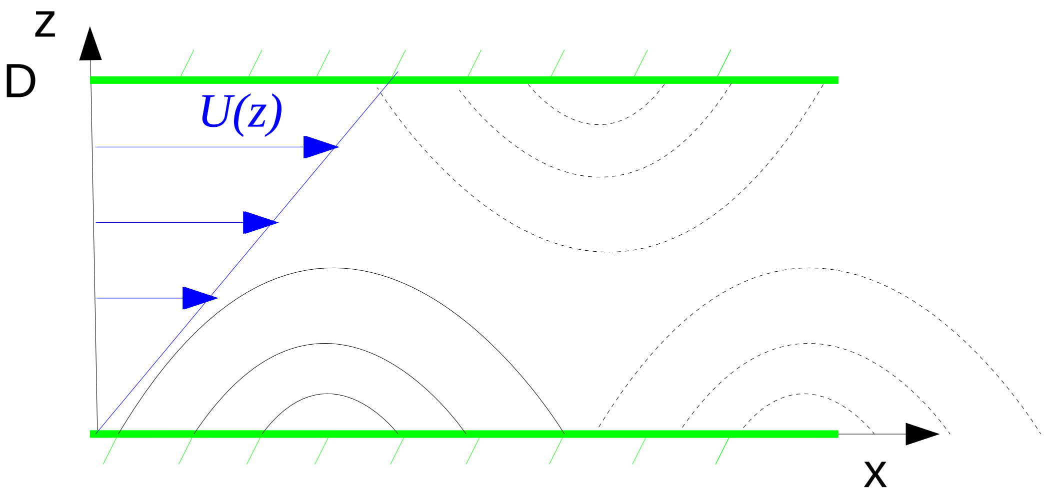
Dispersion relation :

$$\omega_T=k\Lambda D-\frac{\Lambda f_0}{N}$$

## c) Baroclinic instability in terms of 2 interacting Eady “edge” waves

## Toy model 2

In the Toy model 2, the upper and lower Eady waves are “phase locked”:




Phase locked :

they have same horizontal wavenumber, ***k***

and absolute frequency, **ω**

$$\omega_T = k\Lambda D - \frac{\Lambda f_0}{N} = \omega_S = \frac{\Lambda f_0}{N}$$

$$\longrightarrow \boxed{k = 2\frac{f_0}{ND},\quad \omega = \frac{\Lambda f_0}{N}}$$

Quite near the values of the fastest growing mode in the full Eady problem

To take into account the interaction between the upper and lower level waves, we let their complex amplitudes $\Phi_s$ and $\Phi_T$ vary in time :

$$\Phi' = \left(\Phi_s(t)\, e^{-2\frac{z}{D}} + \Phi_T(t)\, e^{+2\left(\frac{z}{D}-1\right)}\right) e^{i\left(\frac{2f_0}{ND}x - \frac{\Lambda f_0}{N}t\right)}$$

$$\left(\frac{\partial}{\partial t} + \Lambda D\frac{\partial}{\partial x}\right)\frac{\partial \Phi'}{\partial z} - \Lambda\frac{\partial \Phi'}{\partial x} = 0$$

$$\frac{\partial}{\partial t}\frac{\partial \Phi'}{\partial z} - \Lambda\frac{\partial \Phi'}{\partial x} = 0$$

$$\longrightarrow \quad \frac{d}{dt}\Phi_T = e^{-2}\left(\frac{d}{dt}\Phi_S + 2i\omega\Phi_S\right), \qquad \frac{d}{dt}\Phi_S = e^{-2}\left(\frac{d}{dt}\Phi_T - 2i\omega\Phi_T\right)$$

$$\longrightarrow \quad \hat{\Phi}_S = A\, e^{\sigma t}, \quad \boxed{\sigma = \frac{2e^{-2}}{\sqrt{1-e^{-4}}}\,\omega \approx 0.27\frac{\Lambda f}{N}}$$

