

Dynamic Meteorology

(WAPE: General Circulation of the Atmosphere and Synoptic Meteorology)

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3) Meridional circulations and the role of the Eddies

a) Tropospheric Hadley and Ferrel cells, Eddy-driven jets

b) “Eulerian” and “transformed Eulerian” mean formalisms

Residual circulation, Eliassen Palm fluxes

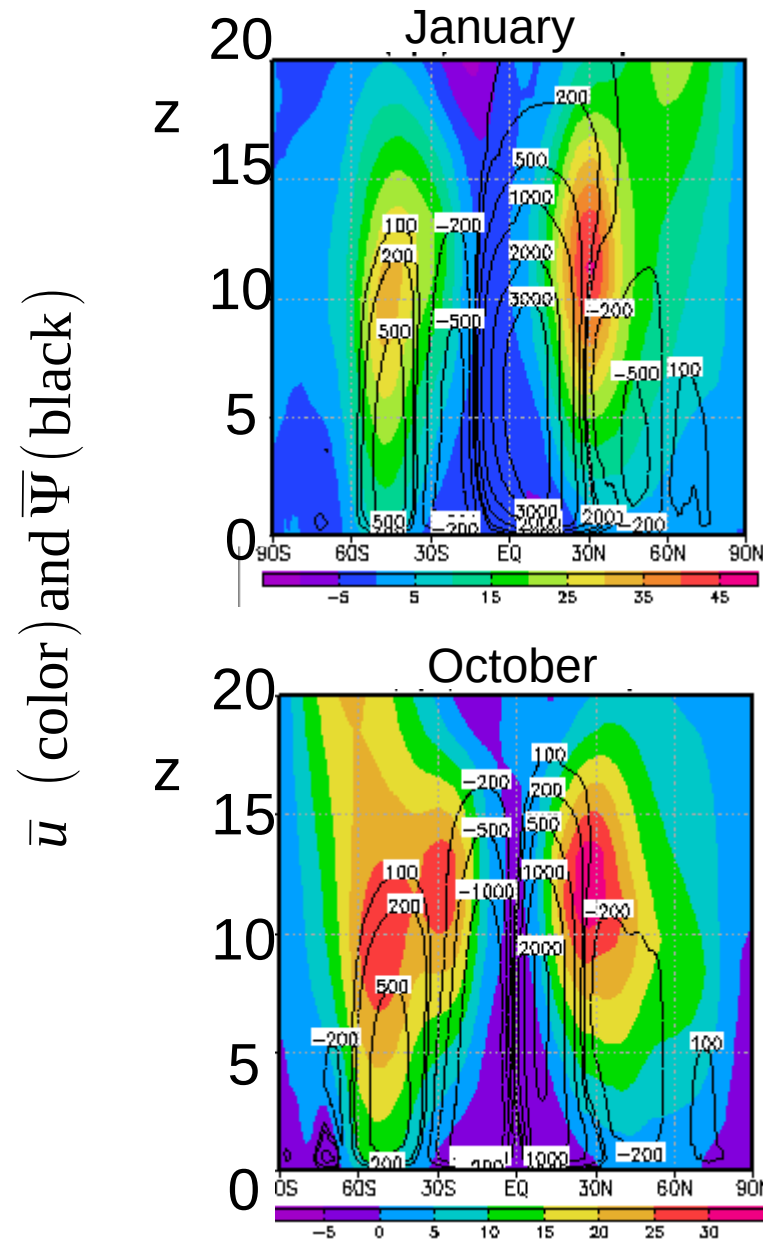
c) Middle atmosphere Brewer-Dobson circulation

Transformed Eulerian mean streamfunction

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

Zonal mean meridional circulations in the troposphere visualized with a streamfunction

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$$\bar{v} = -\frac{1}{\rho_0 \cos \phi} \frac{\partial \bar{\Psi}}{\partial z} \quad \bar{w} = +\frac{1}{\rho_0 a \cos \phi} \frac{\partial \bar{\Psi}}{\partial \phi}$$

- As expected from the shallow water model, the winds are positive at the subtropical end of the Hadley cells
- In the midlatitude regions, there seems to be a Cells which upper branch points toward the tropics, these are the Ferrel Cell
- Even poleward, there seem to be polar cells with upper branch toward the poles
- During Equinox, the Hadley and Ferrel cells are almost symmetric
- Note in the SH the presence of a secondary jet in the midlatitudes.
- Since this jet is not located at the exit of the Hadley cell, this secondary jet is “Eddy driven”.

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

Wave-mean flow interaction equations in the **Eulerian mean formalism**

Zonal mean: $\bar{u}(\varphi, z, t) = \frac{1}{2\pi} \int_0^{2\pi} u d\lambda$ Disturbance: $u'(\lambda, \varphi, z, t) = u - \bar{u}$

Angular momentum budget :

$$\bar{u}_t + \left[(a \cos \phi)^{-1} (\bar{u} \cos \phi)_\phi - f \right] \bar{v} + \bar{u}_z \bar{w} = \bar{X}$$

$-(a \cos^2 \phi)^{-1} (\overline{u'v'} \cos^2 \phi)_\phi$
 Rossby waves

$-\frac{1}{\rho_0} (\rho_0 \overline{u'w'})_z$
 Gravity waves in Midlatitudes
 Equatorial waves in the tropics

Geostrophic approximation: $f \bar{u} \approx -\frac{1}{a} \bar{\Phi}_\phi$ (midlatitudes only!)

Hydrostatic approximation : $\bar{\Phi}_z = \frac{RT}{H}$

Zonal mean meridional circulation satisfies mass conservation :

$$\frac{1}{a \cos \phi} (\bar{v} \cos \phi)_\phi + \frac{1}{\rho_0} (\rho_0 \bar{w})_z = 0$$

Thermodynamics :

