A stochastic parameterization of non-orographic gravity waves: Formalism and impact on the equatorial stratosphere

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[1] A formalism is proposed to represent a broadband spectrum of Gravity Waves (GWs) via the superposition of a large ensemble of statistically independent monochromatic ones. To produce this large ensemble at a reasonable numerical cost, we use the fact that the life cycles of the waves needed to be parameterized in General Circulation Models (GCMs) have time scales that largely exceed the time step of the model. We can therefore launch few waves with characteristics chosen randomly at each time step, and make them having an effect on a longer time scale by applying an AR1 relation between the gravity waves drag at a given time and that at the next time step. The stochastic GW parameterization is applied to a GCM in the tropics, and its additional drag causes a realistic Quasi-Biennial Oscillation (QBO). The more realistic wind structure also results in a better representation of the large scale equatorial waves, like the Rossby Gravity Waves (RGWs) with periods around 4–5 day. **Citation:** Lott, F., L. Guez, and P. Maury (2012), A stochastic parameterization of non-orographic gravity waves: Formalism and impact on the equatorial stratosphere, *Geophys. Res. Lett.*, *39*, L06807, doi:10.1029/2012GL051001.

1. Introduction

[2] The fact that Numerical Weather Prediction Models (NWPMs) and General Circulation Models (GCMs) need stochastic parameterizations of the physical processes that occur at the subgrid scales is now well established [*Palmer et al.*, 2005]. A practical motivation is that without stochastic parameterizations, the multiple forecasts done with NWPMs to make ensemble predictions do not spread enough. There is also more physical reasons: the stochastic parameterizations represent better the unpredictable aspects of the subgrid-scale dynamics. As an illustration, the fact that the GWs observed in situ in the low stratosphere are very intermittent [*Hertzog et al.*, 2008], justifies the introduction of stochastic effects in the non-orographic GW parameterizations done by *Piani et al.* [2004] and *Eckermann* [2011].

[3] In the work by *Piani et al.* [2004] the inclusion of stochastic effects is done on the Doppler spread parameterization of *Hines* [1997], which belongs to the GW schemes that treat globally and at each time an entire spectrum of GWs, a technique that aims to better take into account the nonlinear nature of the GWs breaking. They found that when the *Hines* [1997] parameterization is made stochastic, the model simulates better the QBO: the stochastic approaches can also help to improve individual climate simulations. *Eckermann* [2011], introduced stochastic effects in the parameterization summarized by *Garcia et al.* [2007], and which is a "multiwave" parameterization, in the sense that it represents the GW field as the superposition of independent monochromatic GWs. The method used by *Eckermann* [2011] consists in treating only one GW at each "physical" time step, and by choosing its amplitude and spectral characteristics randomly. With this technique, the multiwaves schemes become more computationally efficient, a clear progress since these schemes need to take into account a large number of GWs.

[4] Nevertheless, the *Eckermann* [2011] approach has a conceptual defect. It considers that each GW acts during one model "physical" time step only, that is often less than one period of the GW considered, and certainly much less than the lifetime of GW packets. This undermines the time-Fourier analysis which is at the basis of the parameterization of non-orographic GWs. *Eckermann* [2011] also noticed that his method can have the more practical defect of producing grid-scale noise. Also, and may be because they were too expansive before being made stochastic the multiwave parameterizations have not been tested in the context of the simulation of the QBO. This is an important issue since the QBO dynamics involves critical levels interactions between GWs and the large-scale flow, and we know that this critical layer dynamics can necessitate a very good spectral resolution to be well solved [*Martin and Lott*, 2007].

[5] Since the late 1990's, there have been many dedicated GCM simulations that produce QBO-type oscillations [see, *Takahashi*, 1999], and some climate models now routinely produces a realistic QBO [*Scaife et al.*, 2000; *Giorgetta et al.*, 2002]. According to these papers, two key factors are at least needed, namely a sufficiently good vertical resolution, and a parameterization of the non-orographic gravity waves supposedly triggered by convection. Accordingly, if we want others to adopt our proposed stochastic GW parameterization, a good motivation is the demonstration that it can help a GCM to simulate a QBO. On top of the QBO, it is also important to measure the impact of the GW parameterization on the resolved waves. One obvious reason is that these waves also contribute to the QBO dynamics [*Holton and Lindzen*, 1972], a second important reason is that the gravest among those waves, like the near 10 day Kelvin waves and the 4–5 day RGWs dominate the tropical variability of the stratosphere at the synoptic scales.