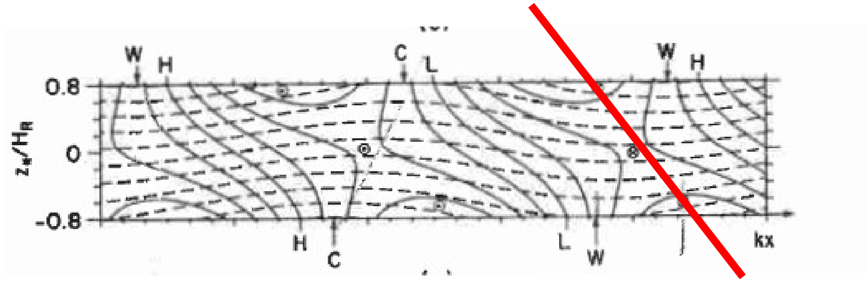
(growth rate $\sigma$, slightly smaller than that of the most unstable Eady mode)

# c) Baroclinic instability in terms of 2 interacting Eady "edge" waves

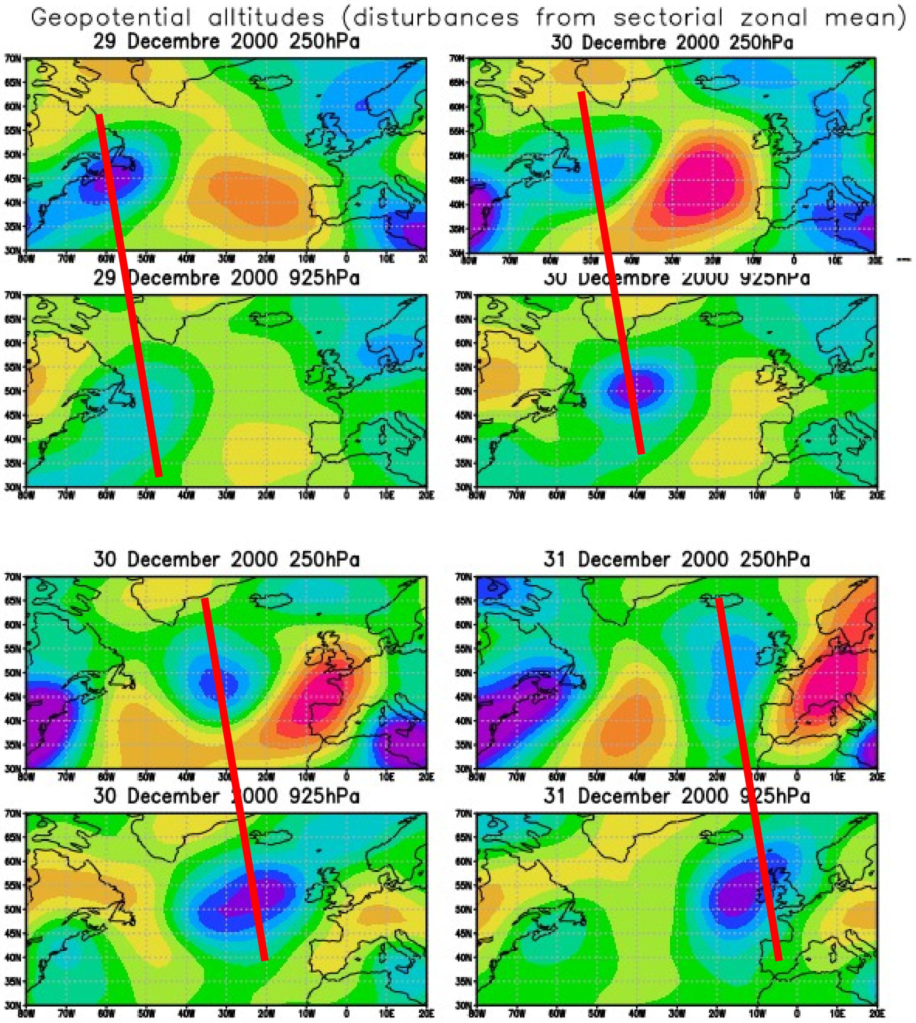
## Toy model 2, realism?