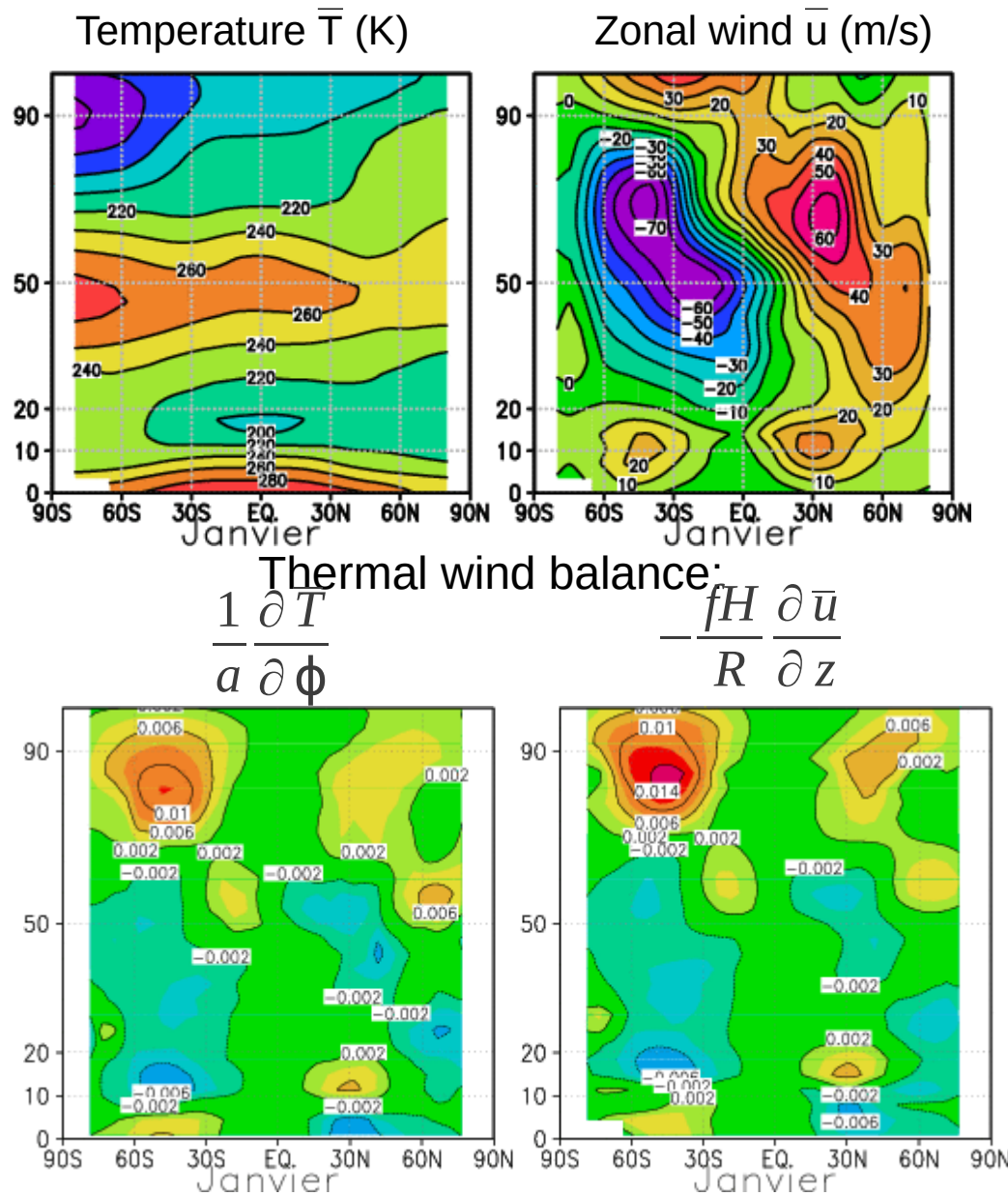
$$\bar{\theta}_t + a^{-1} \bar{v} \bar{\theta}_\phi + \bar{w} \bar{\theta}_z = \bar{Q}$$

$-(a \cos \phi)^{-1} (\overline{v'\theta'} \cos \phi)_\phi - \frac{1}{\rho_0} (\rho_0 \overline{w'\theta'})_z$
 Baroclinic instabilities (troposphere)

 Rossby waves (upper troposphere middle atmosphere)

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

Thermal wind balance between the zonal mean zonal wind and the zonal mean temperature



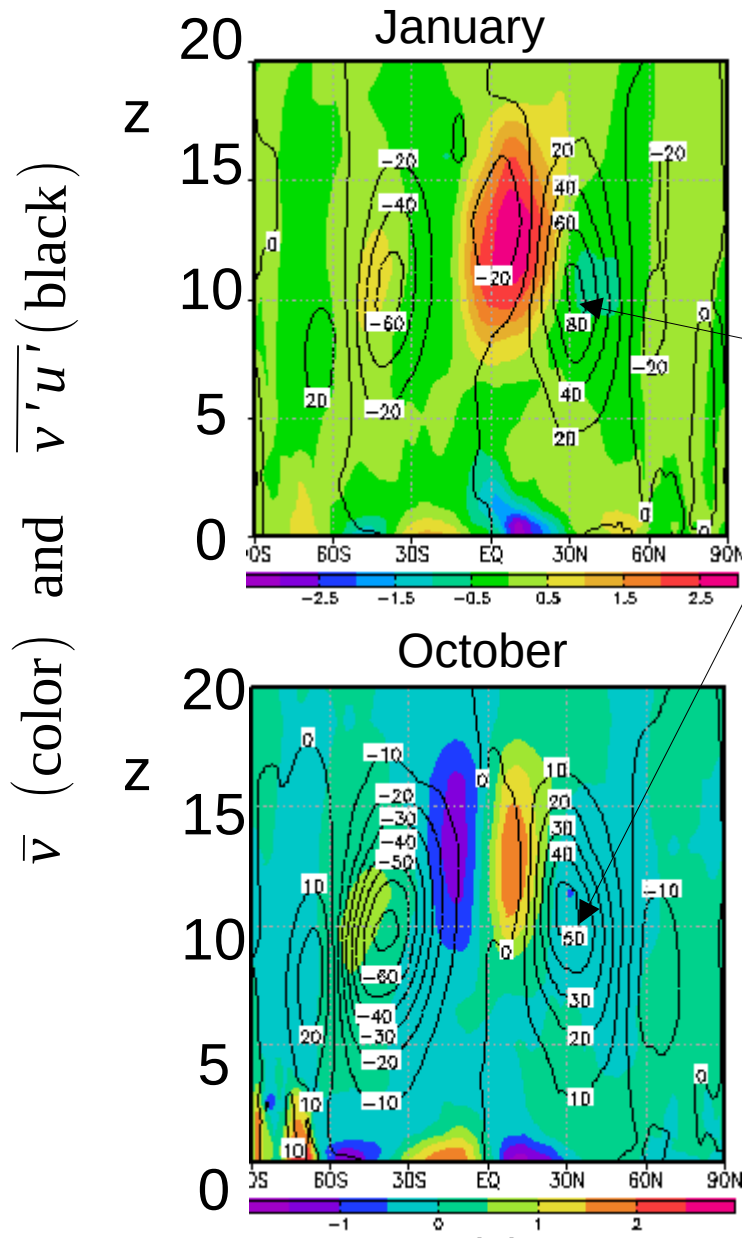
- Can we really separate the thermal and mechanical forcings?
- The thermal wind balance translates the strong links that exist in the midlatitude between dynamics and thermodynamics
- In January, the increase of T with latitude in the SH mesosphere permits the Easterly jet to close at the mesopause.s
- Imagine now that one decelerates the jet at the mesopause to close the westerlies, the Temperature gradient in latitude should be positive below to satisfy the thermal wind balance.
- We will see that these mechanical forcings by the waves are extremely important, and drive the atmosphere out of radiative equilibrium.

