Dynamic Meteorology

(General Circulation of the Atmosphere and Synoptic Meteorology)

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Aymeric Spiga

7) Troposheric equatorial variability

a) Mean and variabilities of the Outgoing Longwave Radiation observed by satellites

Level 0 statistics: variance teleconnections and EOFs

b)The inter-annual El-nino or southern "oscilllation"

Level 1 statistics: PCs spectra and co-spectra, composites

c) The intraseasonnal or Madden Julian "oscilllation"

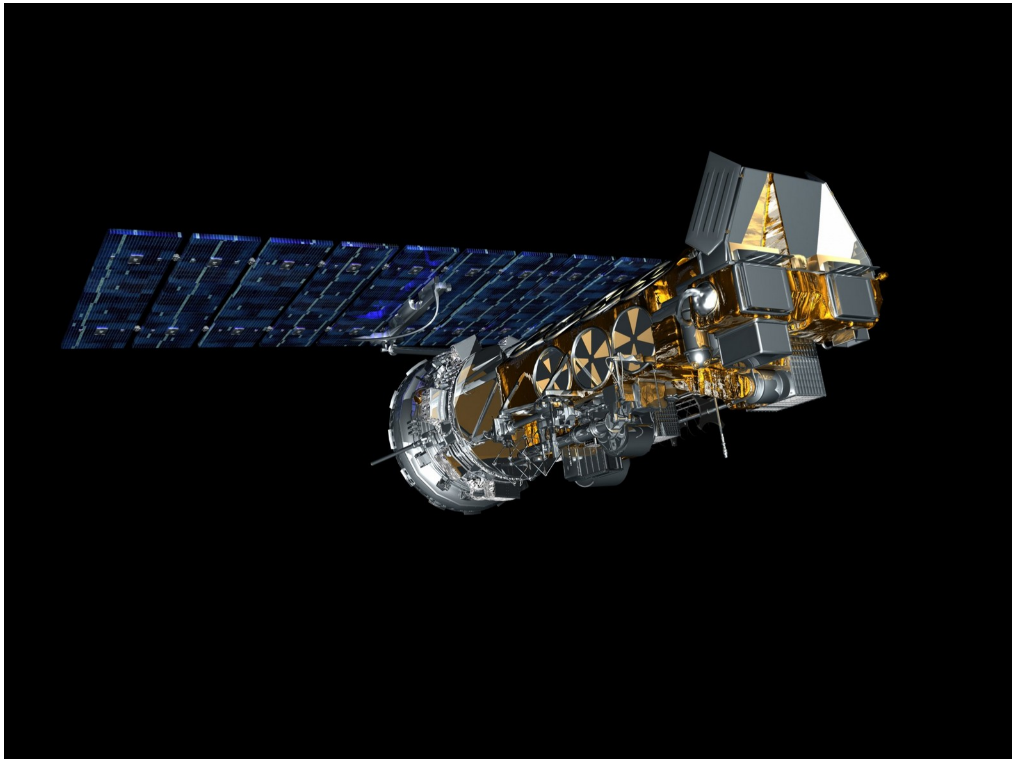
d) The convectively coupled equatorial waves

Level 1 statistics: time-longitude spectra, and composites

e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

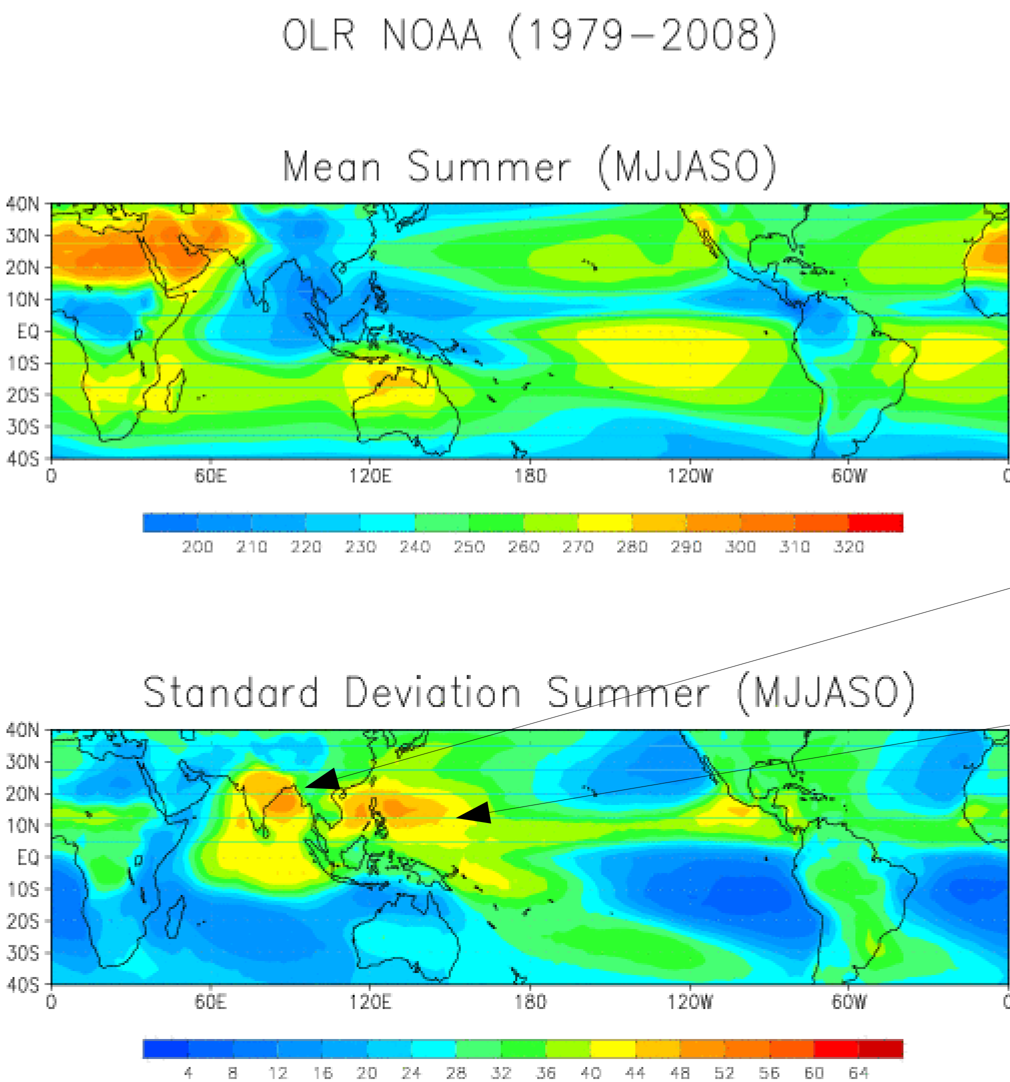
a) Mean and variabilities of the observed outgoing longwave radiation (OLR)
Level 0 statistics: variance teleconnections and EOFs



Satellite NOAA14-17

The outgoing longwaves are primarily coming from the highest optical obstacle, and measure
The black body T° at the top of the obstacle. For instance if there are thick high level clouds
The OLR will be low because at high altitude the T° is low.

a) Mean and variabilities of the observed outgoing longwave radiation (OLR)
Level 0 statistics: variance teleconnections and EOFs



In summer, the mean minimal values are over the monsoon regions, where the precipitations are the largest, the highest values over the desertic lands.

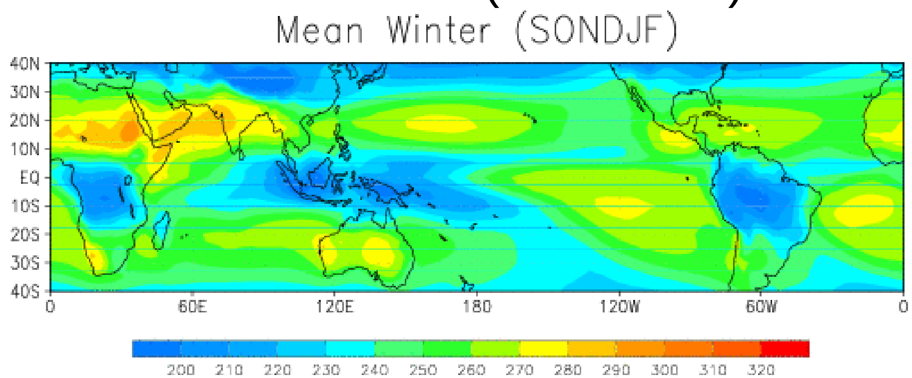
In summer, the variabilities are also the largest where the mean precipitations are the largest. Note the maxima in variability over the bay of Bengal (Indian monsoon) and the South Eastern Asia (Asian monsoon)

a) Mean and variabilities of the observed outgoing longwave radiation (OLR)

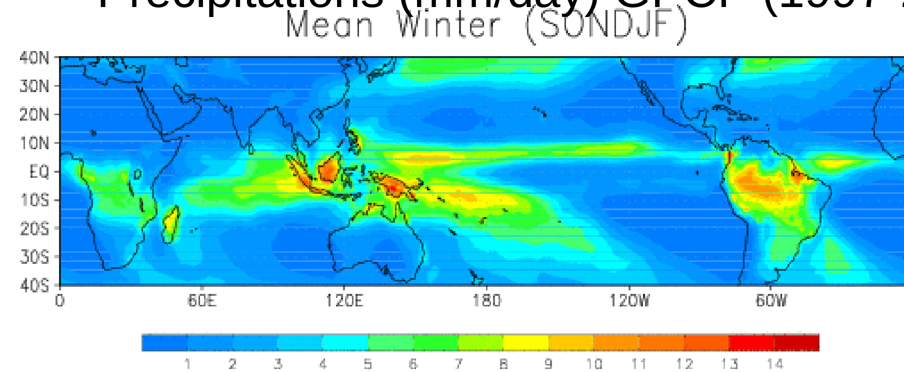
Level 0 statistics: variance teleconnections and EOFs

