TEF, ZOOM,
model coupling, feedback analysis

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Models

appearance
physics, chemistry, biology, economy
mathematics
numerics
software

Mathematics = common language (hopefully)

- we consider models belonging to mathematics and numerics worlds: **Models = sets of equations**
- always try to exhibit links with upper layers
- (Sub-)System = model $\oplus$ variables

Sensitivity, coupling, feedback

Sensitivity = response to a change in forcings

Coupling = response of a variable φ_1 to a change of model relative to another variable φ_2

Feedback = response of a variable to the closure of a loop

Importance of partitionning into labelled submodels

Partitionning and coupling

η_A $\quad$ $\partial_t \eta_A = G_A(\eta_A, \theta_S)$

Atmosphere

θ_S

φ_S

Ocean

η_O $\quad$ $\partial_t \eta_O = G_O(\eta_O, \varphi_S)$

- For every sea surface temperature $\theta_S \longrightarrow$ one (unique) solution for η_A
- For every surface flux $\varphi_S \longrightarrow$ one (unique) solution for η_O

Transfer models = Supplementary conditions restauring the behaviour of the complete system.

Usual coupling technique in GCMs :

Usual coupling technique in GCMs :	temperature continuity:	$\theta_S = T_{S(\eta_O)}$
	flux continuity at surface:	$\varphi_S = \Psi_{S(\eta A)}$

TEF ; global structure

Two kinds of models

- cells = models relative to parts of the system (partial models ; well posed problems)
- transfers = coupling and matching models

Two sets of variables

- state variables ($\vec{\eta}_\alpha$)
- transfer variables ($\vec{\varphi}$)

Equations :

Unsteady model		steady model
$\partial_t \vec{\eta}_\alpha = \vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi})$	for each cell α	$\vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi}) = 0$
$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$	for transfers	$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$

TLS: Tangent Linear System

TEF is mainly designed to study properties of the Tangent Linear System (TLS).
TLS is obtained by linearizing equations in the vicinity of a reference state $\vec{\eta}_{ref}$.

$\delta\eta$ $\quad$ $\eta(t_0 + \delta t) = \eta(t_0) + \delta\eta$

$\eta(t_0)$ $\quad$ $\vec{\eta}_i$

$\vec{\eta}_{i+1}$

$\vec{\eta}_{i+2}$

$\eta(t) = \eta_{ref}(t) + \Delta\eta(t)$

$\Delta\eta$

$\eta_{ref}(t)$

TLS is used in three ways:

- for dynamic models and small time displacement: TLS is used in the time marching simulation procedure.
- for steady model iterative solving procedure: Newton Raphson method makes use TLS.
- for pertubation studies.

Cells

Unsteady model		steady model
$\partial_t \vec{\eta}_\alpha = \vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi})$	for each cell α	$\vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi}) = 0$

η

$$\delta\vec{\eta} = \vec{\eta}(t_0 + \delta t) - \vec{\eta}(t_0)$$

$$\delta\vec{\varphi} = \vec{\varphi}(t_0 + \delta t) - \vec{\varphi}(t_0)$$

$\eta_\alpha(t)$

$$\delta\vec{\eta}_\alpha = \delta\vec{\eta}_{\alpha,dec} + F \;.\; \delta\vec{\varphi}$$

$\eta_{\alpha,dec}(t)$

F = influence matrix

η_0

t_0

$t_f = t_0 + \delta t$

Transfers

$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$

$\vec{\eta}_\alpha(t_0 + \delta t) - \vec{\eta}_\alpha(t_0)$ is considered as a pertubation:

$\delta\vec{\varphi} = \sum_\alpha \partial_\alpha f \; . \; \delta\vec{\eta}_\alpha$

where: $\partial_\alpha f = \dfrac{\partial \vec{f}}{\partial \vec{\eta}_\alpha}$

Coupling equation: state variable elimination

Cell and transfer equations:

$$\begin{cases} \delta\vec{\eta}_\alpha = \delta\vec{\eta}_{\alpha,dec} + F \,.\, \delta\vec{\varphi} \\ \delta\vec{\varphi} = \sum_\alpha \partial_\alpha f \,.\, \delta\vec{\eta}_\alpha \end{cases} \tag{1}$$

Elimination of state variables:

$$\begin{cases} (1 - \sum_\alpha \partial_\alpha f \,.\, F_\alpha)\delta\vec{\varphi} = \delta\vec{\varphi}_{ins} \\ \vec{\varphi}_{ins} = \sum_\alpha \partial_\alpha f \,.\, \delta\vec{\eta}_{\alpha,dec} \end{cases} \tag{2}$$

- When cells are insensitive ($F_\alpha = 0$), then $\delta\vec{\varphi} = \delta\vec{\varphi}_{ins}$
- The coupling matrix $\mathcal{C} = \sum_\alpha \partial_\alpha f \,.\, F_\alpha$ represents the effect of cell feedbacks.

Time discretized feedback gains

Example : **Ocean/atmosphere thermodynamic coupling**

1D problem with 2 interface variables (sea surface temperature and heat flux).

Question: is the feedback important ? what constraint does it impose on the coupling method ?