CIRA January climatologies

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

Momentum fluxes

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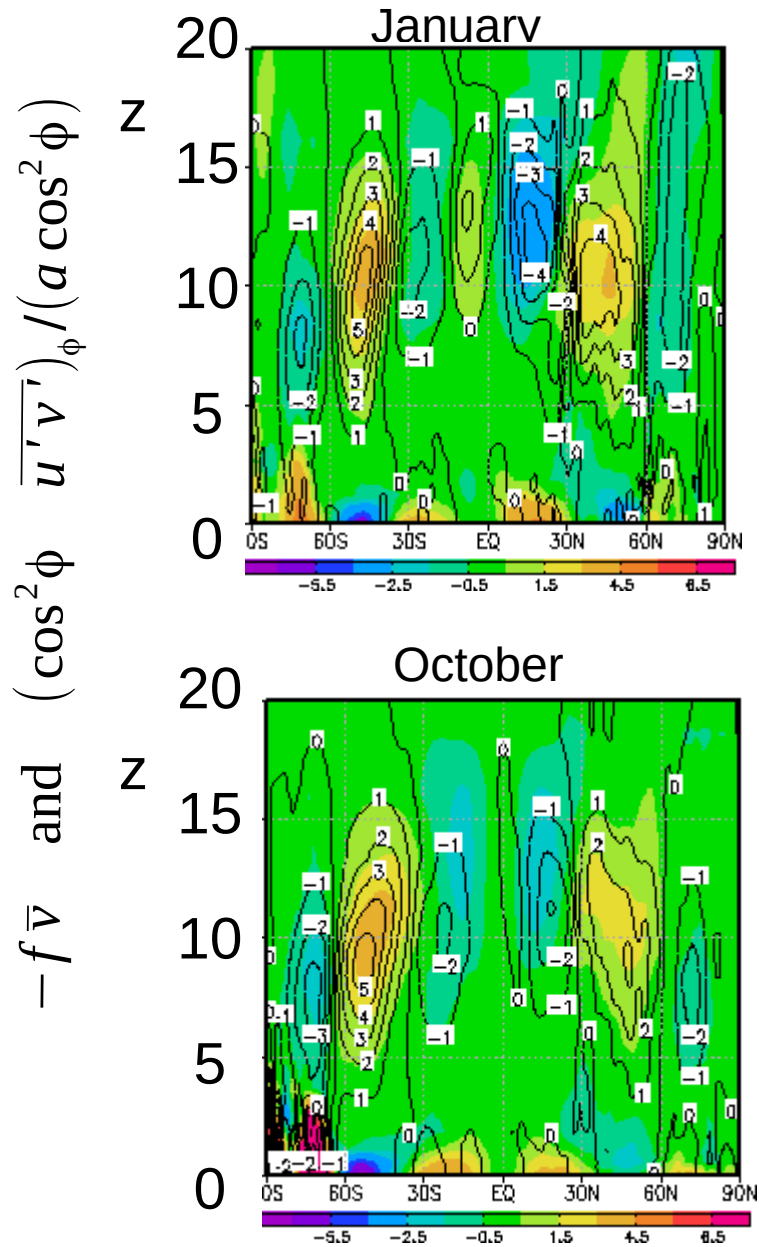
- The amplitude of the meridional wind is much less than that of the zonal wind (meridional circulations are slow)
- Note the flux of momentum due to the Eddies from the subtropics to the midlatitude in the upper troposphere

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

Momentum fluxes divergence in good part equilibrated by the Coriolis torque:

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$$-(a \cos^2 \phi)^{-1} (\overline{u'v'} \cos^2 \phi)_\phi \approx -f \bar{v}$$



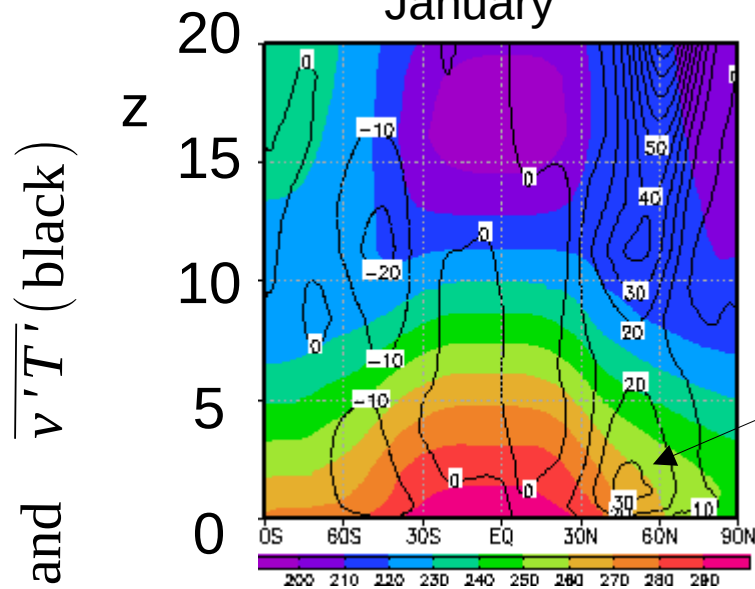
- The Hadley cell seems in good part Eady-driven.
- The Ferrel Cell is entirely related to the presence of the Eddies

a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

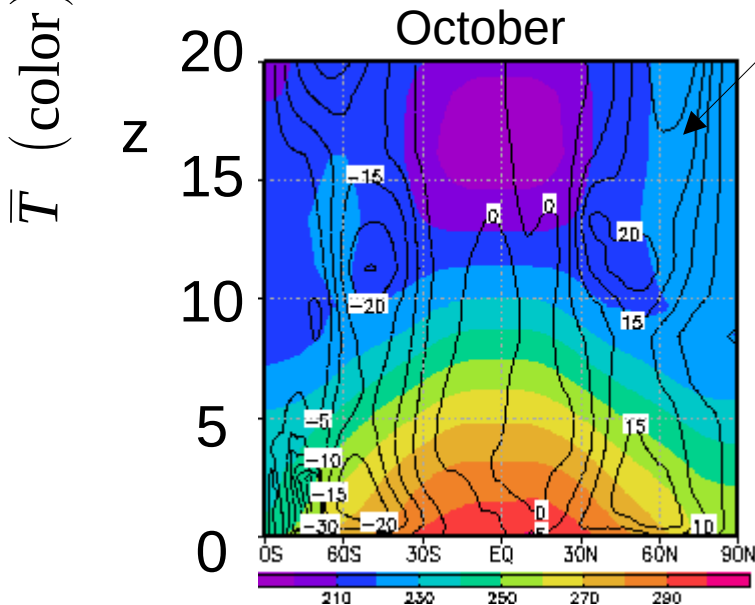
Thermal fluxes

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January



October



- Substantial poleward flux of temperature at all altitudes in the midlatitudes
- Near the surface this warms the polar regions through diabatic effects
- But there are also substantial fluxes in the upper troposphere and lower stratosphere.
- In these regions the diabatic effects are not strong

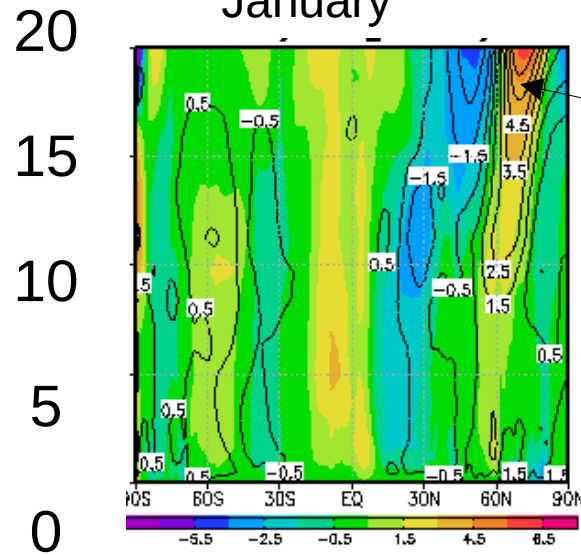
a) Tropospheric Hadley and Ferrel Cells, Eddy-driven jets

In the midlatitudes the thermal fluxes in good part equilibrated by vertical ascent