[6] The purpose of the present paper is to give a formalism that generalizes the stochastic method of *Eckermann* [2011], and in order that at each time step and at each place a multiwaves parameterization can represent a very large number of waves at a very low cost. We will then show that the parameterization we propose can help to produce a QBO,

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and to improve the explicit simulation of the gravest equatorial waves.

2. General Formalism

[7] To build a non-orographic GW parameterization, we generally assume that the disturbances with horizontal scales below some subgrid scales Δx, Δy can not be explicitly solved by the model, and need to be parameterized. GW theory also indicates that these disturbances have life cycles which duration Δt can be around 1 day. This rough estimate is an approximate time scale measuring for instance the time of travel of a mid-frequency GW through the neutral atmosphere. This is also roughly the characteristic time scale of the peaks in GW drag measured during field experiments [*Scavuzzo et al.*, 1998]. Therefore, it seems a priori reasonable to represent the unresolved GWs at each grid point by a spectrum specified via a time versus horizontal space triple discrete Fourier series over the subgrid scale volume $\Delta x \Delta y \Delta t$. In reality however, none of these scales is well known: if Δx and Δy are eventually comparable to the model grid scales δx and δy, the temporal scale Δt can largely exceeds the time step δt of the model. This, added to the uncertainties about the mesoscale dynamics that produces the waves, tell that a stochastic formalism is more adapted than pure Fourier series. We will therefore consider that at each time t the vertical velocity field can be represented by a sum of GWs w'_n,

$$w' = \sum_{n=1}^{\infty} C_n w'_n, \tag{1}$$

where the C_n's are normalization coefficients such that

$$\sum_{n=1}^{\infty} C_n^2 = 1. \tag{2}$$

Up to this point, this representation is very near the Fourier formalism, which can be recovered by choosing suitably the w'_n. Nevertheless, and for the reason mentioned before, we will partly chose them randomly.

[8] We will then assume that each of the w'_n's are independent realizations so they can be treated independently one from the others. This last assumption is probably adapted when there are critical levels, since the linear dynamics predicts quite well what occurs near them. It is less adapted when the GWs break far from critical levels because this is a very nonlinear process. We will come back to this point in the conclusion. Once the realizations in (1) are considered independent, each C_n^2 can be viewed as the probability that the wave field is given by the GW w'_n. This generalizes the intermittency coefficient ϵ introduced by *Alexander and Dunkerton* [1999].

[9] In the following, we apply this formalism to a very simple multiwave parameterization. To specify the w'_n we actually consider monochromatic waves,

$$w'_n = \Re\left\{\hat{w}_n(z) e^{z/2H} e^{i(k_n x + l_n y - \omega_n t)}\right\}, \tag{3}$$

where the wavenumbers k_n, l_n, and frequency ω_n are chosen randomly. In (3), $H = 7$ km is a middle atmosphere characteristic vertical scale and z is the log-pressure altitude $z = Hln(P_r/P)$, where $P_r = 1023$ mb. To evaluate $\hat{w}_n$, we will impose its amplitude randomly at a given launching altitude z_0, $\hat{w}_n(z_0)$, and then iterate from one model level, z_1, to the next, z_2, by a WKB approximation,

$$\hat{w}(z_2) = \hat{w}(z_1)\sqrt{\frac{m(z_1)}{m(z_2)}} e^{-i\int_{z_1}^{z_2}\left(m(z) - i\frac{\mu m^3}{\rho\Omega}\right)dz}. \tag{4}$$

In (4), we have dropped the n-index for conciseness, m is a vertical wavenumber, and the minus sign in the exponential ensures that the wave propagates upward. Still in (4), we have also introduced explicitly a constant vertical viscosity μ acting on the GWs only. It controls the GW drag vertical distribution near the model top.Actually, the efficiency of the dissipative attenuation in (4) is related to the kinematic viscosity $\nu = \mu/\rho$: it increases rapidly with altitude since ρ is the density $\rho = \rho_r e^{z/2H}$ where, $\rho_r = 1$ kgm^{-3}. We have also made the Hydrostatic approximation and we will take the WKB non-rotating approximation for m in the limit $H \rightarrow \infty$,