<u>Eady unstable wave. T</u>he significance of the upper level disturbances located around the Tropopause, e.g. near where the midlatitude jet is at a maximum.

Most unstable Eady mode (Gill, p. 559)

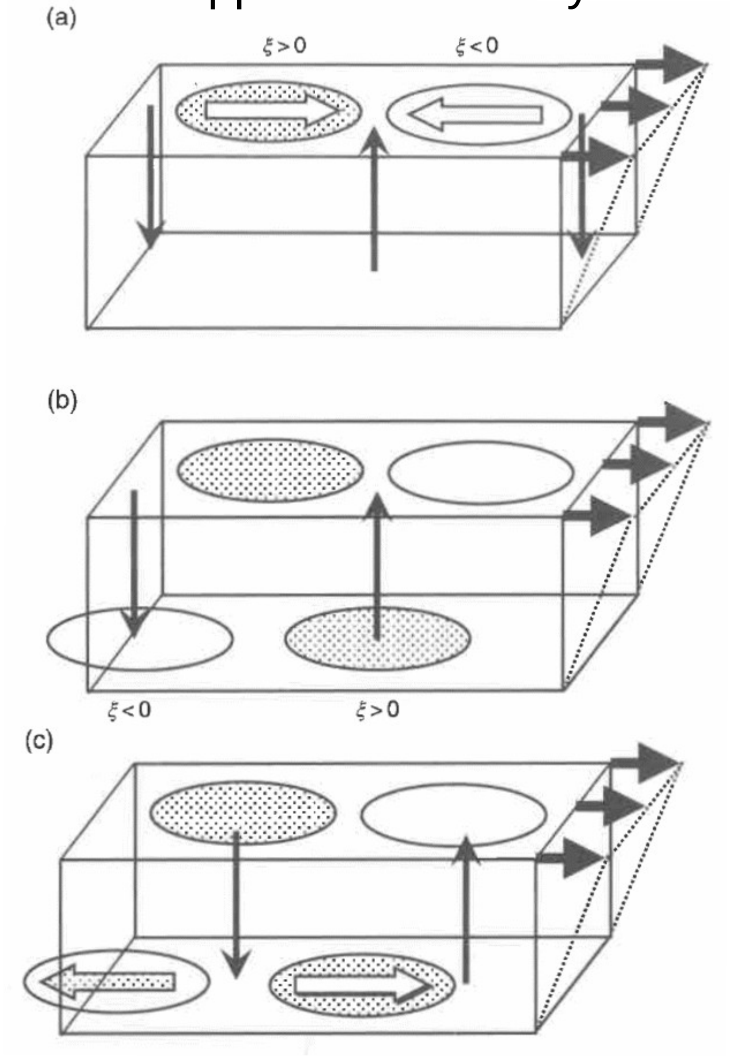



Potential temperature $b_0$+b (dashed) and meridional wind $v_g$ (solid)



# d) Interpretation and more realistic base state

How an upper level vorticity anomaly triggers a surface disturbance




Hoskins and James (2014)

The ageostrophic circulation accompanying an upper level wave, for instance evaluated via the Omega equation (S.5.8) :

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{f_0^2}{N^2}\frac{\partial^2 w}{\partial z^2}=\vec{\nabla}_H\cdot\vec{S}_q$$

Produces vortex stretching at the surrface (see the relative vorticity equation S5.7) :

