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NONLINEAR EVOLUTION OF AN UNSTABLE STRATIFIED SHEAR LAYER

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ABSTRACT

Churilov and Shukhman /1/ have investigated the nonlinear development of Kelvin Helmholtz instabilities in a weakly supercritical stratified shear flow when the interaction is viscous In that case, the role of the Prandtl number is predominant. This last result is investigated again through nonlinear numerical simulations in which the dissipations are represented by Newtownian cooling and Rayleigh friction.

INTRODUCTION

The study of the nonlinear evolution of weakly unstable small amplitude disturbances in a non-rotating stratified unbounded shear flow was analytically initiated in /2/ and continued in /1/ An important role in the development of the disturbances is played by the critical level, where the wave horizontal phase velocity coincides with the horizontal mean flow velocity Then, the regime of the critical layer depends on the largest of the unsteady scale $L_t=\gamma$, the viscous scale $L_v=(kRe)^{-1/3}$ and the nonlinear scale $L_n=A^{2/3}$. Here, k stands for the horizontal wave number of the most unstable mode, γ is its growth rate and A its amplitude Re is a Reynolds number For $L_v \gg \max(L_n, L_t)$, the interaction is viscous and the nonlinear effects are controlled by the Prandtl number $Pr=\nu/\kappa$ (ν is the viscosity and κ is the thermal conduction) For Pr>1, the growth rate of the unstable mode becomes larger than the linear one the disturbance is destabilized through nonlinear interactions. That result is mostly linked to a feedback influence of the modification of the stability of the mean flow due to the action of the wave For Pr<1, opposite effects are observed.

The purpose of this study is to determine if that property also exists when dissipations are represented by Newtownian cooling and Rayleigh friction To do this, we determine the fully nonlinear evolution of Kelvin Helmholtz instabilities with the help of the numerical model presented in /3/ for different values of the Prandtl number Pr=a/b Here a is the Rayleigh friction and b is the Newtownian cooling

THE MODEL

We consider the nonlinear development of a Kelvin Helmholtz instability in a Drazin profile $\hat{U}(\hat{z})=U_0\tanh(\hat{z}/d)$, $N^2(\hat{z}) = N_0^2$, where ^ indicates dimensional variables Introducing d, U_0, d/U_0 and $N_0^2 d$ as units of length, speed, time and buoyancy, the system of equations in the Boussinesq approximation is written in dimensionless form:

$$(\partial_t + U\,\partial_x)\,\Delta\psi - d^2U/dz^2\,\partial_x\psi + J\,\partial_x\varphi + a\,\Delta\psi = +\{\psi,\Delta\psi\} \quad (1)$$

$$(\partial_t + U\,\partial_x)\,\varphi - \partial_x\psi + b\,\varphi = +\{\psi,\varphi\} \quad (2)$$

Where ∂_i stands for partial derivative whith respect to the variable i, Δ stands for the Laplacian $\Delta\psi = \partial_{xx}\psi + \partial_{zz}\psi$ ψ and φ are the stream function ($\partial_z\psi=u$, $\partial_x\psi=-w$) and the buoyancy force associated to the disturbance, $\{f,g\} = \partial_x f\,\partial_z g - \partial_z f\,\partial_x g$ is used to express the nonlinear terms, U is the normalized initial mean wind and J is the initial minimum Richardson number $J= N_0^2 d^2 / U_0^2$ The initial flow is weakly unstable: $\mu = (0\ 25-J) \ll 1$
The boundary conditions are $\psi \to 0$ for $z \to \pm\infty$ They are numerically simulated by introducing sponge layers at the top and at the bottom of the field In the horizontal direction, the system is periodic

To describe the instability, the disturbance is decomposed as the sum of a disturbance of the horizontal mean field and a disturbance for which the horizontal mean is zero

$$\varphi(x,z,t) = \bar{\varphi}(z,\tau) + \sum_{l\neq 0,\,-\infty}^{+\infty} \varphi_l(z,\tau)\, e^{i l k(x-ct)} \quad (3)$$

Here $\tau=\mu t$ is a long time scale The fundamental mode (l=1) of the system is the most unstable one ($k^2=0\ 5$, c=0, /4/), and it reaches a critical level at z=0. The distortion of the mean flow and the secondary modes (l=2,3, .) are nonlinearly generated by this fundamental mode Note that the onset of such unstable modes has often been observed in realistic atmospheres (see for instance /5/)