$$m = \frac{N||\vec{k}||}{\Omega}. \tag{5}$$

In (5), $\Omega = \omega - \vec{k}\cdot\vec{u}$ is the intrinsic frequency, $\vec{u}$ the large-scale horizontal wind, and N the Brunt-Vaisala frequency. We then follow *Lindzen* [1981] and limit the prediction in (4) to amplitudes which do not exceed the breaking amplitude w_s,

$$||\hat{w}|| \leq \hat{w}_s = \frac{\Omega^2}{||\vec{k}||N} e^{-z/2H} S_c \frac{k^*}{||\vec{k}||} \tag{6}$$

or to $\hat{w} = 0$ when Ω changes sign to treat critical levels. In (6) the amplitude w_s is that beyond which the waves convectively overturn, and the term $k^*/||\vec{k}||$ on the right, is to take into account that each of the individual waves is not supposed to occupy the entire domain $\Delta x \Delta y$, but only a fraction of it. We consider that this fraction is related to the ratio between a minimum horizontal wavenumber (for instance $k^* \approx 1/\sqrt{\Delta x \Delta y}$), and the wave horizontal wavenumber amplitude $||\vec{k}||$. From the WKB expression (4) and the polarization relation between $\hat{u}$ and $\hat{w}$ (not shown) we can deduce the EP flux:

$$\vec{F}^z(k, l, \omega) = \Re\left\{\rho_r \frac{\vec{\hat{u}}\hat{w}^*}{2}\right\} = \rho_r \frac{\vec{k}}{2||\vec{k}||^2} m(z)||\hat{w}(z)||^2. \tag{7}$$

It does not vary with altitude if we take for $\hat{w}$ its WKB approximation in (4), but varies if we take the saturated value in (6). To treat a large number of waves at a given time t, we launch at each time step δt a finite number of waves M, and compute the tendencies due to them, $\rho^{-1}\partial_z \vec{F}^z_{n'}$, where $n' = 1, M$. As they are independent realizations the averaged tendency they produce is the average of these M tendencies. We then redistribute this averaged tendency over the longer time scale Δt by first rescaling it by $\delta t/\Delta t$ and second by using a lag-one Auto Regressive (AR-1) relation between the GW tendencies at two successive time steps:

$$(\partial_t \vec{u})_W^{t+\delta t} = \frac{\delta t}{\Delta t}\sum_{n'=1}^{M} \frac{\left(\partial_z \vec{F}^z_{n'}\right)^t}{M\rho} + \frac{\Delta t - \delta t}{\delta t}(\partial_t \vec{u})_W^t. \tag{8}$$

In other words, and at each time step, we promote M new waves by giving them the largest probability to represent the GW field, and degrade the probabilities of all the others by the multiplicative factor $(\Delta t - \delta t)/\Delta t$. If we express the cumulative sum underneath the AR-1 relation in (8), we recover the formalism for stochastic waves infinite superposition in (1) by taking

$$C_n^2 = \left(\frac{\Delta t - \delta t}{\Delta t}\right)^p \frac{\delta t}{M \Delta t}, \tag{9}$$

where p is the nearest integer that rounds $(n - 1)/M$ towards the left.

3. Implementation and Impact on the Zonal Mean Flow

[10] To test the parameterization in the context of the simulation of the QBO, we take the stratospheric version of the LMDz-GCM [*Lott et al.*, 2005] on its 1.875° by 3.75° horizontal grid and extent its vertical resolution from 50 levels up to 80 levels. In this new configuration, the model top is still near z = 65 km, but the vertical resolution is around 600 m between z = 15 km and z = 35 k, instead of being between 1 km and 2 km in *Lott et al.* [2005]. Note that the model version in the work by *Lott et al.* [2005] has the GW parameterization due to *Hines* [1997], and the orographic GW parameterization described by *Lott* [1999]. Both are left unchanged since they largely improve the model performances in the midlatitudes. Note that this 80 level version, do not simulate a QBO (not shown), but has a midlatitude climate which is very near that in *Lott et al.* [2005] (not shown).