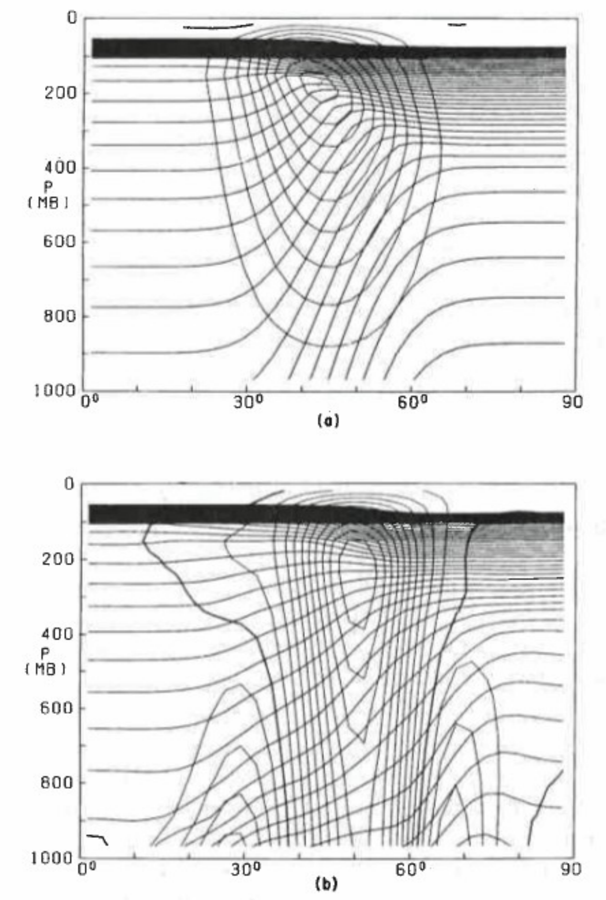
$$D_g\left(\frac{\partial v_g}{\partial x}-\frac{\partial u_g}{\partial y}\right)=f_0\frac{\partial w}{\partial z}$$

# d) Interpretation and more realistic base state

Zonal mean zonal wind and Temperature

Initial

Final



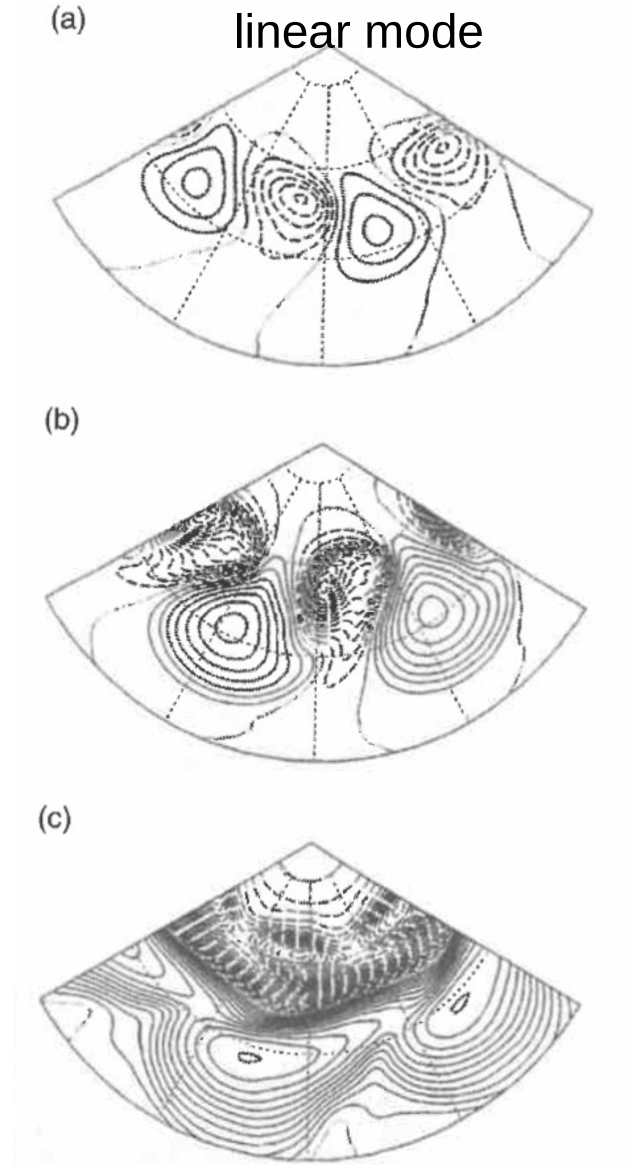

Nonlinear evolution, primitive equations on the sphere (Simmons Hoskins, 1980)

Surface pressure nonlinear evolution associated with the most unstable linear mode