Here we shall consider the nonlinear evolution of the most unstable mode in a flow for which J=0 22, a/k=0 02 and for three different values of the Newtownian cooling b/k=0 01 , 0 02 , 0 04

To initialize the system, a disturbance with horizontal wavenumber k is introduced Later, the most unstable mode associated with that wavenumber begins to dominate the solution. When transient effects associated with the initialization have disappeared, the growth rate of the instability is constant if the amplitude of the mode is small enough to make the nonlinear terms negligible This is the starting point of the results we shall present Thereafter, since the unstable mode continues to grow, nonlinear effects begin to be significant

LINEAR RESULTS

As long as the amplitude of the mode is small, its growth rate γ is the linear one In that case, it is observed that in the three different experiments, the value of γ_{lin} slightly increases when the Prantl number decreases This effect has already been noticed in /3/. It is small for small dissipation rates and we have found that $\gamma_{lin}/k \simeq 0.0083$ in all three experiments to a good approximation. Note that we verify $\gamma_{lin}<\max(a,b)$, so that the dissipation mechanisms control the critical layer interaction.

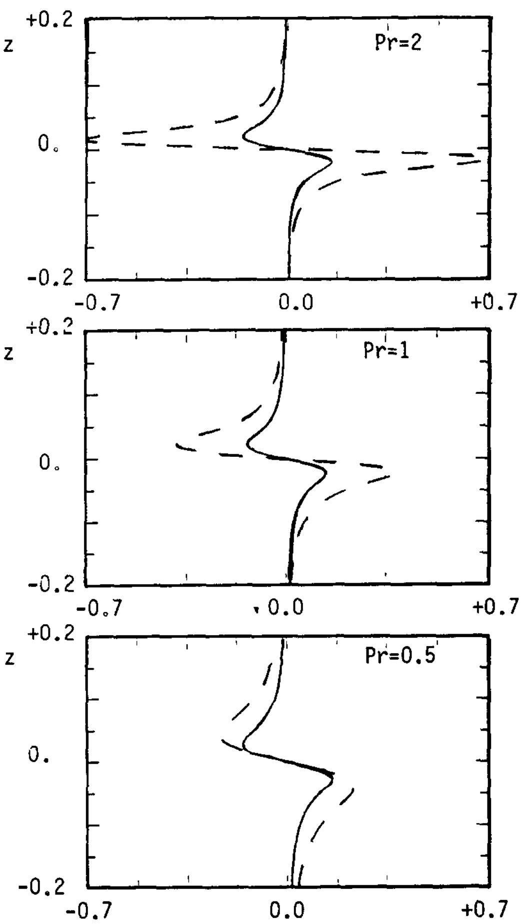
From the structure of the fundamental mode obtained from those simulations, and as long as the situation remains linear, it is possible to analyse the action of the fundamental mode of the wave on the mean flow:

$$\partial_t \bar{u} + a\,\bar{u} \simeq (2\gamma + a)\,\bar{u} \simeq -\tfrac{1}{2}\,\mathrm{Real}(d(w_1^* u_1)/dz) \qquad (4)$$

$$\partial_t \bar{\varphi} + b\,\bar{\varphi} \simeq (2\gamma + b)\,\bar{\varphi} \simeq -\tfrac{1}{2}\,\mathrm{Real}(d(w_1^* \varphi_1)/dz) \qquad (5)$$

Furthermore, the modification of the stability of the mean flow through the action of the wave in the vicinity of the critical level is given by :

$$(\delta Ri) \simeq J\,(d\bar{\varphi}/dz - 2\,d\bar{u}/dz) \text{ for } z\simeq 0 \qquad (6)$$



$-\frac{1}{2}\mathrm{Real}(d(w_1^* u_1)/dz)$ (——)

$-\frac{1}{2}\mathrm{Real}(d(w_1^* \varphi_1)/dz)$ (— —)

Figure 1: Action of the fundamental mode of the instability on the horizontal mean wind and the horizontal mean stratification as a function of z for various Prandtl number The unchanged parameters of the simulations are J=0.22, k^2=0 5, c=0 and a/k = 0 02