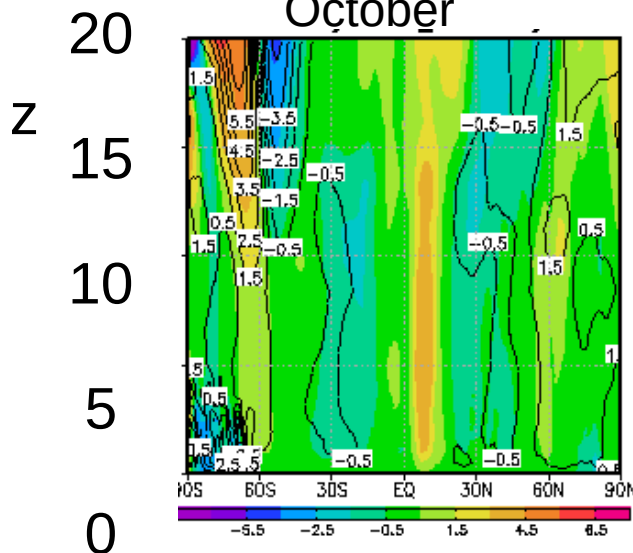
$\bar{w}\bar{\theta}_z$ (color) and $-(a\cos\phi)^{-1}(\overline{v'\theta'}\cos\phi)_\phi$ (black)

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January



October



- One sees ascent in the tropics as expected in presence of zonal mean diabatic heating

$$\bar{w}\bar{\theta}_z \approx \bar{Q}$$

- The vertical « Eulerian mean » motions in the midlatitudes is in part equilibrated by eddy heat flux divergence in the lower stratosphere

$$\bar{w}\bar{\theta}_z \approx (a\cos\phi)^{-1}(\overline{v'\theta'}\cos\phi)_\phi$$

- In midlatitudes, Eddy Heat fluxes contribute as much as the Eddy momentum fluxes to the meridional circulation.
- The fact that there are substantial Eddy fluxes (thermal and dynamical) on both momentum and thermodynamic equations, whereas the thermal wind balance enforce constraint between wind and Temperature render the separation between momentum and thermal diagnostics quite misleading to describe eddy- mean flow interactions!

b) “Transformed Eulerian” mean formalisms

Wave-mean flow interaction equations in the **Transformed Eulerian mean formalism**

**Residual
mean
Circulation :**

$$\overline{w^*} = \overline{w} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left(\cos \phi \frac{\overline{v' \theta'}}{\overline{\theta}_z} \right)$$

$$\overline{v^*} = \overline{v} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\rho_0 \frac{\overline{v' \theta'}}{\overline{\theta}_z} \right)$$

Thermodynamics :

$$\overline{\theta}_t + a^{-1} \overline{v^*} \overline{\theta}_\phi + \overline{w^*} \overline{\theta}_z = \overline{Q}$$

$$- \frac{1}{\rho_0} \left(\rho_0 \overline{v' \theta'} \frac{\overline{\theta}_\phi}{a \overline{\theta}_z} + \rho_0 \overline{w' \theta'} \right)_z$$

Very small, at
least well above
the boundary layer

Angular momentum budget :

$$\overline{u}_t + \left[(a \cos \phi)^{-1} (\overline{u} \cos \phi)_\phi - f \right] \overline{v^*} + \overline{u}_z \overline{w^*} = \overline{X} + (\rho_0 a \cos \phi)^{-1} \vec{\nabla} \cdot \vec{F}$$

Geostrophic approximation: $f \overline{u} \approx -\frac{1}{a} \overline{\Phi}_\phi$

$$\left(\vec{\nabla} \cdot \vec{F} = \frac{1}{a \cos \phi} \frac{\partial \cos \phi \overline{F}^\phi}{\partial \phi} + \frac{\partial \overline{F}^z}{\partial z} \right)$$

Hydrostatic approximation : $\overline{\Phi}_z = \frac{R \overline{T}}{H}$

Transformed Eulerian mean
meridional circulation
satisfies mass conservation :

$$\frac{1}{a \cos \phi} (\overline{v^*} \cos \phi)_\phi + \frac{1}{\rho_0} (\rho_0 \overline{w^*})_z = 0$$

**Eliassen
Palm
Flux:**

$$\overline{F}^\phi = \rho_0 a \cos \phi \left(-\overline{v' u'} + \overline{v' \theta'} \frac{\overline{u}_z}{\overline{\theta}_z} \right)$$

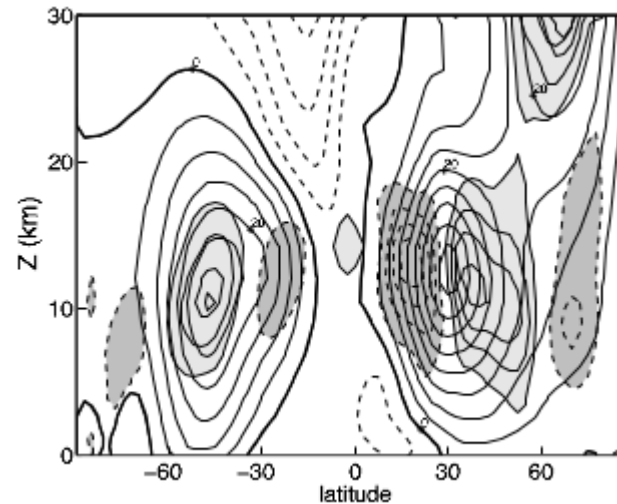
$$\overline{F}^z = \rho_0 a \cos \phi \left(\frac{\overline{v' \theta'}}{\overline{\theta}_z} \left(f - \frac{(\overline{u} \cos \phi)_\phi}{a \cos \phi} \right) - \overline{w' u'} \right)$$

b) Transformed Eulerian mean diagnostics

Forcing components of the Eady-driven jets in the troposphere

$$\frac{1}{a \cos \phi} \frac{\partial \cos \phi \overline{F^\phi}}{\partial \phi}$$

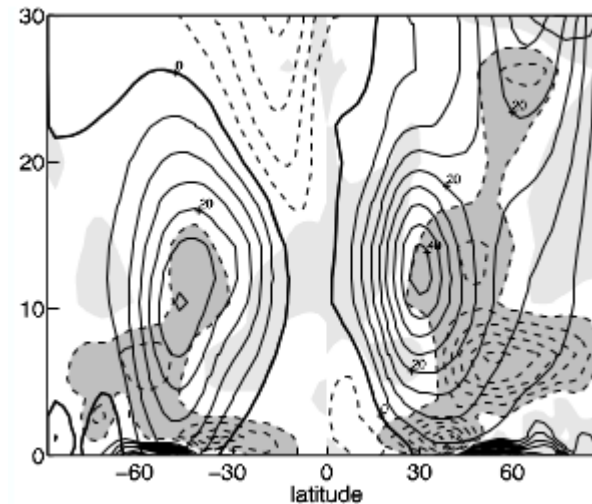
$\overline{F^\phi}$ essentially related to $\overline{v'u'}$