4 days

6 days

9 days



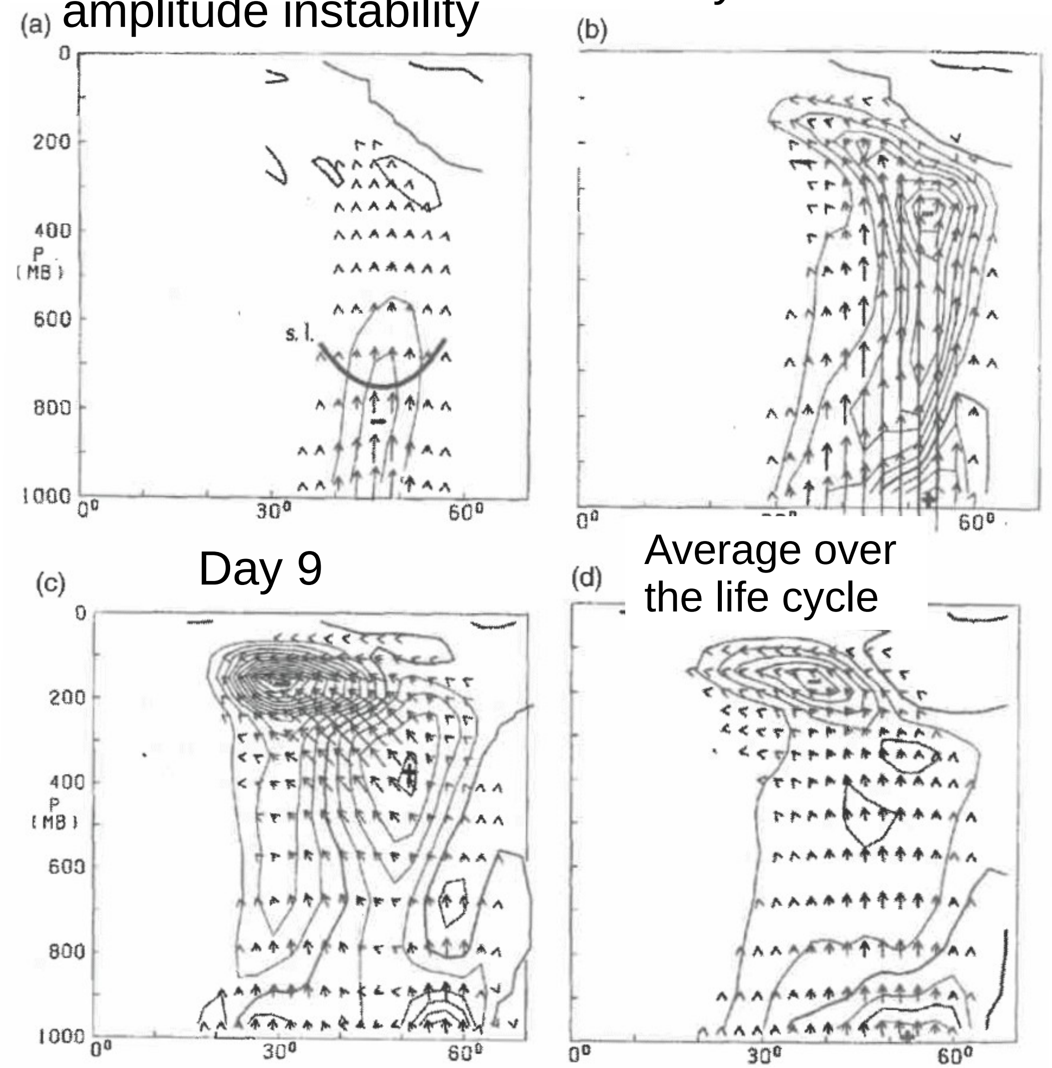
# d) Interpretation and more realistic base state