Figure 1 displays the vertical structure of these forcing terms as a function of z and for different values of the Prandtl number In the three experiments, it is observed that the wave tends to take horizontal momentum and buoyancy from the mean flow above the critical level (z=0) and to restore the same quantity below. Thus, it tends to decrease both $d\bar{u}/dz$ and $d\bar{\varphi}/dz$ in the vicinity of the critical level According to (6), it appears that near the critical level, **the wave stabilizes the mean flow by decreasing the mean shear, whilst at the same time, the wave destabilizes it by decreasing the mean stratification.**
For Pr=1 (Figure 1b), comparing the shears of the action of the wave on the mean horizontal velocity and on the mean buoyancy, it is observed that

$$-\mathrm{Real}(d^2(w_1^* u_1)/dz^2) \simeq -\tfrac{1}{2}\,\mathrm{Real}(d^2(w_1^*\varphi_1)/dz^2) < 0 \text{ for } z\simeq 0 \tag{7}$$

Furthermore, since b=a, the associated changes in $d\bar{\varphi}/dz$ and $d\bar{u}/dz$ are similarly damped and we can presume that:

$$(\delta Ri) \simeq J\,(d\bar{\varphi}/dz - 2\,d\bar{u}/dz) \simeq 0 \quad \text{for } z\simeq 0. \tag{8}$$

For b>a (Pr<1) (figure 1c) the thermal exchanges occur on a thicker layer than the exchanges of momentum Thus, the associated shear of the action of the wave on the mean buoyancy is larger than in the preceeding case and it is found that

$$-\mathrm{Real}(d^2(w_1^* u_1)/dz^2) < -\tfrac{1}{2}\,\mathrm{Real}(d^2(w_1^*\varphi_1)/dz^2) < 0 \text{ for } z\simeq 0 \tag{9}$$

Furthermore, since b>a, the associated changes in $d\bar{\varphi}/dz$ are more damped than those of $d\bar{u}/dz$. This effect adds to the preceding one and it is presumed that the mean flow will be stabilized in the vicinity of the critical level

$$(\delta Ri) \simeq J\,(d\bar{\varphi}/dz - 2\,d\bar{u}/dz) > 0 \quad \text{for } z\simeq 0. \tag{10}$$

For, b<a (figure 1a) opposite effects occur.

NONLINEAR RESULTS

Figure 2 displays the temporal evolution of the nonlinear growth rate of the mode under study as compared to the linear one for various Prandtl numbers We find again that the disturbance is more stable in the nonlinear case than in the linear one if Pr<1 For Pr>1, it is more unstable, while for Pr=1, as compared to the two other experiments, it is observed that the nonlinear growth rate remains closer to the linear On that figure, choosing characteristic scales of the mean flow representative of those existing in the real atmosphere, the evolution of the unstable mode is also represented using dimensional variables

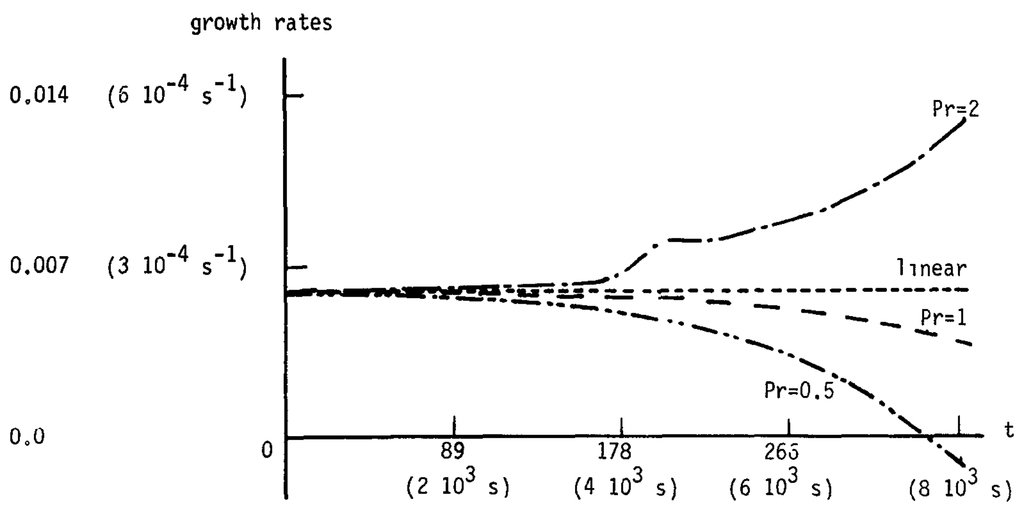
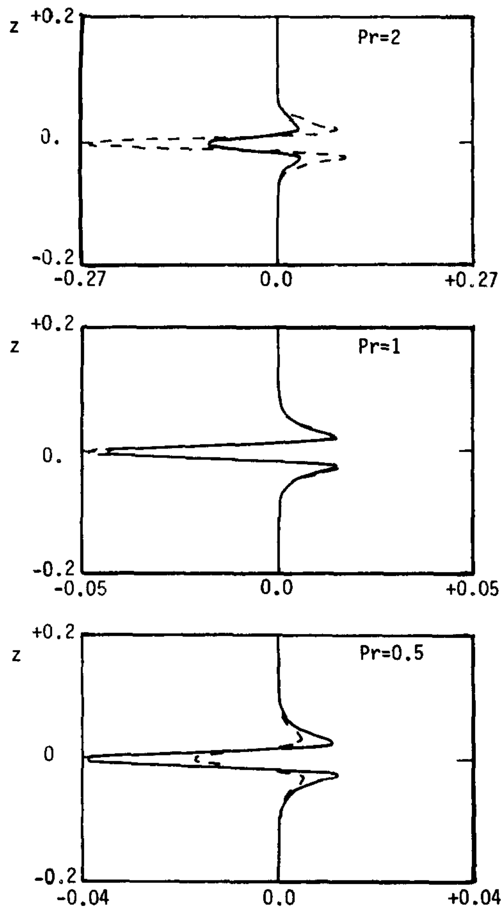