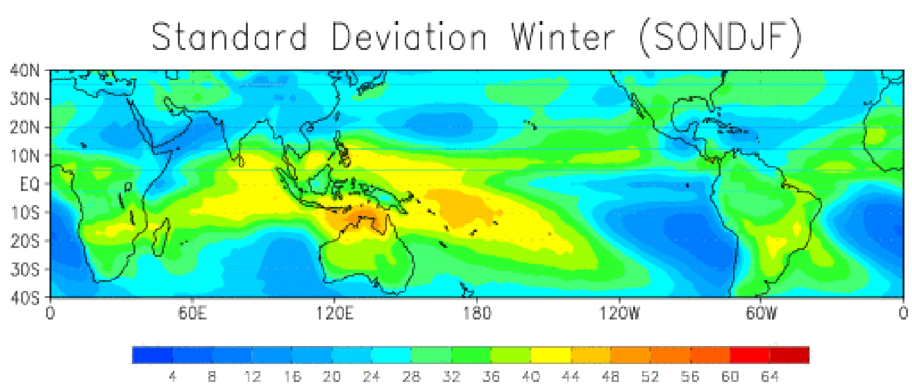
In (NH) winter the monsoon regions are now over southern America and South Equatorial Africa, but the Largest mean convection occurs over the oceanic Indonesian continent. Over this large sector, essentially located over the ocean and centered at the equator, the variabilty is also extremely large. Air sea interactions certainly play a central role in this variability

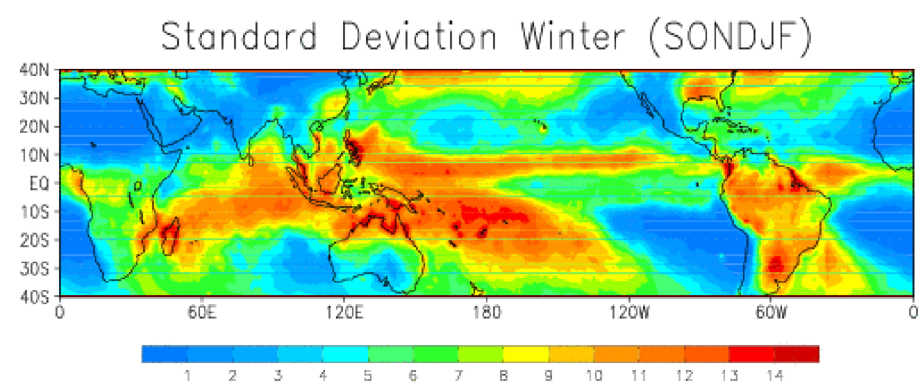
OLR NOAA (1979-2008)



Precipitations (mm/day) GPCP (1997-2008)



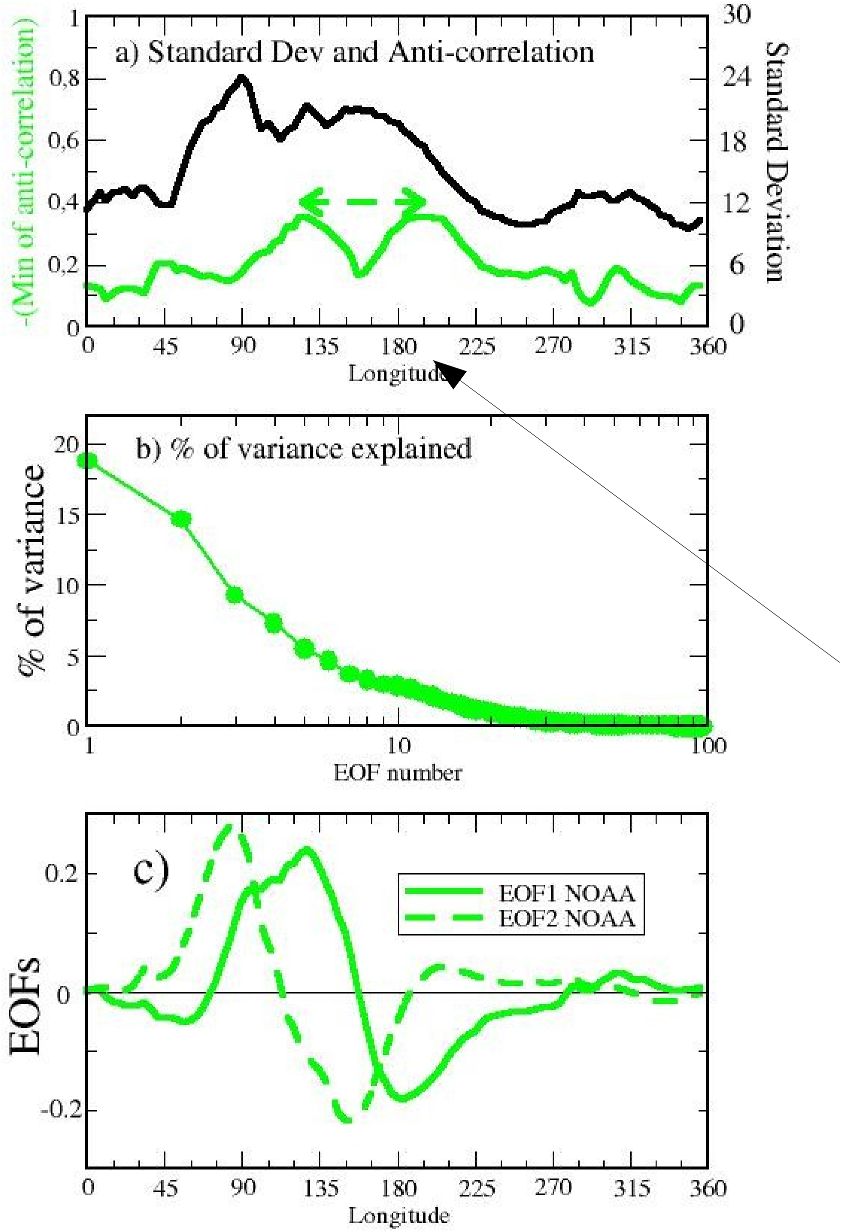




Precipitations and OLR give very comparable informations

a) Mean and variabilities of the observed outgoing longwave radiation (OLR)

Level 0 statistics: variance teleconnections and EOFs



To focus attention on the Equatorial variability,
we average the OLR over the Eq band (10°S-10°N)
and calculate its variance
(after substraction of the
Annual cycle and using 3-day mean data).

We find again the a broad maxima of variancelargely
Covering the maritime continent and the western
Pacific.

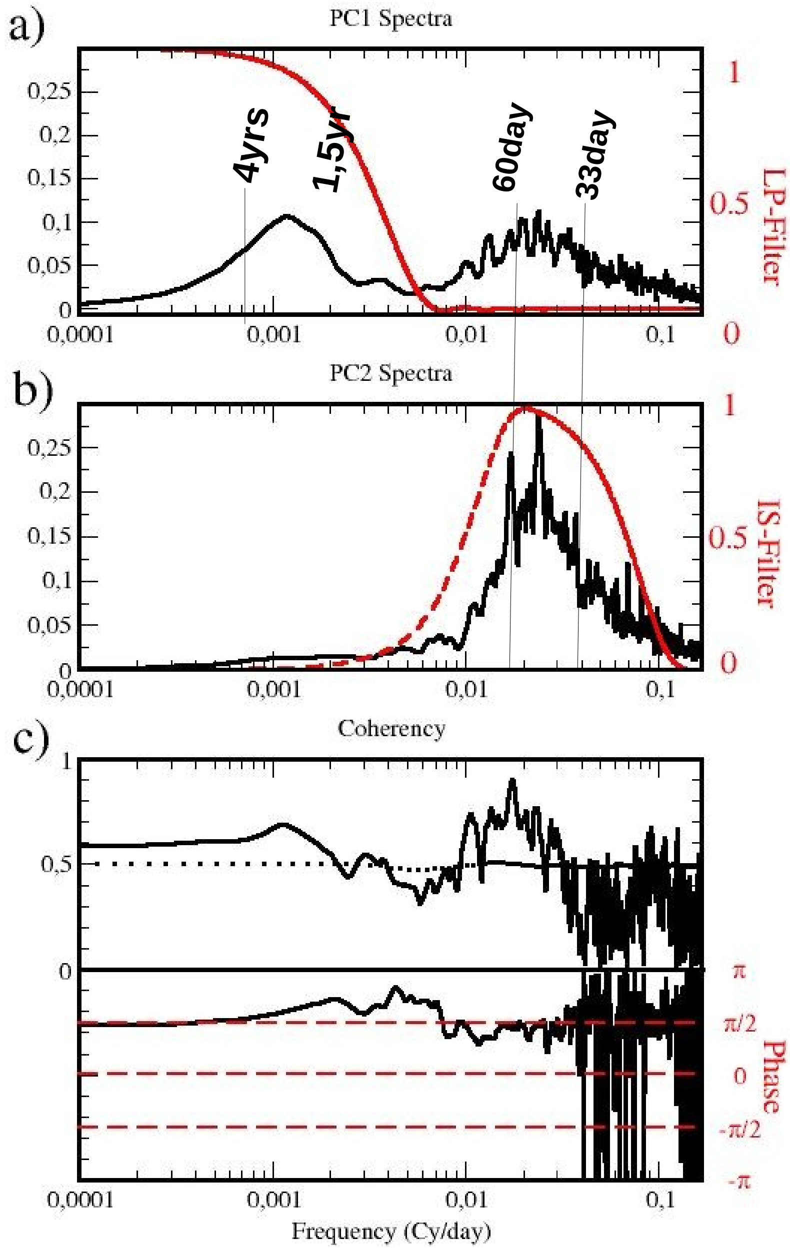
The maxima of anticorrelation shows that the
maritime continent and the central Pacific are in phase
opposition.

The first 2 EOFS have center of actions well
Located in the broad maxima of variance identified
Before. They represent around 35 % of the variance
(datas every 3 days.

The first EOF seems to capture the
Anticorrelation between the maritime continent and
The central Pacific

b) The inter-annual El-nino or southern "oscilllation"

Level 1 statistics: PCs spectra and co-spectra, composites



Spectral analysis of the PCs 1 and 2.