Atm. $\vec{\eta}_A$

$$\delta\vec{\eta}_A = \delta\vec{\eta}_{A,dec}(\delta t) + \vec{F}_A(\delta t)\delta\theta$$

$\delta\theta = \partial\vec{f}_O \,.\, \delta\vec{\eta}_O$ $\quad\theta\quad$ $\quad\varphi\quad$ $\delta\varphi = \partial\vec{f}_A \,.\, \delta\vec{\eta}_A$

$$\delta\vec{\eta}_O = \delta\vec{\eta}_{O,dec}(\delta t) + \vec{F}_O(\delta t)\delta\varphi$$

Ocean $\vec{\eta}_O$

State variable elimination:

$$\begin{cases} \delta\varphi = \partial\vec{f}_A \,.\, \vec{F}_A \,\delta\theta + \partial\vec{f}_A \,.\, \delta\vec{\eta}_{A,dec} \\ \delta\theta = \partial\vec{f}_O \,.\, \vec{F}_O \,\delta\varphi + \partial\vec{f}_O \,.\, \delta\vec{\eta}_{O,dec} \end{cases} \tag{3}$$

Elimination of $\delta\varphi$

$$(1-\partial\vec{f}_O \,.\, \vec{F}_O \,\partial\vec{f}_A \,.\, \vec{F}_A)\,\delta\theta = \partial\vec{f}_O \,.\, \delta\vec{\eta}_{O,dec} + \partial\vec{f}_O \,.\, \vec{F}_O \,\partial\vec{f}_A \,.\, \delta\vec{\eta}_{A,dec} \tag{4}$$

Time discretized feedback gains - 2

$$\begin{cases} (1-g)\delta\theta = \delta\theta^*_{ins} \\ g = \partial\vec{f}_O \, . \, \vec{F}_O \; \partial\vec{f}_A \, . \, \vec{F}_A \\ \delta\theta^*_{ins} = \partial\vec{f}_O \, . \, \delta\vec{\eta}_{O,dec} + \partial\vec{f}_O \, . \, \vec{F}_O \; \partial\vec{f}_A \, . \, \delta\vec{\eta}_{A,dec} \end{cases} \tag{5}$$

F_A ∂f_A

θ φ

F_O ∂f_O

Open loop: system made insensitive to θ by setting $\vec{F}_A = 0$. Equation reads $\delta\theta = \delta\theta^*_{ins}$.
Closed loop: the gain g comes in: g describes the effect of the feedback.

Dynamic coupling analysis

(= exact solving of TLS)

Classically: linear differential system $\longrightarrow$ Laplace transform.

Here, we use Borel transform instead:

$$\mathcal{B}(f)_{(\tau)} = \tilde{f}_\tau = \frac{1}{\tau}\int_0^\infty exp(-\frac{t}{\tau})f(t)dt \tag{6}$$

Why Borel transform ? Because it is very easy to compute: any computer code, that uses Crank-Nicolson scheme to compute $\delta\varphi_{CN}(\delta t)$ (such as ZOOM), can compute numerically $\delta\tilde{\varphi}(\tau)$,thanks to:

$$\delta\varphi_{CN}(\delta t) = 2\delta\tilde{\varphi}(\delta t/2)$$

Dynamic coupling analysis - 2

Example : **influence of high cloud cover on troposphere temperature**

$\longleftarrow$ High cloud cover H_{CC}

T $\longleftarrow$ Troposphere temperature

Elimination of all variables except H_{CC} and T.

$$(1 - \mathcal{C})\delta\vec{\varphi} = \delta\vec{\varphi}_{ins}$$

Row corresponding to T:

$$(1 - C_{11}(\tau))\delta\tilde{T}(\tau) - C_{12}(\tau)\delta\tilde{H_{CC}}(\tau) = \delta\tilde{T}_{ins}(\tau)$$

Dynamic coupling analysis - 3

T equation

$$\delta\tilde{T}(\tau) = \frac{1}{1 - C_{11}(\tau)}\delta\tilde{T}_{ins}(\tau) + \frac{C_{12}(\tau)}{1 - C_{11}(\tau)}\delta\tilde{H_{CC}}(\tau)$$

Main point: coefficients C_{1i} are of the form $\partial_\alpha f_1 ... F_{\beta j}$; they are independent of H_{CC} model ($= \partial_{alpha} f_2$). $\Rightarrow$ Influence of H_{CC} onto T is explicit. Inverting $\mathcal{B}$ yields:

$$\delta T(t) = ... + \mathcal{B}^{-1}(\frac{C_{12}}{1 - C_{11}}) * \partial_t(\delta H_{CC}(t))$$

- coupling coefficient $\mathcal{B}^{-1}(\frac{C_{12}}{1-C_{11}})$ is the response of T to a step in H_{CC} at $t = 0$.
- limit for large t = steady state coupling coeffcient
- direct simulation confirms that it takes approximately 10 years for the system to reach its equilibrium

Saliant features

apart from the fact TEF and ZOOM do perform simulations

- model splitting and coupling is an efficient modelling tool
- model splitting enables coupling and feedback analysis
- importance of studying the Tangent Linear System
- feedback gains are properties of the TLS (not effects of non-linearities)
- coupling coefficients describe the response of a variable to a change in the model corresponding to another variable
- feedback gains describe the effect of the closure of a feedback loop
- full dynamic analysis of TLS yields characteristic times as well as values of coupling coefficients and feedback gains