Decelerate the subtropical jet near the tropopause accelerates in the midlatitude (potentially yielding an upper level eddy driven jet)

$$\frac{\partial \overline{F^z}}{\partial z}$$

$\overline{F^z}$ essentially related to $\overline{v'T'}$

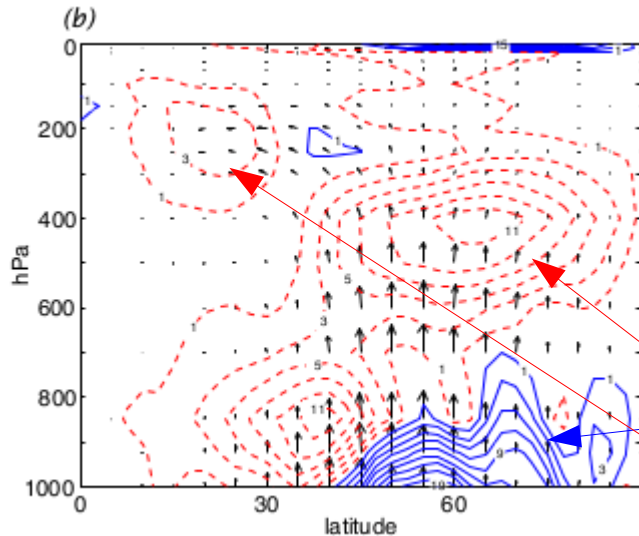


Decelerates in the midlatitude near the tropopause acceleration near the surface : « barotropisation » of the jet

Observed NH winter from ERA40 (Figs. 12.18, Vallis 2005)

b) Transformed Eulerian mean diagnostics

Application to the Eady-driven jets in the troposphere



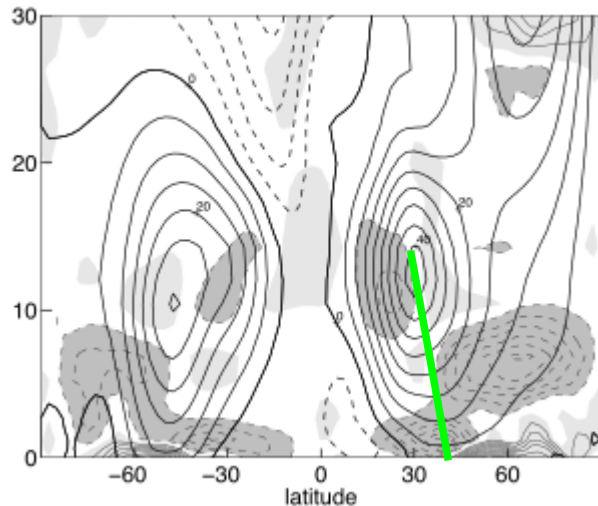
$$\vec{F} \text{ and } \frac{\vec{\nabla} \cdot \vec{F}}{\rho_0 a \cos \phi}$$

The dominant effect of the eddies is to make the subtropical jet more barotropic in the midlatitudes :

Dynamical acceleration at low levels
In the NH midlatitudes,

compensated by

a deceleration near the tropopause
in the NH midlatitude and on the southern
flank of the subtropical jet



$$\bar{U} \text{ and } \frac{\vec{\nabla} \cdot \vec{F}}{\rho_0 a \cos \phi}$$

Tilt with altitude of the maxima in the westerlies

Observed NH winter from ERA40 (Figs. 12.16, 12.17, Vallis 2005)

c) The middle atmosphere Brewer-Dobson circulation

Rather than starting from angular momentum, we can look more directly at trace species: H_2O , O_3

Satellite measurements in the stratosphere

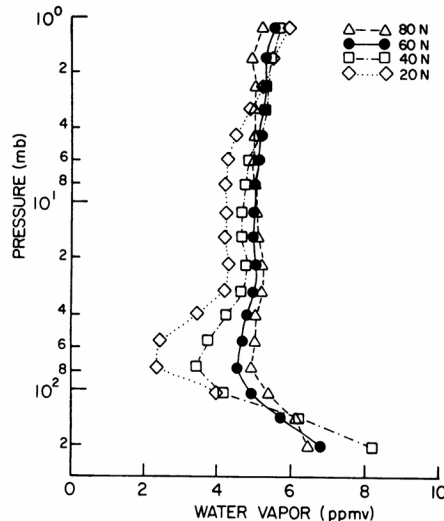
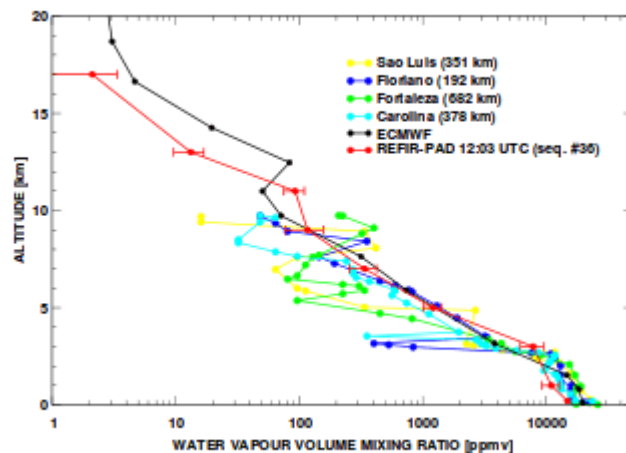


Fig. 1.5. Vertical profiles of water vapor mixing ratio at several latitudes measured by the LIMS instrument on the *Nimbus 7* satellite for May 1-26, 1979. [From Remsberg *et al.* (1984b). American Meteorological Society.]

- From lecture 1
- Water vapor (H_2O) presents Rapid decay with altitude, very weak values (almost uniform) in the stratosphere.
- Strong greenhouse impact in the troposphere.
- And minimum value at the equatorial tropopause

Balloon measurements in the troposphere:

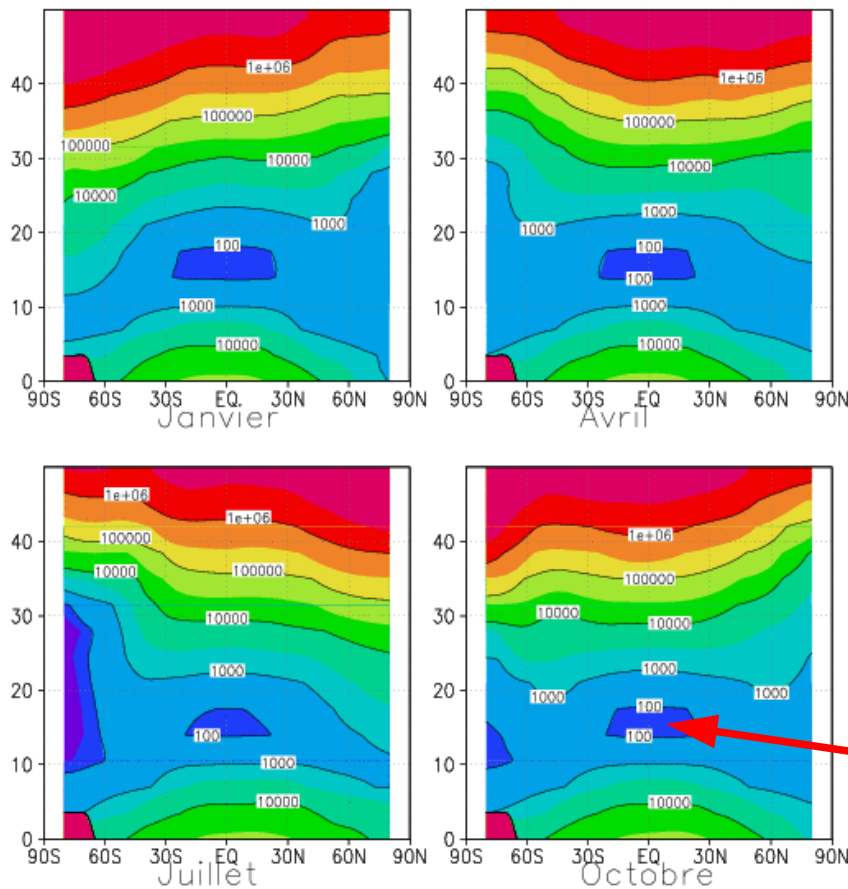


c) The middle atmosphere Brewer-Dobson circulation

Evidence on trace species: H₂O

Stratosphere is very poor in water vapor (few ppmv).
not related to the saturated values of the water vapor mixing ratios