The PC1 shows enhanced variabilities in the inter-annual band (ω^{-1}>1yr not the PC2)

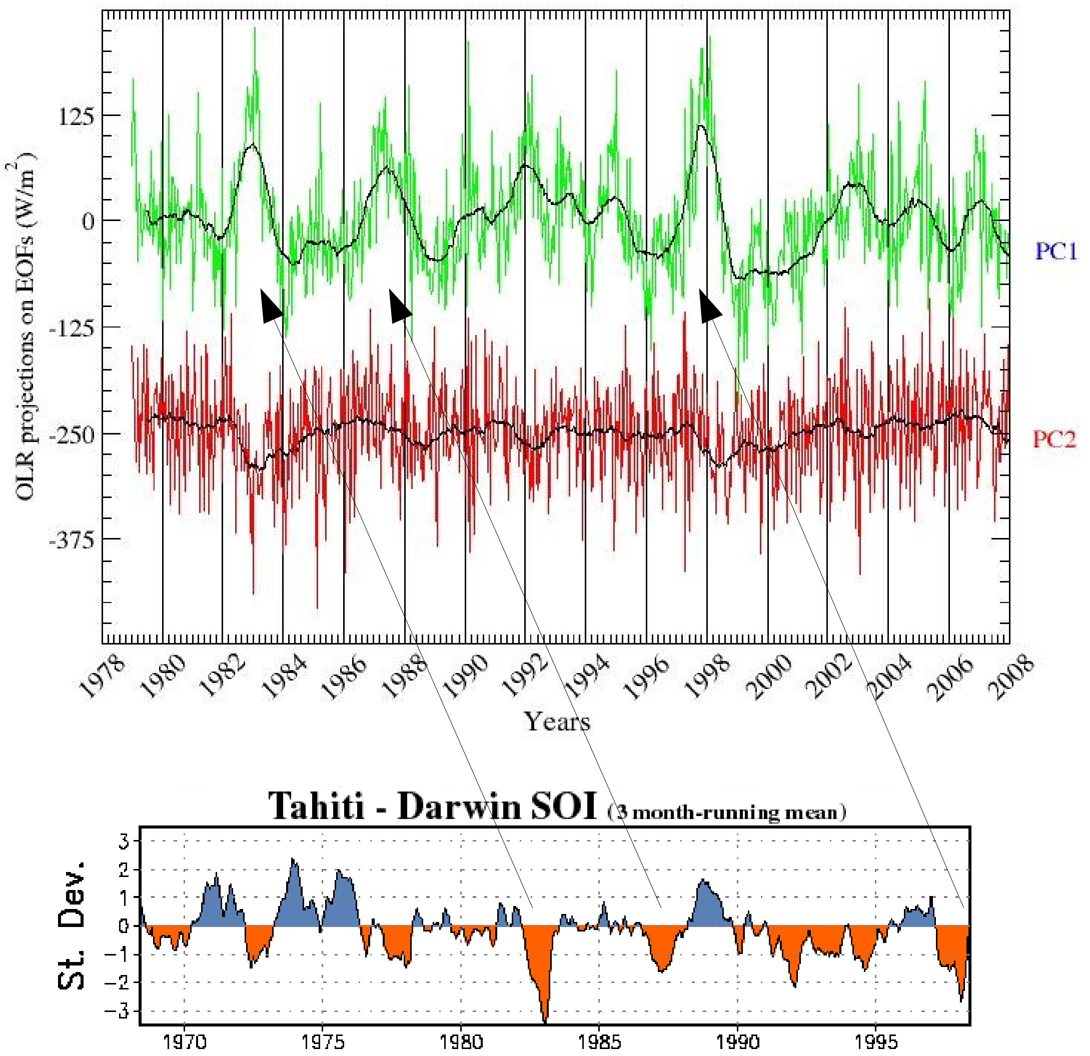
The PC1 and 2 show enhanced variabilities in the intra-seasonnal band (100day>ω^{-1}>10days)

In the intra-seasonnal band the PC1 and 2 are significantly correlated, and in lead-lag quadrature

The interannual and intra-seasonal signals are extracted using non-recursive filters which Transfer function are shown in solid red

b) The inter-annual El-nino or southern “oscilllation”

Level 1 statistics: PCs spectra and co-spectra, composites



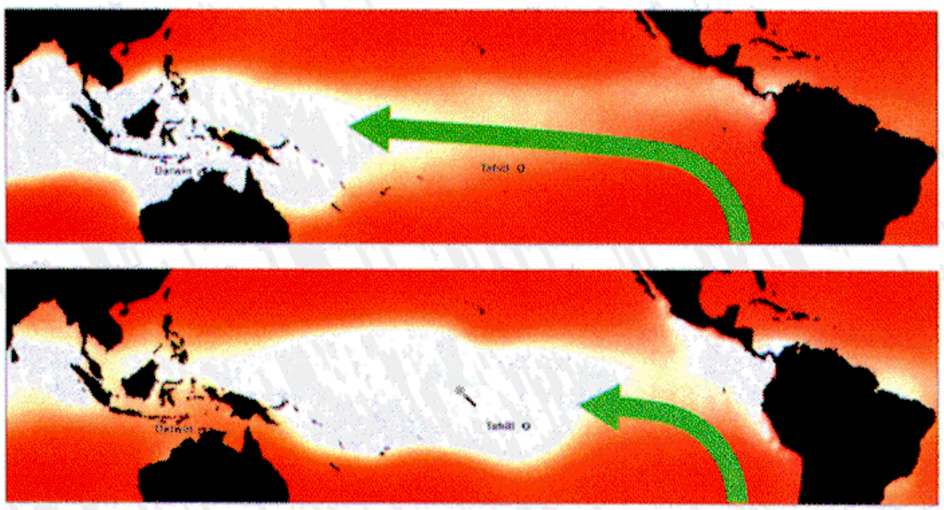
The low pass filtered PC1 (black) is a smoothed version of the raw one.

It presents large peaks, that matches the peacks of the ENSO Index Given by the averaged Sea surface Temperature over central and Eastern Pacific

b) The inter-annual El-nino or southern “oscilllation”

Level 1 statistics: PCs spectra and co-spectra, composites

If the interannual variability has some form of oscillatory components, there is some hope that we can predict its evolution a long time in advance and using statistical tools
But the modern series are two short (like our OLR data here) so we use proxy,
like here the pressure difference betwwen Tahiti and Darwin

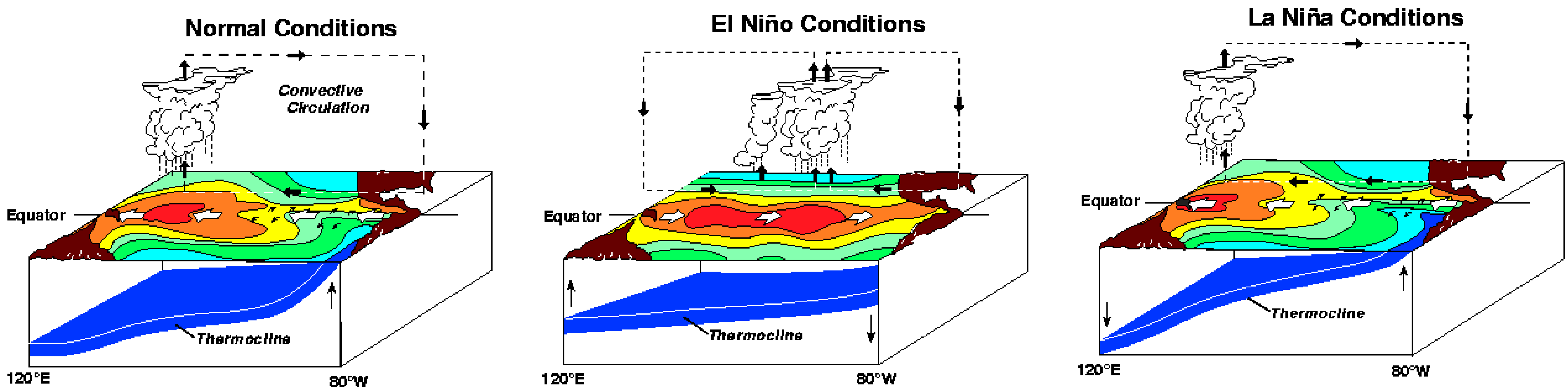


At the Equator, the winds blow from high to low pressure (the Coriolis force is small)
Durin El-nino, the shift in precipitations toward the central pacific correspond to a reduction of the trade winds between Darwin and Tahiti.

This pressure difference is measured until the end of the 19th century and validate over the satellite period

b) The inter-annual El-nino or southern “oscilllation”

Level 1 statistics: PCs spectra and co-spectra, composites



Normal situations:
The convection is centerd over the Pacific warm Pool

The warm pool is itself feede by the trade winds

El-Nino situation
The warm pool shifts toward the central Pacific,
The trade winds reduced and are less efficient in “pushing” the warm water towrd the western Pacific

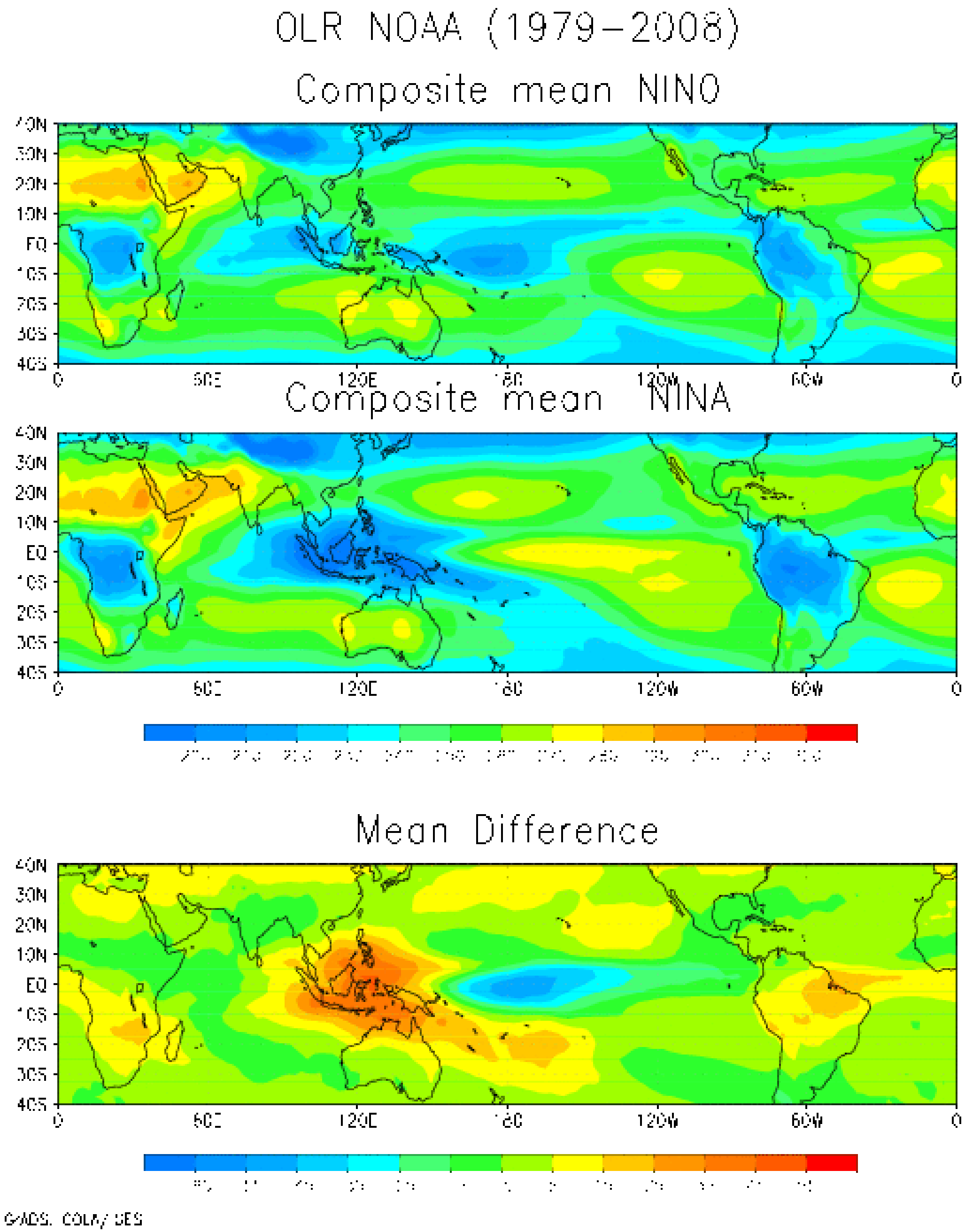
(a example of positive feedback)