Day 0 ?
Initial small
amplitude instability

Day 6

Day 9

Average over
the life cycle



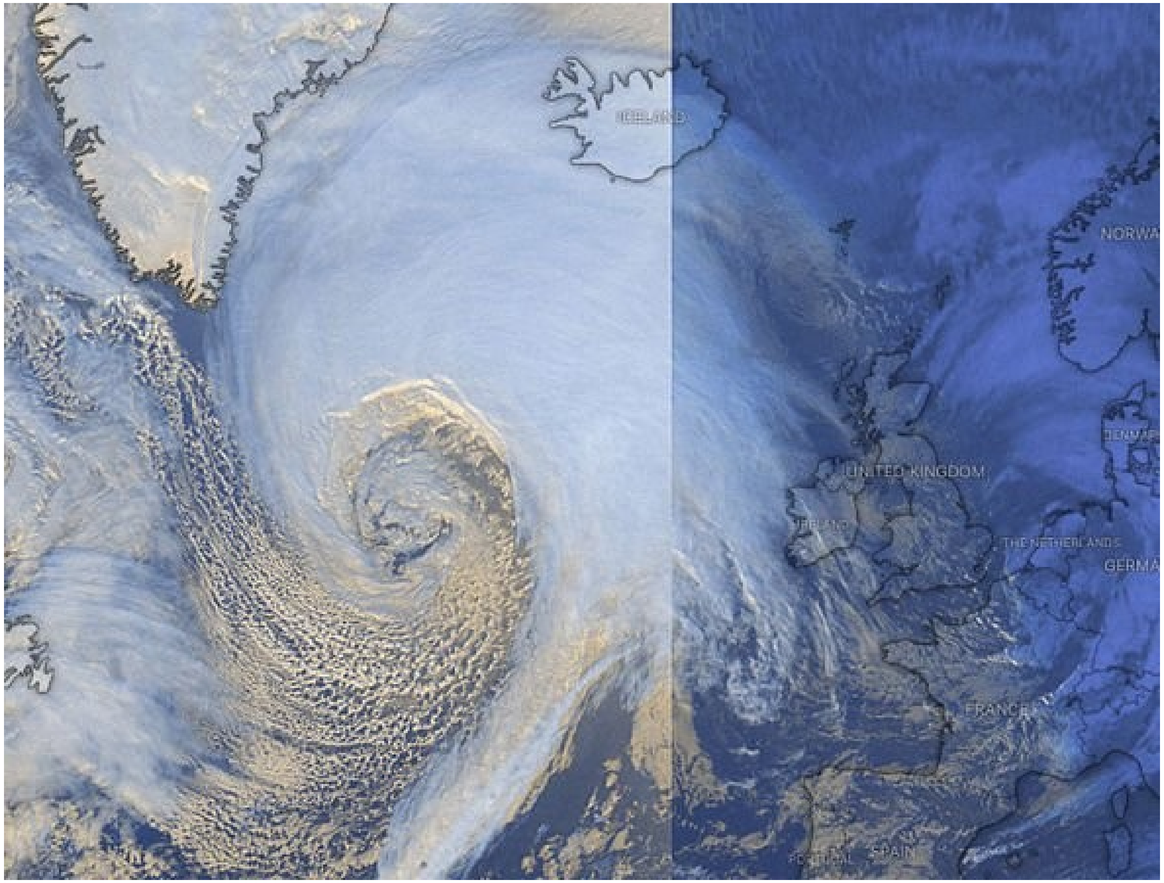

EP-flux vector during the evolution
Here the QG aproximation
But is almost the form used in the
Edmon et al. (1980) :

$$\overline{F}^{\phi} = -\rho_0 a \cos\phi\, \overline{u_g' v_g'}$$

$$\overline{F}^{z} = \rho_0 a \cos\phi\, f_0 \frac{\overline{v_g' \theta'}}{\overline{\theta}_z}$$

# d) Interpretation and more realistic base state

Depression Denis (12-14 Fev 2020)