Données CIRA, $\mathcal{V}_{v \text{ sat}}$

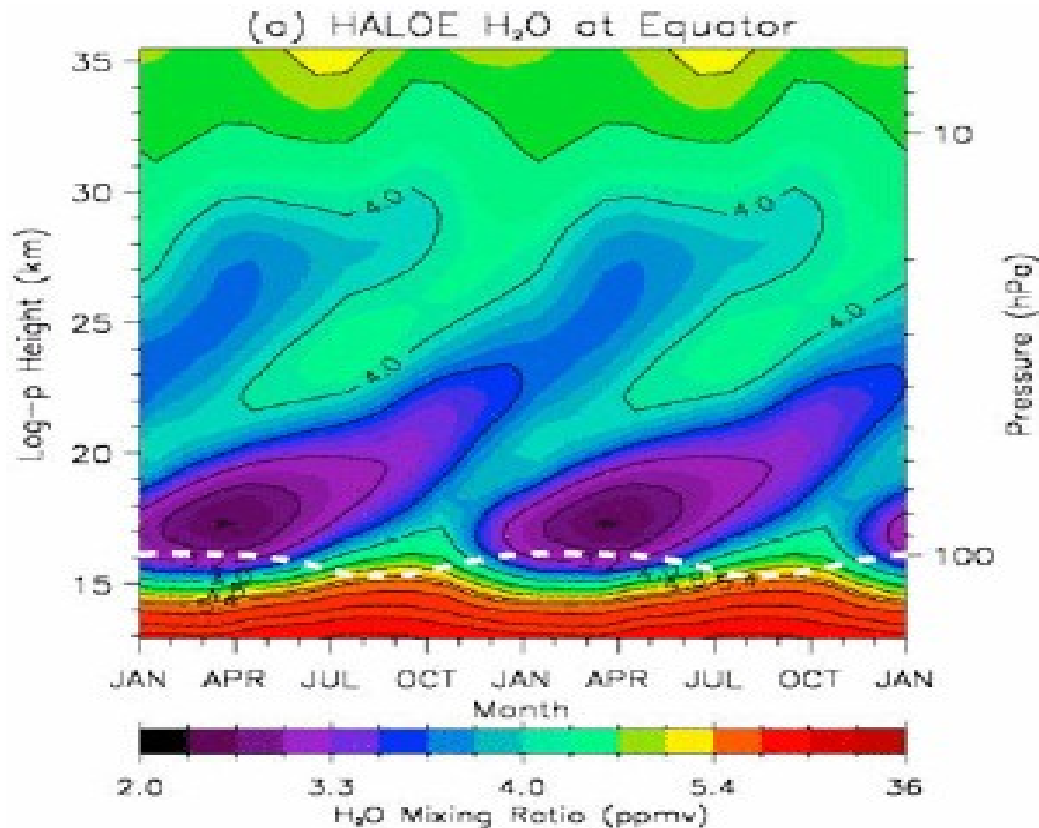


- Number of molecules per unit volume:
 n_v
- Volume mixing ratio (ppmv):
$$\mathcal{V}_v = n_v / n_A = p_v / p_A$$
- In the troposphere, $\mathcal{V}_v$ is around 10000ppmv, it is near the max value:
$$\mathcal{V}_{v \text{ sat}} = p_{v \text{ sat}} / p_A \quad \text{avec:}$$
$$p_{v \text{ sat}} \approx 1.E^{+5} \exp\left(13.7 - \frac{5120}{T}\right)$$
- In stratosphere: 2-6 ppmv
- The only region where such low values can be obtained during all seasons is the equatorial tropopause
- Small scale processes are nevertheless still missing: 100ppmv is still too much

c) The middle atmosphere Brewer-Dobson circulation

Evidence on trace species: H₂O

Satellites observations of H₂O (HALOE) The « tape recorder »



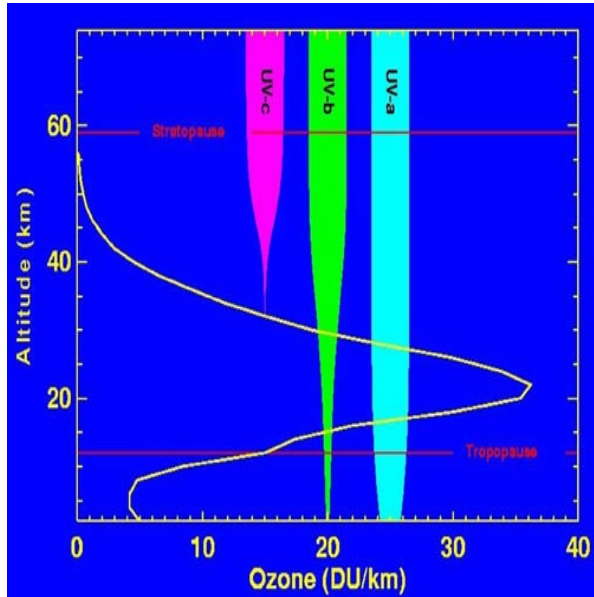
- Note the slow ascent of the tropical air, and which conserves quite well the value of $\mathcal{V}_v$ it acquires at the tropical tropopause.
- The values of $\mathcal{V}_v$ increases as the air rises via horizontal diffusion and because of the methane (CH₄) oxidation
- Park et al. JGR 2004

c) The middle atmosphere Brewer-Dobson circulation

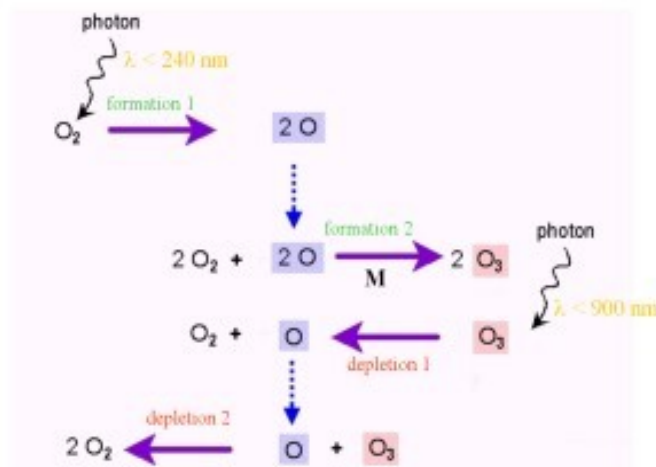
Evidence on trace species: O_3

La production de l'Ozone (O_3)

Penetration
Of the
UV-a, UV-b,
UV-c



Chapman
cycle
(1930):