La Nina situation:
The warm pool is even more confined to the western Pacific and maritime continent.

Issues de http://www.pmel.noa.gov/tao/elnino

b) The inter-annual El-nino or southern “oscilllation”

Level 1 statistics: PCs spectra and co-spectra, composites

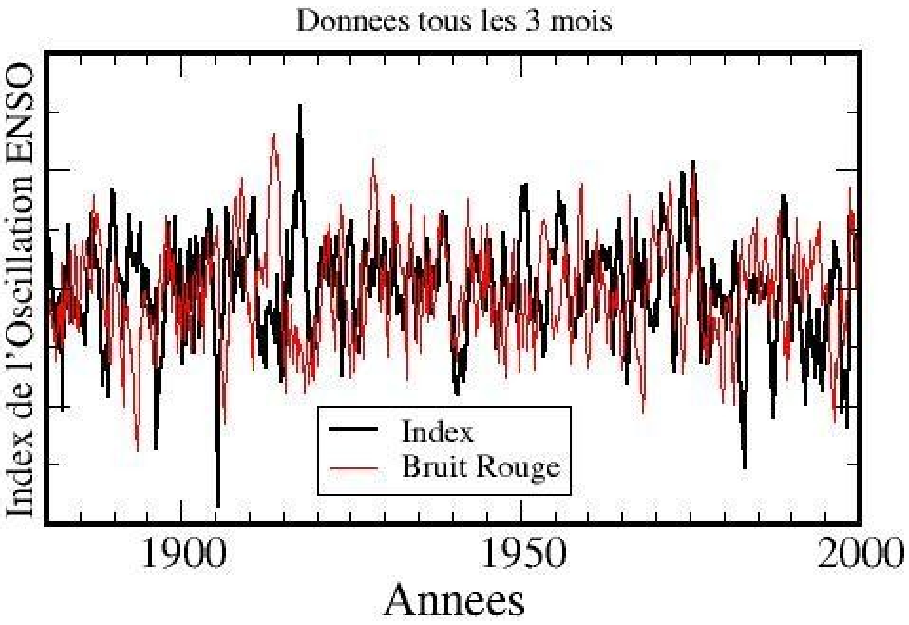


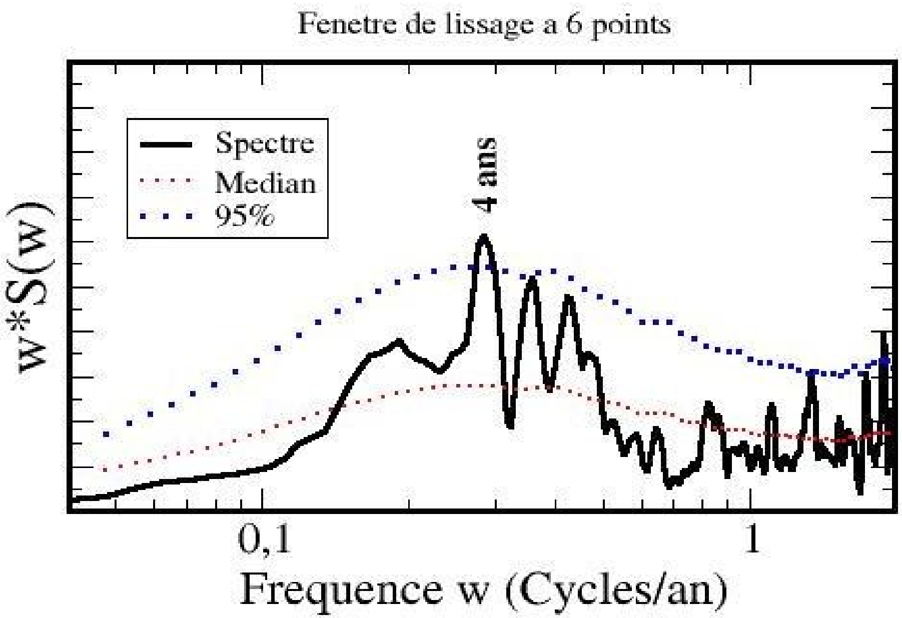
OLR composites keyed
to the low pass PC1 illustrates
Well that during ENSO the
mean precipitations are
considerably reduced over the western
Pacific and maritime continent and
Increasd over the central Pacific
(here averages over winter, DJF)

b) The inter-annual El-nino or southern “oscilllation”

Level 1 statistics: PCs spectra and co-spectra, composites

Evidence that there exists inter-annual oscillations of the ENSO Index





The ENSO index behaves almost like a red-noise

Nevertheless, it presents weakly significant peaks at 2 and 4 years.

The reduced amplitude of this peaks make that purely statistical predictions are not much reliable.

But the inter-annual variability is large, so we still need to understand what produces it.

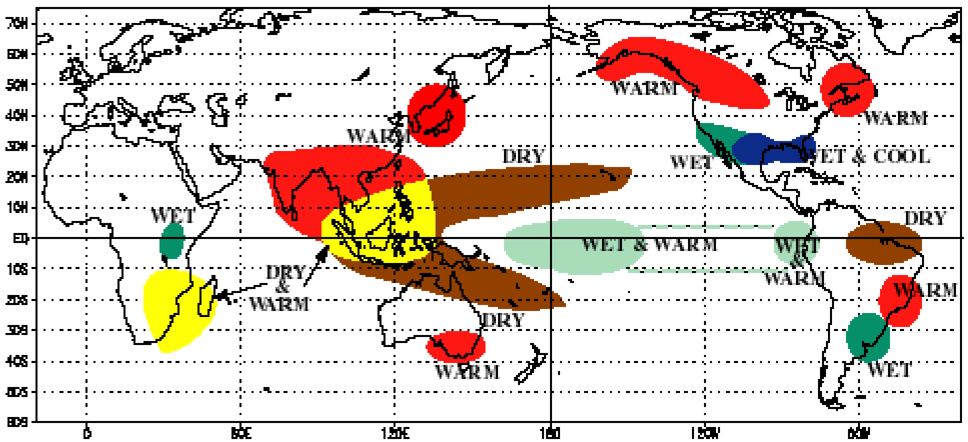
May be the term oscillation is usurpated (or old fashion!)

b) The inter-annual El-nino or southern "oscilllation"

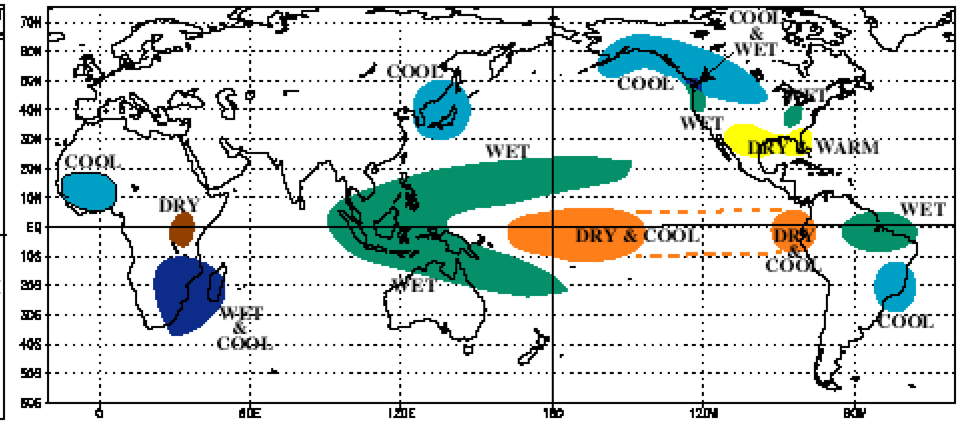
Level 1 statistics: PCs spectra and co-spectra, composites