Figure 2: Nonlinear evolution of the growth rate of the instability for various Prandtl numbers. The unchanged parameters of the simulations are. J=0 22, k^2=0 5, c=0 and a/k = 0 02 Introducing U_0=10ms^{-1} and N_0^2=4 10^{-4}s^{-2} , the dimensional parameters of that simulation are: d=234m, $\hat{k}$=3 10^{-3}m^{-1}, $\hat{a}$=6 10^{-5} s^{-1}

Figure 3 represents the changes in $2d\bar{u}/dz$ and $d\bar{\varphi}/dz$ after 20 periods (t=178) of the calculations. Confirming the preceding analysis, it is observed that the stabilization of the unstable mode for Pr<1 is linked to a stabilisation of the mean flow in the vicinity of the critical level (figure 3c) since $2d\bar{u}/dz < d\bar{\varphi}/dz$ there For Pr=1, both modifications of $2d\bar{u}/dz$ and $d\bar{\varphi}/dz$ are close to each other (figure 3b) while for Pr>1, the mean flow is destabilised near the critical level since $2d\bar{u}/dz > d\bar{\varphi}/dz$ there (figure 3c)



$2d\bar{u}/dz$ (——) $d\bar{\varphi}/dz$ (— —)

Figure 3: Nonlinear modification of the mean wind shear and the mean stratification at t=178 The unchanged parameters of the simulations are J=0 22, k^2=0 5, c=0 and a/k = 0 02

The analysis presented here does not involve the incidence of secondary modes (φ_2, φ_3,) on the nonlinear evolution Nevertheless, comparing fully nonlinear simulations to quasilinear ones (i e , simulations for which those secondary modes are filtered away) it has been observed that their influence is weak compared to that of the modification of the stability of the mean flow (as long as the interaction is weakly nonlinear) Similar results have been obtained in /1/, although in that case the dissipation mechanisms were viscosity and thermal conduction

CONCLUSION

The influence of Newtownian cooling and Rayleigh friction on the development of a Kelvin-Helmholtz instability in a stratified shear layer strongly depends on the amplitude of the disturbance As long as it is small enough to be linear, a decrease in the Prandtl number a/b weakly destabilizes the flow (it increases the growth rate of the most unstable mode) When the nonlinear effects become significant, and when the damping dominates the unsteadiness of the unstable mode, a decrease in the Prandtl number nonlinearly stabilizes the evolution of that mode

Note that these nonlinear properties have to be considered in the discussions concerning the onset of (linear) instabilities when the Richardson number is everywhere larger than 0 25 and when there is dissipation /6/ and /7/ In reality, these modes generally exist at small Prandtl number. In fact, for at least some of those instabilities, it has been observed that their amplitude remains very small because of the nonlinearities

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