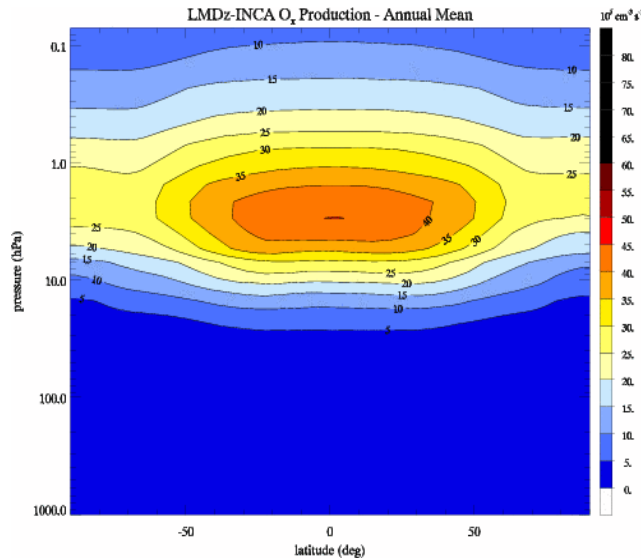


- **UV-c** are absorbed by O_2 between 40km and 60km. This photolysis yields molecular oxygen free radicals O .
- **These free radicals combine rapidly with O_2 to form Ozone O_3 .**
- The Ozone absorbs the **UV-b** above 20km to give O
- In most cases O recombines back with O_2 to give O_3 , there is chemical heating.
- The complete family $O+O_3$ (O_x) nevertheless has a relatively long lifetime in the low stratosphere.
- The Chapman cycle explains the O_x production and the heating by O_3
- It does not explain well the O_3 (Rôle of other free radicals than just O)

c) The middle atmosphere Brewer-Dobson circulation

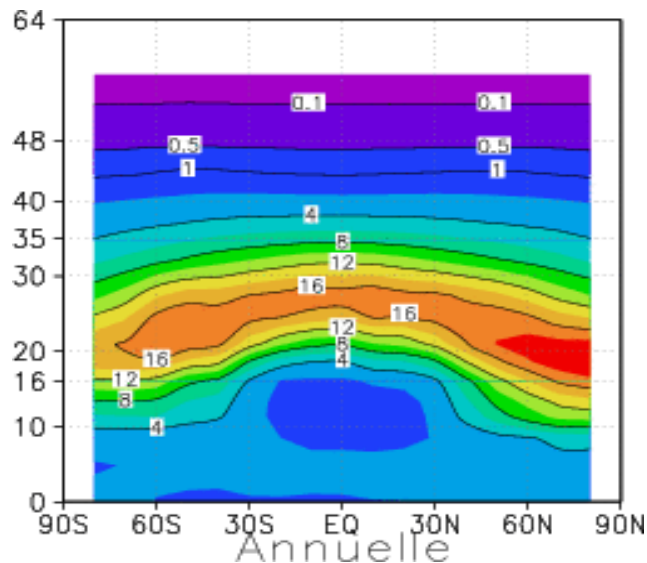
Evidence on trace species: O₃

Annual production of O_x



The production of ozone (O_x)
Atmospheric photo-chemical model INCA,
operational in LMDz
(Thanks to D. Hauglustaine)

Ozone concentration (in DU/km)

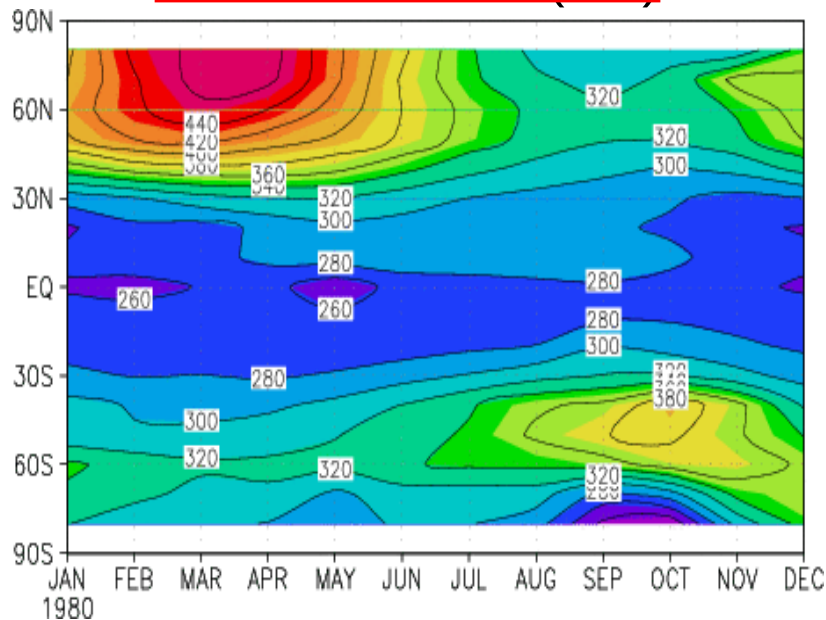


The Ozone is not located where it is
produced

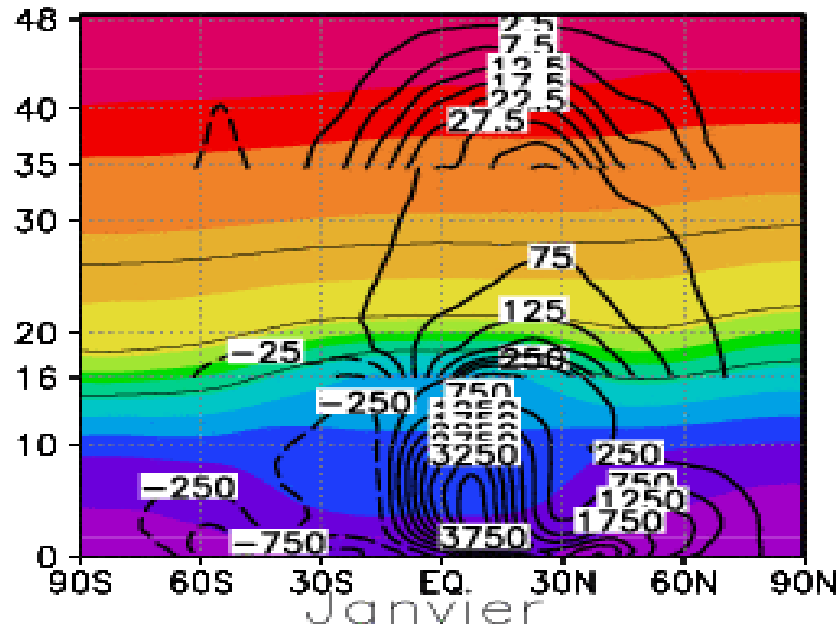
c) The middle atmosphere Brewer-Dobson circulation

Evidence on trace species: O₃

Annual cycle of the Ozone column in Dobson Units (DU)



- The air filled with newly produced O₃ travels from the zones of Ozone production (near and above the tropical stratopause) and transported toward the mid and high latitudes
- Note in April there is a strong concentration of O₃ at the North Pole! The Ozone has cumulated during winter
- Note also the O₃ deficit near the south pole in October: this is the Ozone hole, related to weak horizontal mixing and to a Brewer Dobson in the southern hemisphere winter not reaching the South pole