Climatic Impacts

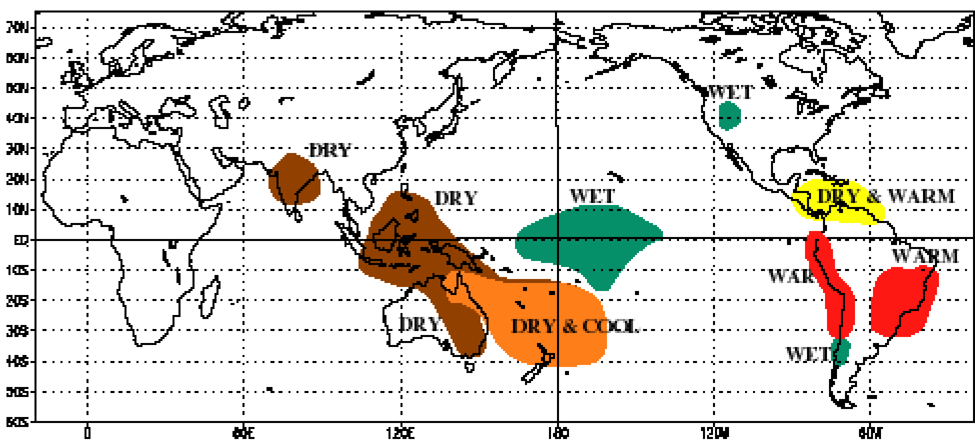
WARM EPISODE RELATIONSHIPS DECEMBER - FEBRUARY



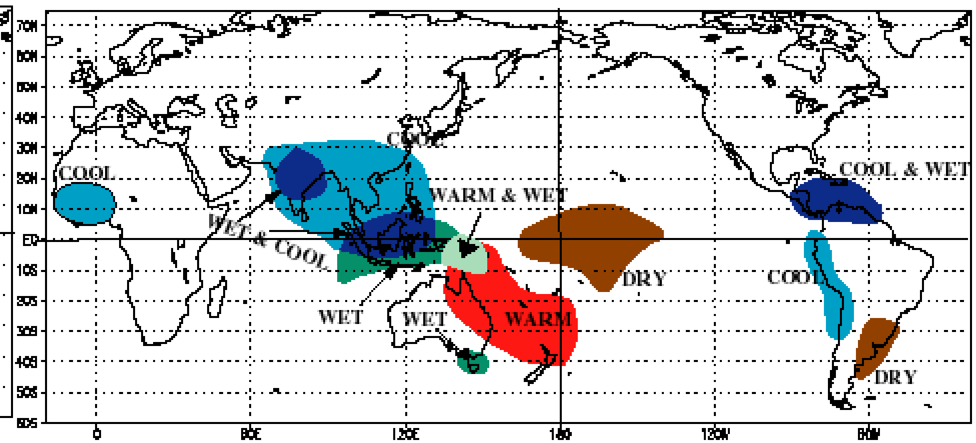
COLD EPISODE RELATIONSHIPS DECEMBER - FEBRUARY



WARM EPISODE RELATIONSHIPS JUNE - AUGUST



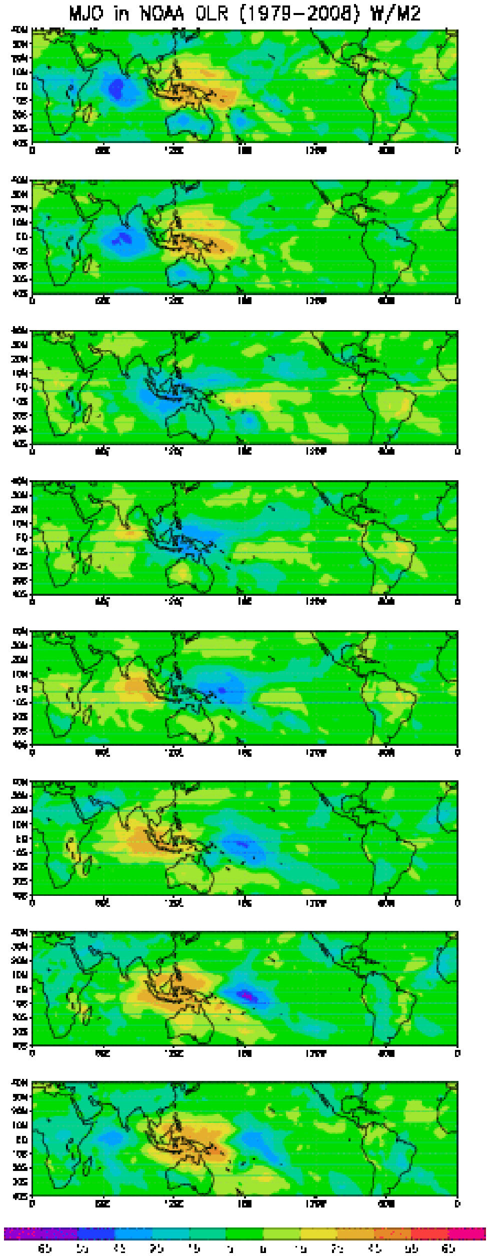
COLD EPISODE RELATIONSHIPS JUNE - AUGUST



Climate Prediction Center
NCEP

Climate Prediction Center
NCEP

c) The intraseasonnal or Madden Julian "oscilllation"



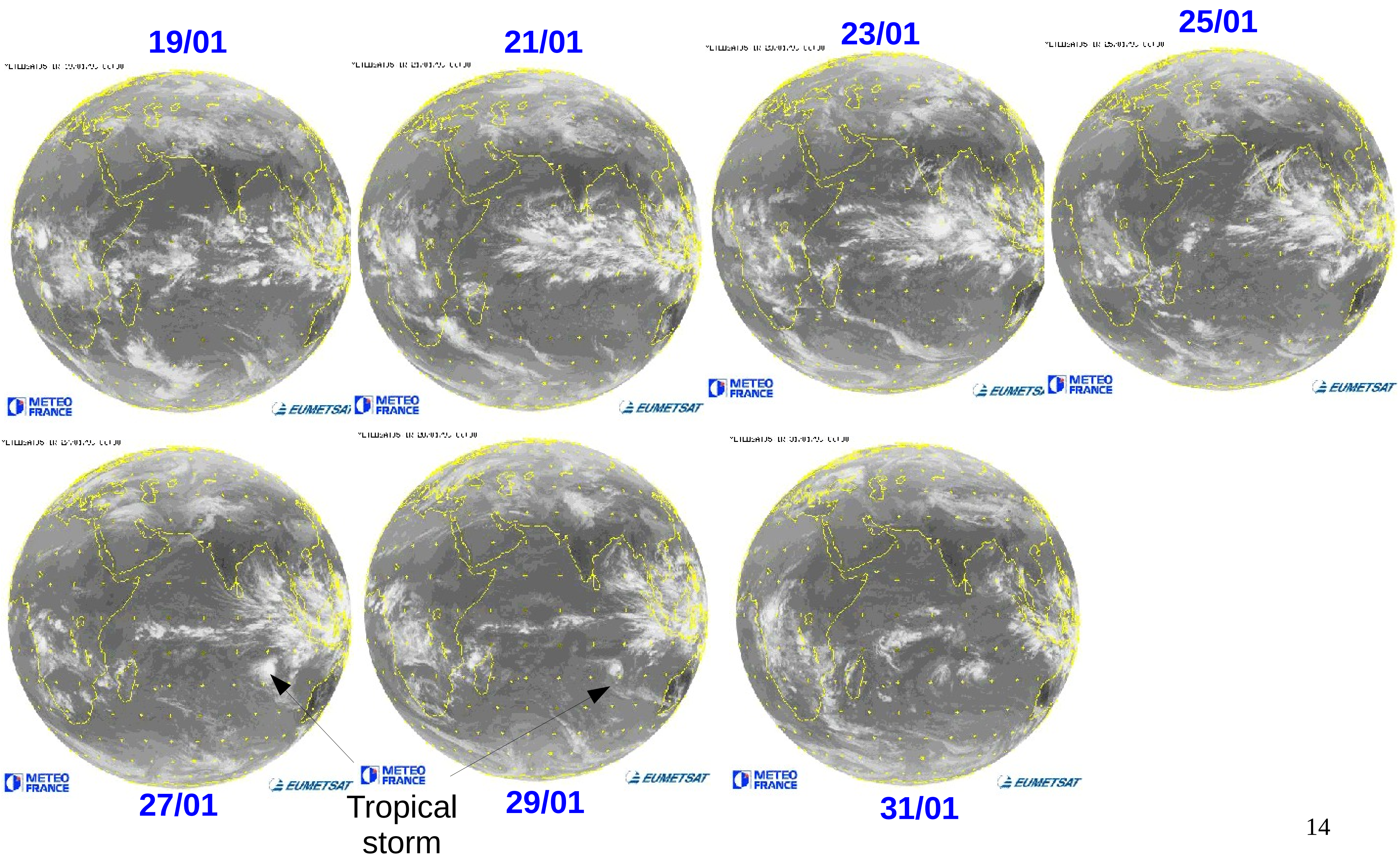
Composites keyed to the PC1 and 2 of OLR filtered
In the intraseasonnal band

Note the slow eastward displacement of the large scale precipitation pattern that is over the Indian ocean toward the western Pacific

Although the composite identify phases in the oscillation the characteristic duration can be well above a month

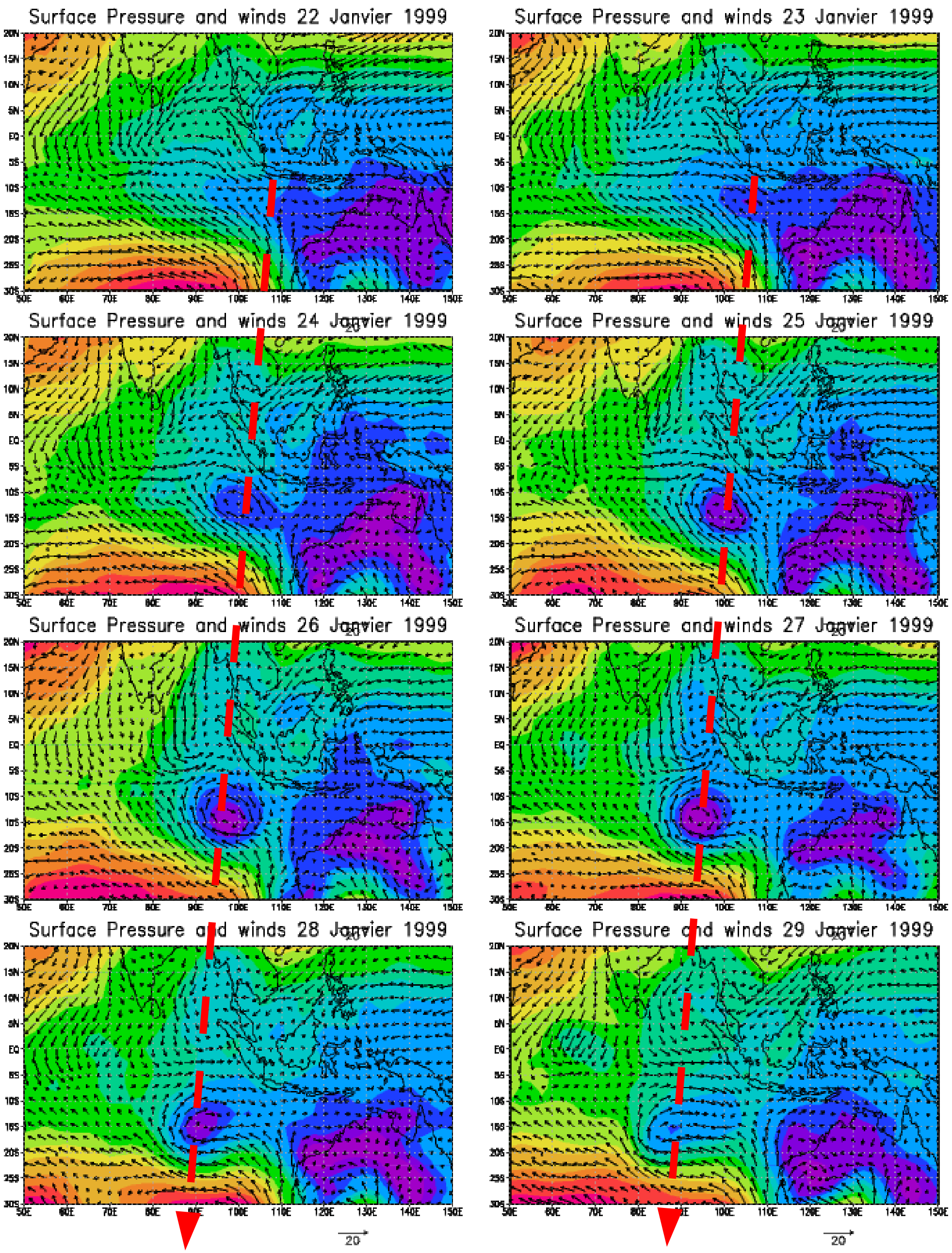
c) The intraseasonnal or Madden Julian “oscilllation”