[11] The fact that our model does not simulate a QBO can be due to the fact that the *Hines* [1997] GW scheme we use launches everywhere and at every time the same amount of waves: there is no enhanced GW fluxes in the regions where there is intense subgrid-scale convection, as often occurs in the tropics. For this reason, and also because we do not want to loose the benefits of prior tunings of the other GW schemes, we add to the model the stochastic GWs described in Section 2, but limit their influence to the tropical regions. More precisely, and for each GW introduced, the parameters are chosen randomly according to the following rules. First, the amplitude of the wave stress $||\vec{F}^z||$ is chosen randomly between 0 and F_M^z = 10 mPa and is imposed at the launching altitude P = 800 hPa. The GW flux is then limited to the equatorial regions by multiply it with a $\cos^8\phi$ taper where ϕ is the latitude. Second, the the horizontal wavenumber amplitude is chosen randomly between $k^* < ||\vec{k}|| < k_s$ with $k_s = 1.\text{km}^{-1}$ and $k^* = 0.01\,\text{km}^{-1}$, and the phase velocity amplitude $C = \omega/k$ is chosen randomly between $C_m < |C| < C_M$, where C_m = 1 m/s and C_M = 30 m/s. In all these random choices, the probability to pick a particular value within the bounds given is constant. We also chose randomly the direction of propagation of the waves but impose $l = 0$. For the other parameters, we take $S_c = 0.75$, and launch M = 8 waves per grid points each physical time step δt = 30 mn. For this value of M, the stochastic GW scheme is as fast as the *Hines* [1997] scheme. We also take for the characteristic time scale of the life-cycle of the waves Δt = 1 day, which means that around $M \times \Delta t/\delta t \approx 400$ monochromatic waves contribute to the wave field each day and at a given horizontal grid point. For the viscosity, we take $\mu/\rho_r = 0.15\,\text{km}^2\,\text{day}^{-1}$, and we verify that for this value the effect of μ on the GW tendency in the QBO region is quite small.

[12] The results for the zonal wind shown in Figure 1 are from an experiment which follows a spin-up period of 3 years where the gravity wave amplitude was slightly larger than F_M^z = 10 mPa and the QBO period around 18 months (not shown). We see that in the model, the QBO period rapidly established at 24 months, that is around 4 months faster than in observations. The simulated eastward winds of the QBO reach 15 m/s typically whereas the westward winds typically reach 30 m/s. This asymmetry is a typical feature of the observed QBO. Also, the descent of the eastward wind is more regular than that of the westward wind, again consistent with observations.

[13] In terms of zonal mean climatology, we have also verified that our scheme does not modify very substantially the zonal mean zonal wind and Temperature in the midlatitudes and polar regions (not shown). To evaluate the grid scale-noise production, we have proceeded to a time longitude spectral analysis of the tendency $(\partial_t \vec{u})_W^t$ stored during one month at each time step and at each place in the QBO region. We have then compared the spectra to the spectra obtained when the two parameters M = 1 and $\Delta t = \delta t$, that is when the scheme considers only one GWs per grid point each time step (as in the work of *Eckermann* [2011]). With M = 1 and $\Delta t = \delta t$ the spectrum of the GWs tendency has very substantial power near the zonal truncature s = 48 and the Nyquist frequency $2/\delta t$, respectively. This injection of variabilities by the GWs scheme at the smallest scales of the model is largely absent when M = 8, Δt = 1 day (not shown).

4. Impact on the Large-Scale Waves

[14] To evaluate the large-scale equatorial waves, we next follow the method described and applied to re-analysis data from *Lott et al.* [2009] and applied to model data from *Maury et al.* [2012]. For the RGWs, the method is based on the very simple theoretical fact that the meridional wind v at a given longitude is of uniform sign when the latitude varies. Therefore, a frequency versus zonal wavenumber spectral analysis of v, averaged between 10°S and 10°N (hereinafter $\langle v \rangle$), reveals the spectral domain where these waves make the largest contribution (not shown but see *Lott et al.* [2009, Figure 1c]). Typically and in the low stratosphere $z \approx 21$ km, the periods are around 4–6 days and the wavenumbers between s = 2 and s = 10.

[15] We then design a frequency-wavenumber band pass filter that largely bound the spectral domain of the RGWs. In the following, we take the filter for RGWs given by *Lott et al.* [2009, Figure 2] and applied it to all fields. We then use the filtered values of $\langle v \rangle$ and select the dates when its maximum amplitude as a function of longitude exceeds a given threshold. The threshold is chosen so that for each map, around one date every year is selected.