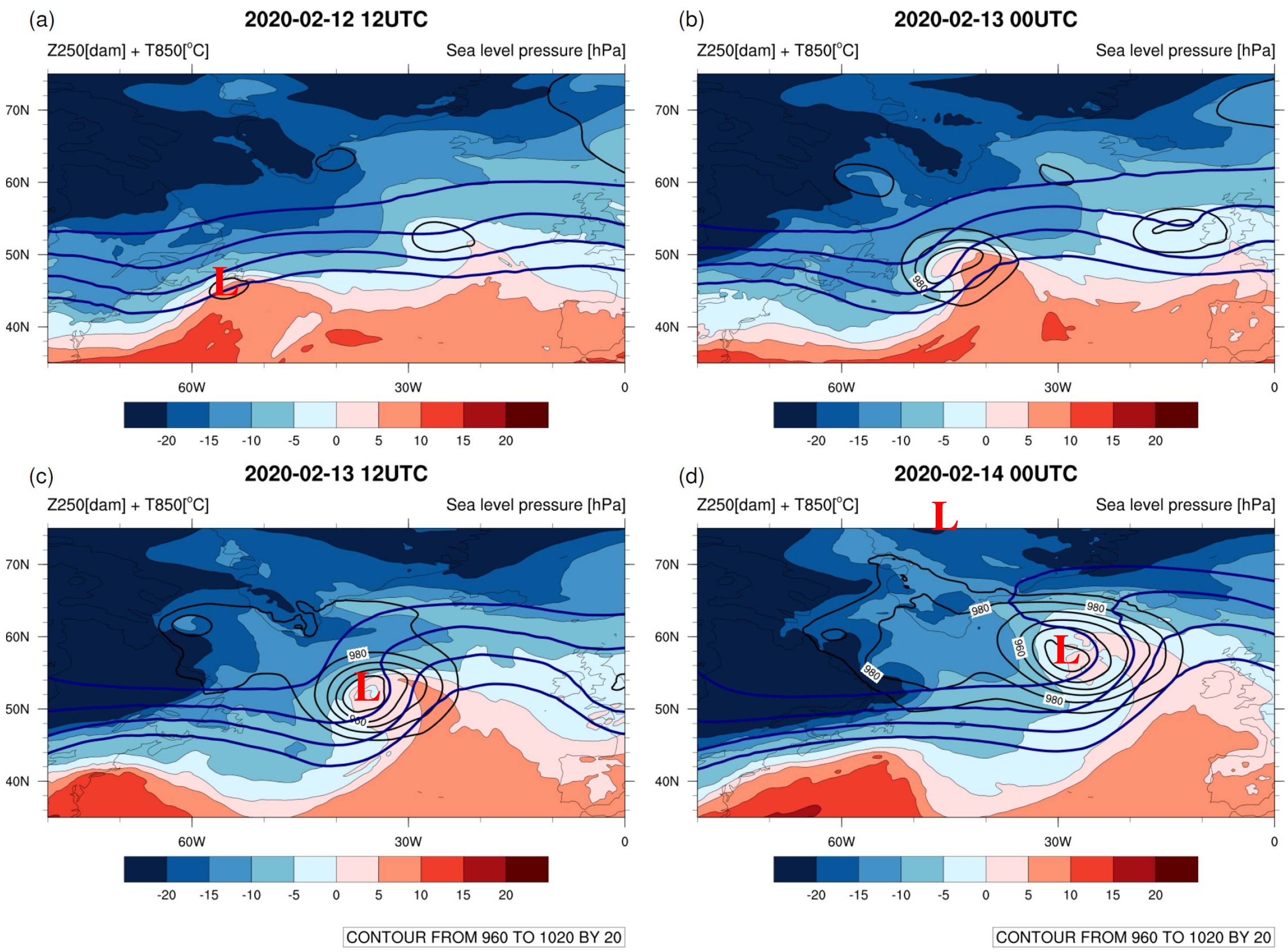



Among the strongest storms ever recorded in the North Atlantic (minimum 920hPa).

Deepening occurred in two phases, we examine here the first, with the genesis of the cyclone.

# d) Interpretation and more realistic base state

Depression Dennis (12-14 Feb. 2020)




**Blue lines: 250hPa Geopotential**

**Black: sea level pressure (every 10hPa)**

**Color: temperature at 850hPa**

(From ERA5 Reanalysis data)