One real case to illustrate the distinction between planetary scale variability and day to day weather variability (visible meteosat data, Jan-Feb 1999).



c) The intraseasonnal or Madden Julian “oscilllation”

The planetary scale variability modulate the onset of synoptic weather events, here a tropical storm that can well propagate westward (some become hurricanes)
Hereagain low-frequency varibilty and synoptic weather should well be distinguished



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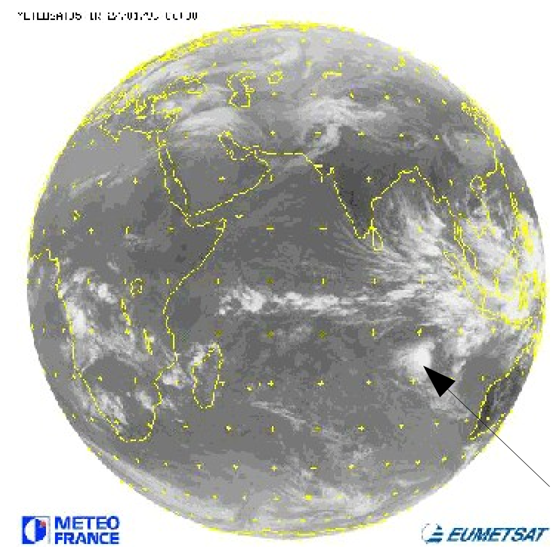
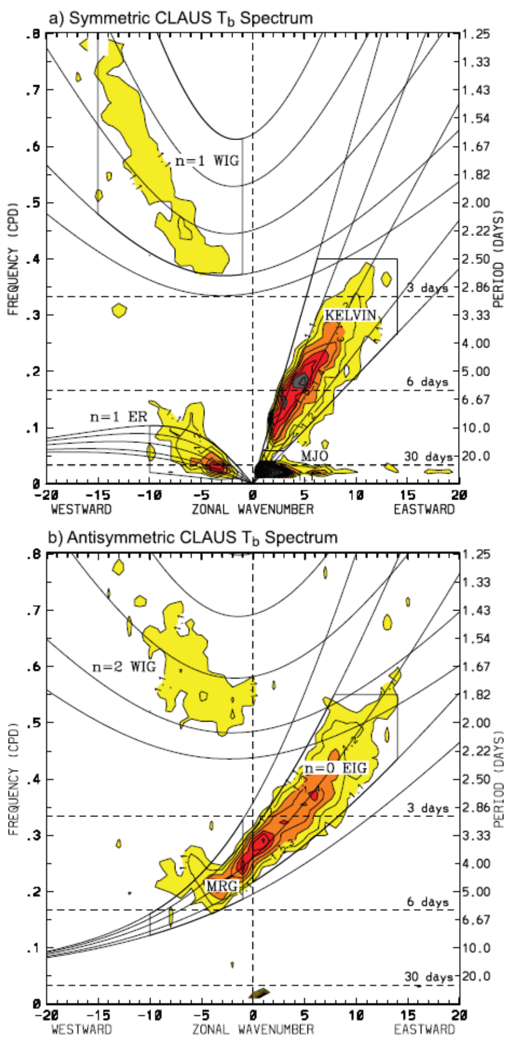


Tropical storm

d) The convectively coupled equatorial waves

Level 1 statistics: time-longitude spectra and composites

Time-frequency spectra of OLR averaged over the equatorial band



Kiladis et al.~(2009)

Symetric and asymmetric signal

$$T_s = \frac{1}{10}\int_0^{10} \left(T(\phi)+T(-\phi)\right) d\phi, \quad T_a = \frac{1}{10}\int_0^{10} \left(T(\phi)-T(-\phi)\right) d\phi$$

Fourier series:

$$T_s(t,\lambda) = \sum_{s=1}^{\frac{nlo}{2}} \sum_{i=1}^{N} \hat{T}_s(s,\omega_i)\, e^{i(s\lambda - 2\pi\omega_i t)}$$

Periodogram:

$$P_{T_s}(s,\omega) = \hat{T}_s \hat{T}_s^*(s,\omega)$$

Very noisy : Spectra are better approximated by smoothed periodograms:

$$S_{T_s} \approx \tilde{P}_{T_s}$$

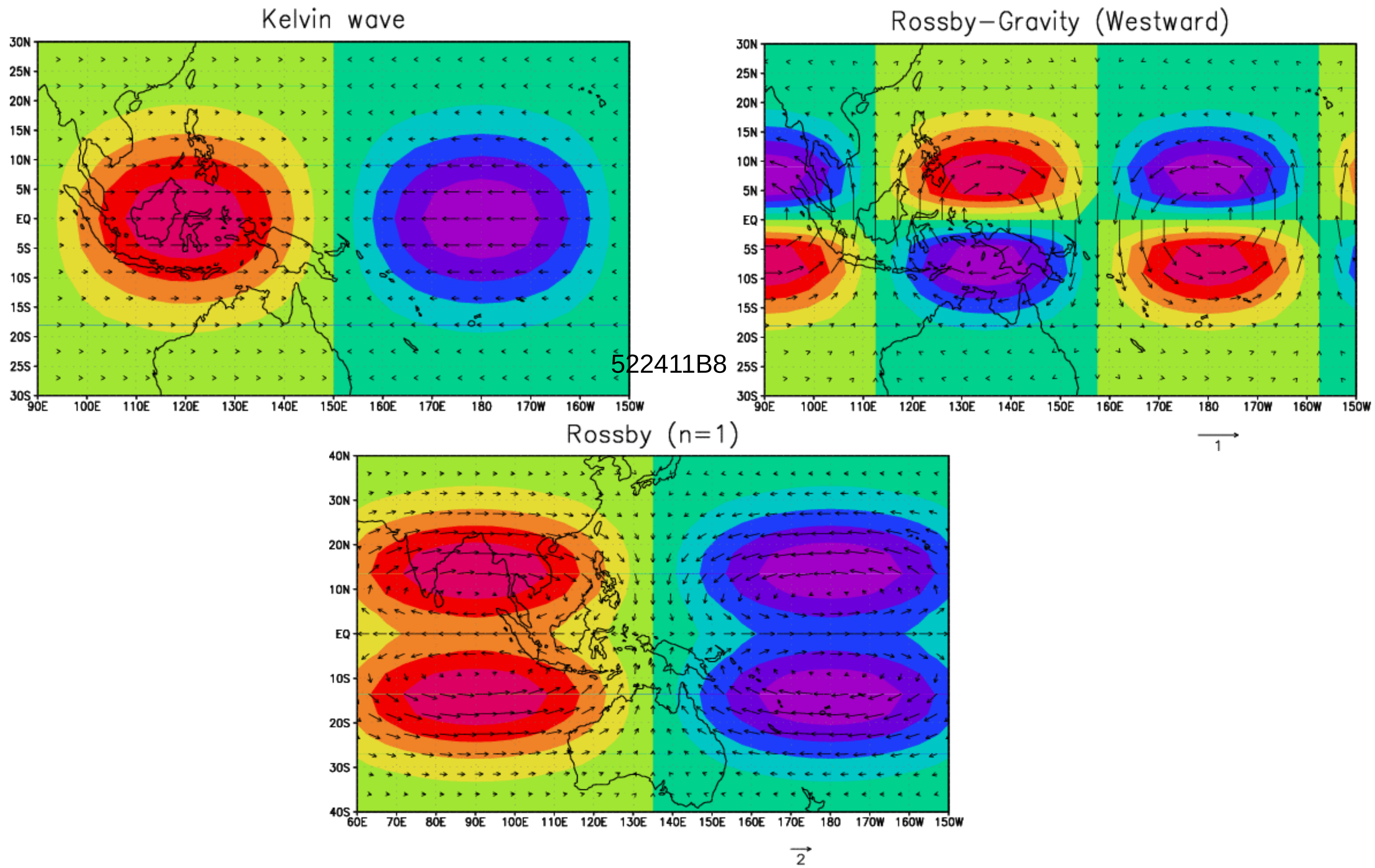
The relative maxima obtained falled between the dispersion curves of the equatorial waves with equivalent depths :

h= 5m, 12m, 25m, 50m and 90m

d) The convectively coupled equatorial waves

Level 1 statistics: time-longitude spectra and composites

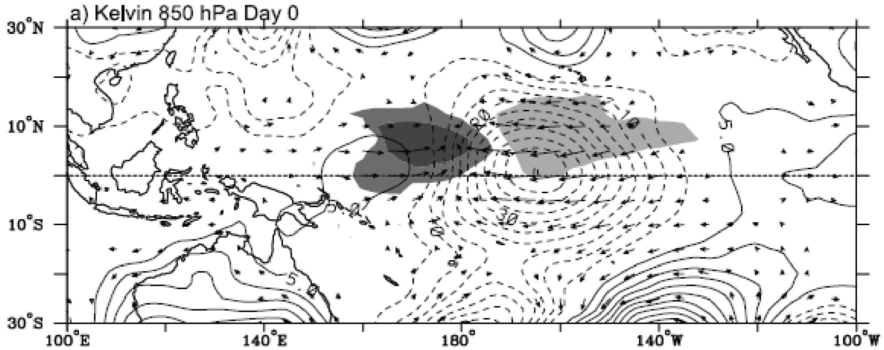
The dispersion curves shown correspond to the Kelvin waves (ν=-1), Rossby Gravity waves (ν=0), Rossby waves (with ν=1),   ect.
Derived from the shallow-water theory of Equatorial waves (Matsuno 1966), but with equivalent depth of 8m, 25m, 25m, 50m, and 90m!