[16] The composite map built by averaging the filtered fields over the selected days are shown in Figure 2. The composite from the ERAI-reanalysis in Figure 2a is that of a characteristic RGWs packet, it corresponds to meridional velocities fluctuations of about 5 m/s and Temperature variations of about 0.5 K. Also the meridional wind is symmetric with respect to the equator, whereas the zonal

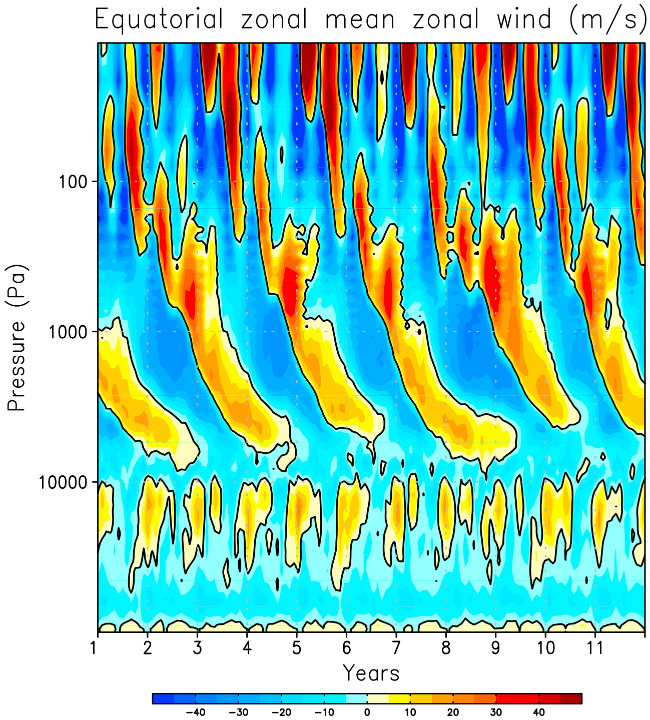

Figure 1. Zonal mean zonal wind averaged between 5S and 5 N, and from the LMDz-GCM with 80 vertical levels and with the stochastic GW parameterization.

wind and the Temperature fields are asymmetric, as expected from RGWs theory. It is important to recall here that our method to extract the RGWs, naturally select the dates when the QBO winds are positive in the low stratosphere (not shown but see *Lott et al.* [2009, Figure 5a]). In this circumstance the intrinsic phase speed of the waves is quite large in amplitude, since the large-scale RGWs have negative absolute phase speed, which tends to decrease their vertical wavenumber and hence their dissipation.

[17] In the model without the stochastic GWs in Figure 2b, the amplitude of the fields are smaller than in the reanalysis, and their equatorial symmetries are in good part lost. These errors are probably due to the fact that the zonal mean zonal wind is essentially westward in the model stratosphere and at all time (not shown). Hence, the RGWs in the model have an intrinsic phase speed which is much reduced in amplitude, which tends to increase the waves vertical wavenumber and their dissipation. With the stochastic GWs in Figure 2c, the RGWs are now much more realistic in amplitude. Also, the patterns of winds and Temperature have the expected symmetries with respect to the Equator. These improvements follow that during the dates selected to build the composite, the zonal mean zonal wind is predominantly positive at the equator (not shown) which favours the RGWs vertical propagation.

[18] A comparable analysis can be conducted for the large-scale Kelvin waves [see, *Maury et al.*, 2012]. We do not show the details here, because the results are that the GCM simulates well these Kelvin waves, and this quite independently of the stochastic GW parameterization (the Kelvin waves are as in the work by *Maury et al.* [2012]). This follows that these Kelvin waves propagate well when the zonal mean zonal wind is negative, a situation that is systematic when there is no QBO in the model, and that occurs more than half of the time when there is a QBO (as is the case in the reanalysis data set and in the GCM with stochastic GWs).

5. Discussion

[19] To use multiwaves GW parameterization at a reasonable cost [*Eckermann*, 2011] has shown that we can launch one monochromatic GW at each time step and at each place, by choosing its properties randomly. As others, we believe that this stochastic approach also has fundamental justifications, so we have tried to improve it and test it for the case of the simulation of the tropical stratosphere.