d) The convectively coupled equatorial waves

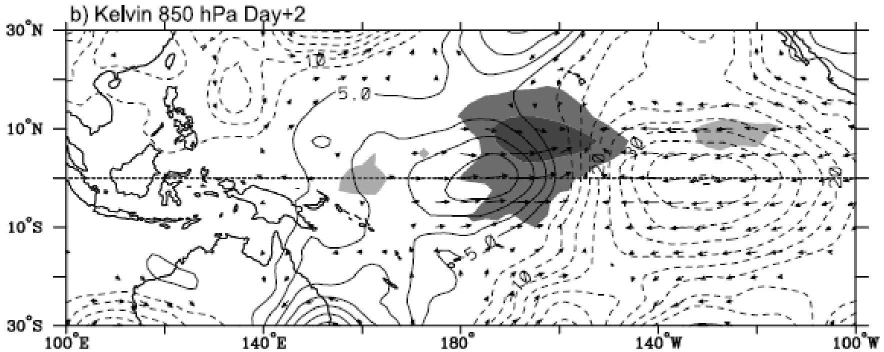
Level 1 statistics: time-longitude spectra and composites

Reconstruction of the signal after filtering all datas (here Brilliance Temperature, Geopotential (contour) and horizontal winds near the ground (850hPa, arrows))



Day 0

Here the band pass Kelvin wave filter is used



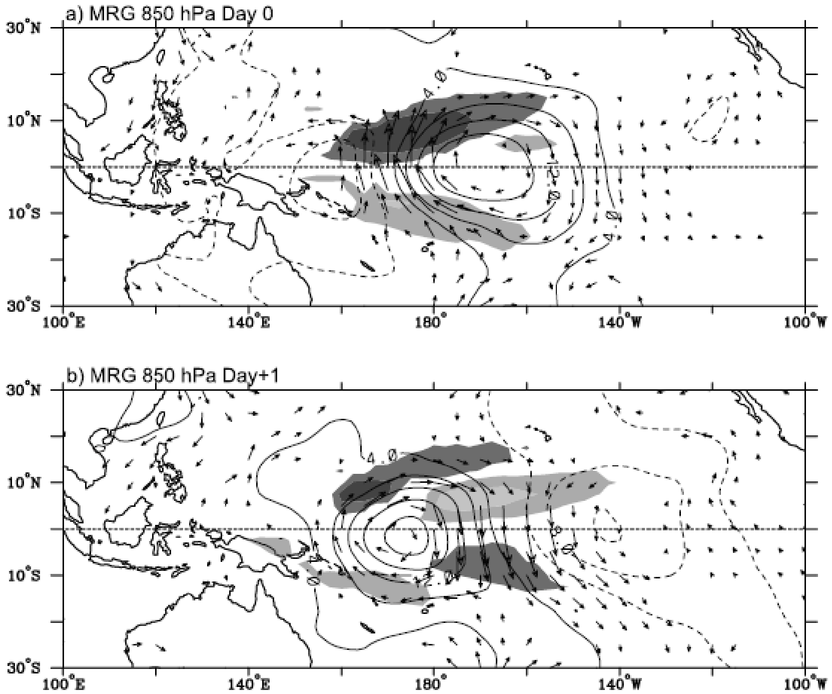
Day 2

The reconstruction is done via correlations with the filtered Brilliance Temperature At 7.5°N, 172°E

Kiladis et al.~(2009)

d) The convectively coupled equatorial waves

Level 1 statistics: time-longitude spectra and composites



Day 0

Day 2

Here the band pass Rossby-gravity wave filter is used

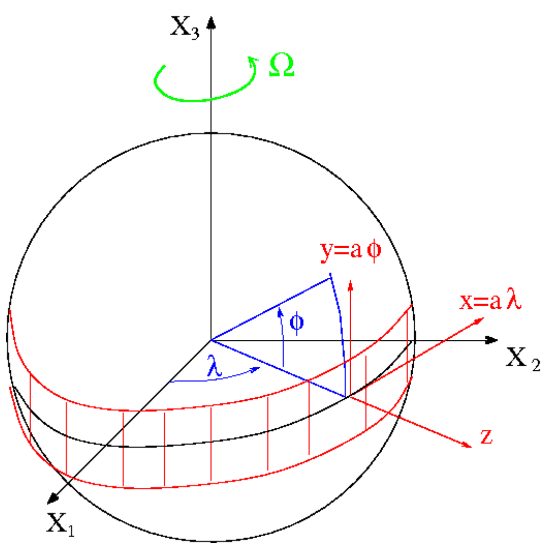
The reconstruction is done via correlations with the filtered Brilliance Temperature At 7.5°N, 172°E

Kiladis et al.~(2009)

e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Equatorial Boussinesq
β plane approximation
(otherwise as in Lecture 5):



Coriolis term: $2\Omega \sin\phi \approx \beta y$

$$\beta = \frac{2\Omega}{a}$$

Sphericity terms:

$$\tan\phi \frac{uv}{a} \quad \text{and} \quad \tan\phi \frac{uu}{a}$$

neglected

$$\frac{Du}{Dt} - \beta y v = -\frac{\partial \Phi}{\partial x}$$

$$\frac{Dv}{Dt} + \beta y u = -\frac{\partial \Phi}{\partial y}$$

$$b = g\frac{\theta}{\theta_s}, \quad \Phi = \frac{p}{\rho_s}$$

$$\frac{\partial \Phi}{\partial z} = b$$

Hydrostatic

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{Db}{Dt} = 0$$

Stratification at rest, $b_0(z)$, $\Phi_0(z)$

Resulting equations :

$$\frac{Du}{Dt} - \beta y v = -\frac{\partial \Phi_e}{\partial x} \qquad \frac{Dv}{Dt} + \beta y v = -\frac{\partial \Phi_e}{\partial y} \qquad \frac{\partial \Phi_e}{\partial z} = b_e$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{Db_e}{Dt} + N^2 w = 0 \qquad N^2 = \frac{db_0}{dz}$$

e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Linearised equations, no background zonal winds, stratification at rest stays given by $\theta_0(z)$

$$\partial_t u' - \beta y v' + \partial_x \Phi' = 0$$

$$\partial_t v' + \beta y u' + \partial_y \Phi' = 0$$

$$\partial_x u' + \partial_y v' + \partial_z w' = 0$$

$$\partial_t \partial_z \Phi' + N^2(z) w' = 0$$

<u>Separation of constants :</u>

u', v' and Φ' have the same vertical structure (say U(z)) and call W(z) the vertical structure of w' :

$$\begin{pmatrix} u' \\ v' \\ \Phi' \end{pmatrix} = U(z) \begin{pmatrix} \tilde{u}(t,x,y) \\ \tilde{v}(t,x,y) \\ \Phi(t,x,y) \end{pmatrix}$$

$$w' = W(z)\tilde{w}'(t,x,y)$$

$$U(z)\left(\partial_t \tilde{u} - \beta y \tilde{v} + \partial_x \tilde{\Phi}\right) = 0$$

$$U(z)\left(\partial_t \tilde{v} + \beta y \tilde{u} + \partial_y \tilde{\Phi}\right) = 0$$

$$U(z)\left(\partial_x \tilde{u} + \partial_y \tilde{v}\right) + W_z(z)\tilde{w} = 0$$

$$U_z(z)\partial_t \tilde{\Phi} + W(z) N^2(z)\tilde{w} = 0$$

e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Vertical structure equation :

$$\frac{d^2 W}{dz^2} + \frac{N^2(z)}{gh} W = 0$$

Case when N^2=cte :

$$W = W_0 e^{imz} \quad \text{where} \quad m^2 = \frac{N^2}{gh}$$

Shallow water equations
(h=equivalent depth)

$$\partial_t \tilde{u} - \beta y \tilde{v} + \partial_x \tilde{\Phi} = 0$$

$$\partial_t \tilde{v} + \beta y \tilde{u} + \partial_y \tilde{\Phi} = 0$$

$$\partial_t \tilde{\Phi} + gh(\partial_x \tilde{u} + \partial_y \tilde{v}) = 0$$

Monochromatic (time longitude) wave :

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\Phi} \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \end{pmatrix} e^{i(kx - wt)}$$

$$-i\omega \hat{u} - \beta y \hat{v} + i k \hat{\Phi} = 0$$

$$-i\omega \hat{v} + \beta y \hat{u} + \partial_y \hat{\Phi} = 0$$

$$-i\omega \hat{\Phi} + gh(i k \hat{u} + \partial_y \hat{v}) = 0$$

Tropospheric application : $e^{imz} \rightarrow \cos mz \ (\text{horizontal wind}) \text{ or } \sin mz (\text{vertical wind})$

$$\text{For } 2\frac{\pi}{m} = 10\,\text{km}, \quad N^2 = 10^{-4}\,\text{s}^{-2}, \quad h \approx 25\,\text{m}$$

e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Kelvin waves $\tilde{v}=0 \quad (\nu=-1)$

$$-i\omega\hat{u}+ik\hat{\Phi}=0$$

$$\beta y\hat{u}+\partial_y\hat{\Phi}=0$$

$$-\omega\hat{\Phi}+ghk\hat{u}=0$$

$$\partial_y\hat{\phi}+\beta\frac{k}{\omega}y\hat{\Phi}=0$$

$$\omega^2-ghk^2=0$$

Frequency and phase speed (shallow water):

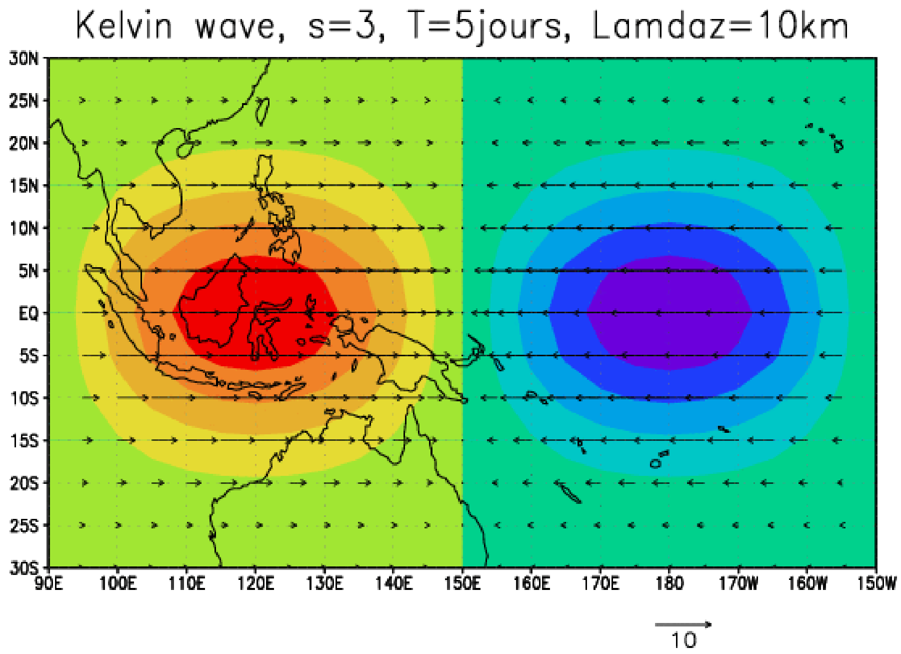
$$\omega=+\sqrt{gh}\,k, \quad \text{or} \quad C_x=\frac{\omega}{k}=\sqrt{gh}=c$$

Meridional structure:

$$\hat{\Phi}=\hat{\Phi}_0 e^{-\frac{\beta}{c}\frac{y^2}{2}}, \quad \hat{u}=\frac{\hat{\Phi}}{\sqrt{gh}}$$

Dispersion relation

$$m^2=\frac{N^2}{gh} \rightarrow \omega^2=\frac{N^2k^2}{m^2}$$



e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Number of nodes between the poles $\nu>-1$ (for the meridional velocity v)

All the equations can be combined to give an equation for the meridional velocity:

$$\partial_y^2 \hat{v} + \left(\frac{\omega^2}{c^2} - k^2 - \frac{\beta k}{\omega} - \frac{\beta^2 y^2}{c^2}\right)\hat{v} = 0$$

Remember Shrodinger (1923)'s equation for the harmonic oscillator

$$\frac{\hbar^2}{2m}\Psi_{xx} + \left(E - \frac{k x^2}{2}\right)\Psi = 0$$

$$c = \sqrt{gh}$$

We search solutions of the form:

$$\hat{v} = e^{-\frac{\beta}{c}\frac{y^2}{2}} V\left(\left(\frac{\beta}{c}\right)^{1/2} y\right)$$

Equation for $V(x),\ x = \left(\frac{\beta}{c}\right)^{1/2} y$

$$\frac{d^2 V}{dx^2} - 2x\frac{dV}{dx} + \left(\frac{\omega^2}{c\beta} - \frac{k^2 c}{\beta} - \frac{kc}{\omega} - 1\right)V = 0$$

Dispersion relation when ν is given :

$$\frac{\omega^2}{c^2} - k^2 - \frac{k\beta}{\omega} = (2\nu+1)\frac{\beta}{c}$$

The Hermite polynomials

Differential equation:

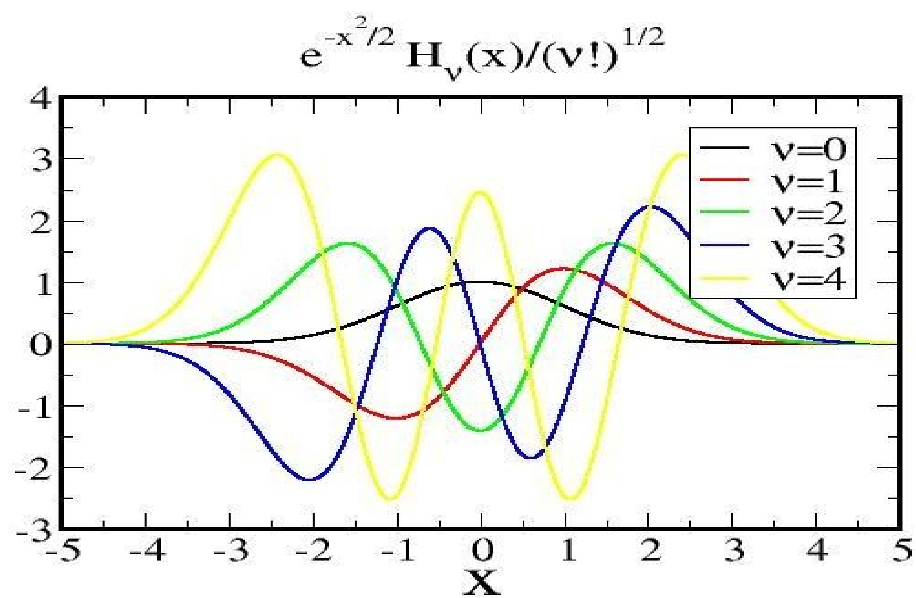
$$H_\nu'' - 2x H_\nu' + 2\nu H_\nu = 0$$

Few examples

They form an orthonormal basis for the scalar product:

$$H_0 = 1,\ \ H_1 = 2x,\ \ H_2 = 4x^2 - 2,\ \ \ldots.$$

$$\langle f, g\rangle = \int_{-\infty}^{+\infty} f(x) g(x) e^{-x^2/2} dx$$



e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

Number of nodes between the poles $\nu=0$: Rossby gravity waves

Dispersion relation:

$$\frac{\omega^2}{c^2}-k^2-\frac{k\beta}{\omega}=\frac{\beta}{c}$$

Factorizes into:

$$\left(\frac{\omega}{c}+k\right)\left(\frac{\omega}{c}-k-\frac{\beta}{\omega}\right)=0$$

$$c=\sqrt{gh}$$

$\frac{\omega}{c}+k=0$ impossible (see Kelvin waves)

Shallow water intrinsic frequency:

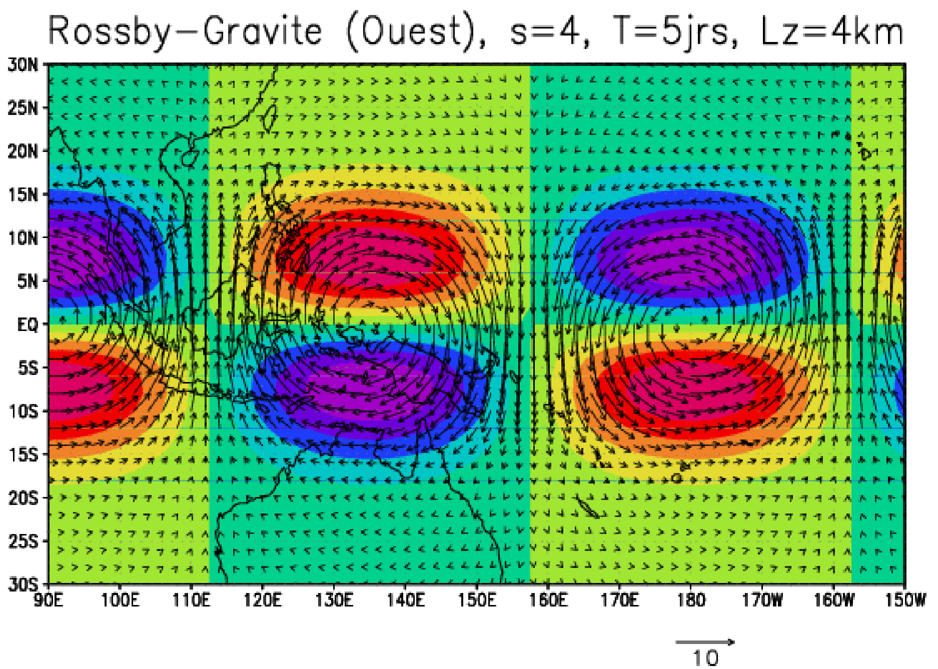
$$\omega=\frac{kc\pm\sqrt{k^2c^2+4\beta c}}{2}$$

The dispersion relation can be derived using

$$m^2=\frac{N^2}{gh}$$

Eastward or westward propagation, the most frequent ones goes westward (in terms of phase speed)

All goes eastward in term of group speed



e) Equatorial waves theory

Toy model 4 : shallow water theory adapted to vertically propagating Equatorial waves

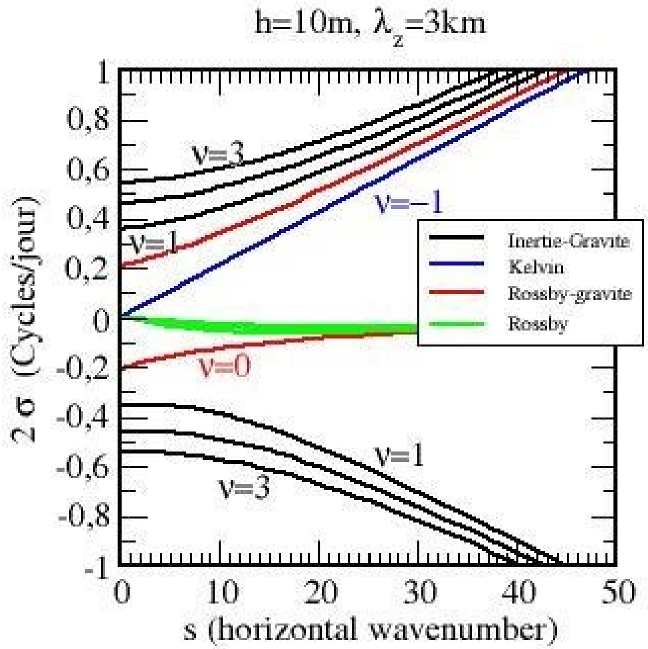
Number of nodes between the poles $\nu>0$: Rossby and gravity waves

The dispersion relation is a third order equation in ω:

$$\frac{\omega^2}{c^2} - k^2 - \frac{k\beta}{\omega} = (2\nu+1)\frac{\beta}{c}$$

$$\nu = 1$$

3 solutions are possible, one westward propagating Rossby wave ($\omega/k<0$) and two gravity waves (eastward or westward)



Example of a Rossby wave

westward propagation