[20] The *Eckermann* [2011] approach has one conceptual weakness. It considers one wave each time step, whereas GWs can have periods and life-cycle duration that lasts much longer than that. This weakness partly undermines the time-Fourier analysis underneath the parameterization of non-orographic gravity waves. Also, launching one different GW at each time step can force substantial grid scale noise,

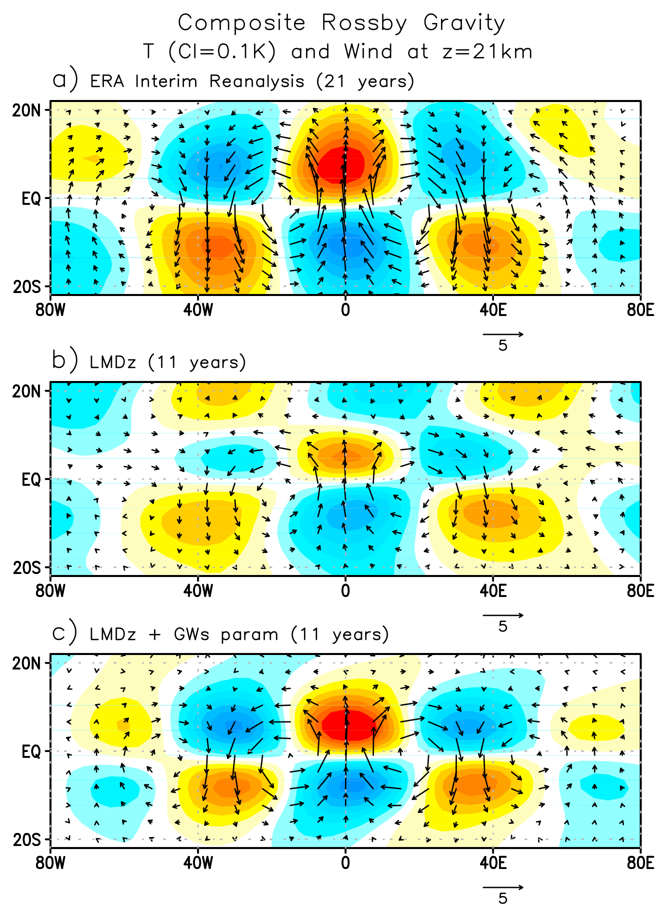

Figure 2. Composite maps of the gravest Rossby-Gravity waves packets reaching the altitude z = 21 km. (a) ERA-Interim for the period 1987–2007, 21 dates selected. (b) LMDz-GCM without the stochastic GWs, 11 dates selected out of 11 years of simulation. (c) LMDz-GCM with the stochastic GWs, 11 dates selected out of 11 years. For the Temperature fields, CI = 0.1 K, and the positive (negative) values are shown with warm (cold) colors. For the wind the arrows shown in each figures show the unit in m/s.

a defect that can become an issue if we want to look at GW effects at more regional scales. To circumvent these problems, we propose a formalism that permits to launch few monochromatic waves at each model time step δt, and to redistribute the tendencies due to these GWs over a much longer time scale (say Δt). This makes that at a given time, many different waves are acting together, which is clearly a benefit when it comes to the treatment of critical levels for instance. We have shown that a very simple multiwave parameterization build using these techniques, can be used on long integrations and in order to produce a QBO. We have also shown that the improvement on the zonal mean zonal wind also results in the improvement of the largest scale equatorial waves.

[21] The model configuration we have adopted, with the *Hines* [1997] scheme left unchanged and our new stochastic scheme applied in the tropical regions only is of course not entirely satisfactory. It is clear that in the near future we should extent the stochastic scheme to the midlatitudes regions, and tune it in order that it can also replace the *Hines* [1997] scheme.

[22] The major disadvantage of the multiwave schemes, compared to the globally spectral schemes [see, e.g., *Hines*, 1997; *Warner and McIntyre*, 1996] is that they treat breaking waves by waves, which somehow contradicts the nonlinear nature of this process. Nevertheless, we can also argue against the globally spectral schemes that the spectra they impose at each time step have to be viewed as the average over large ensembles of periodograms coming out of individual realizations. If we take into account that observations often show well defined wave packets [*Hertzog et al.*, 2008], it is likely that in reality these periodograms are quite narrowbanded. As our stochastic parameterization potentially produced such narrowbanded periodograms, it could be interesting to test if the spectra they yield when we proceed to large ensemble means could converge toward those used in the globally spectral schemes.

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