- “pseudo” Lagrangian streamfunction Ψ^* :

$$\frac{\partial \Psi^*}{\partial z} = -\rho_0 \cos \phi v^*$$

it transports the air from the upper stratosphere to the lower stratosphere and polar regions

c) The middle atmosphere Brewer-Dobson circulation

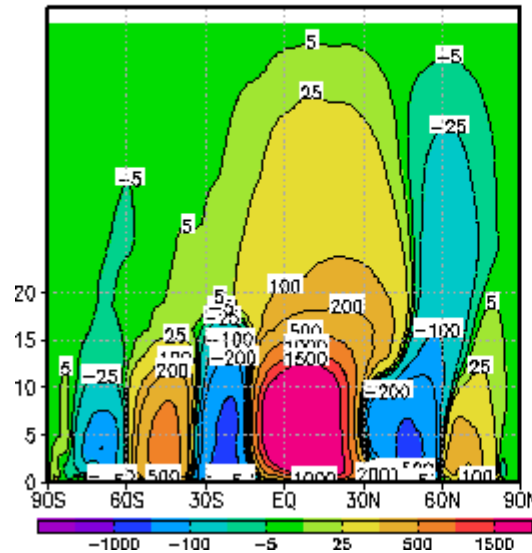
Limit of the Eulerian formalism

The Eulerian formalism can not explain the transport of O₃ from the equatorial stratopause to the lower stratosphere in the polar regions: illustration here using streamfunctions.

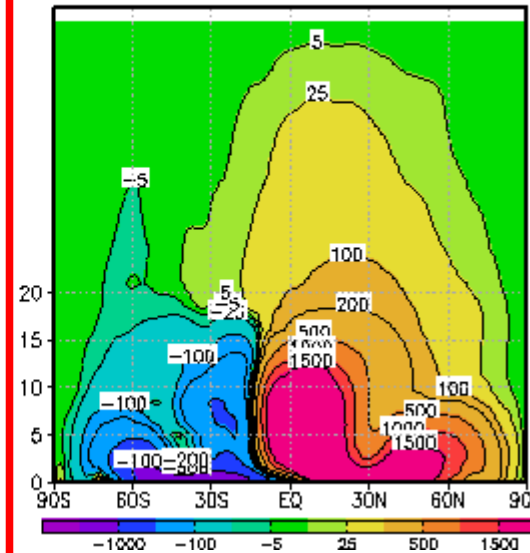
One needs to take into account Lagrangian averaging, or here its approximation, the transformed Eulerian mean

ERA40, January 1981–1985

Eulerian mean streamfunction



Transformed eulerian mean streamfunction



$$\frac{\partial \bar{\Psi}}{\partial z} = -\rho_0 \cos \phi \bar{v}$$

$$\frac{1}{a \cos \phi} \frac{\partial \bar{\Psi}}{\partial \phi} = +\rho_0 \bar{w}$$

$$\frac{\partial \bar{\Psi}^*}{\partial z} = -\rho_0 \cos \phi \bar{v}^*$$

$$\frac{1}{a \cos \phi} \frac{\partial \bar{\Psi}^*}{\partial \phi} = +\rho_0 \bar{w}^*$$

Residual mean circulation (definition):

$$\bar{v}^* = \bar{v} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\rho_0 \frac{\overline{v' \theta'}}{\bar{\theta}_z} \right)$$

$$\bar{w}^* = \bar{w} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left(\cos \phi \frac{\overline{v' \theta'}}{\bar{\theta}_z} \right)$$

c) The middle atmosphere Brewer-Dobson circulation

The middle atmosphere is not in radiative equilibrium

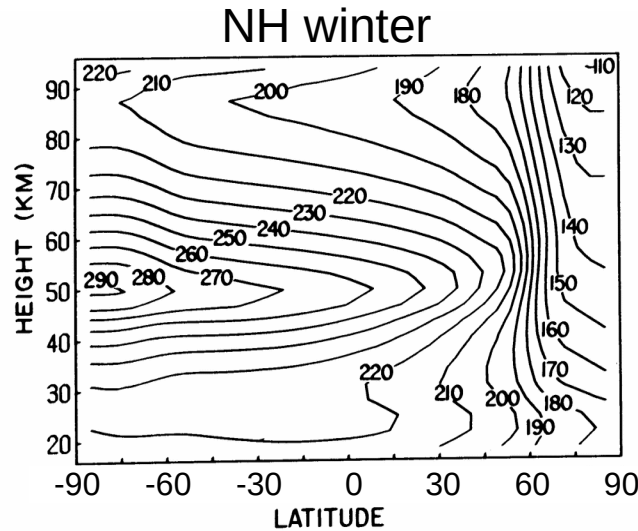


Fig. 2.34. Radiative equilibrium temperature distribution for northern (left) summer solstice. [From Wehrbein and Leovy (1982), with permission.]

- Temperature and winds resulting from radiative equilibrium and thermal wind balance.
- In January, T decreases from summer pole to the winter pole as expected
- The winds are eastward in winter and westward in summer
- The winds are much too strong, and the jets do not close when approaching the mesopause

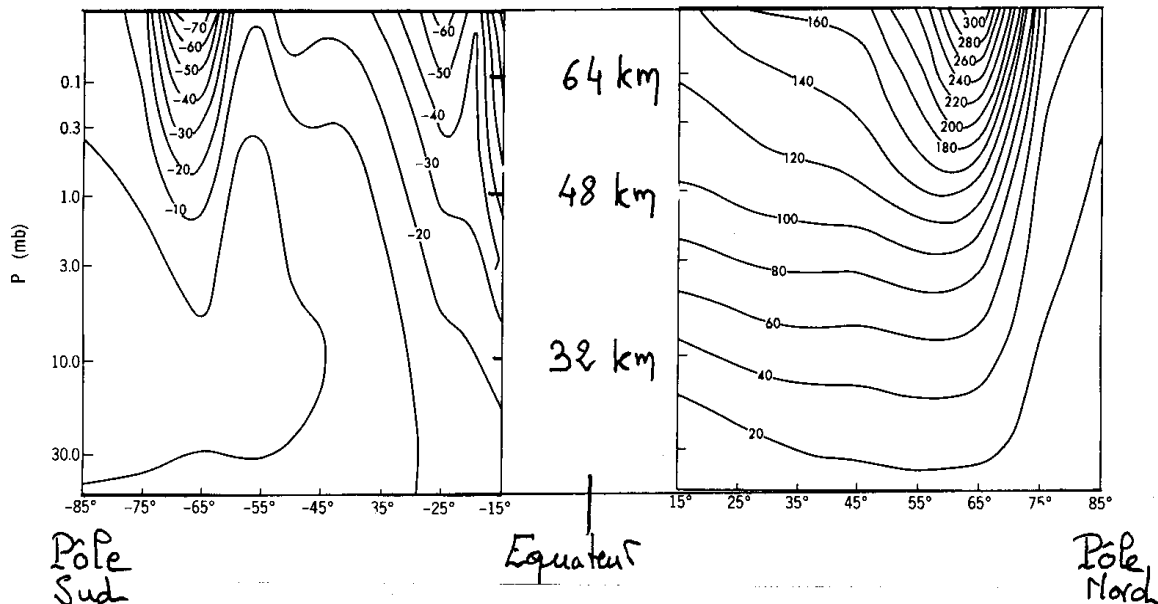
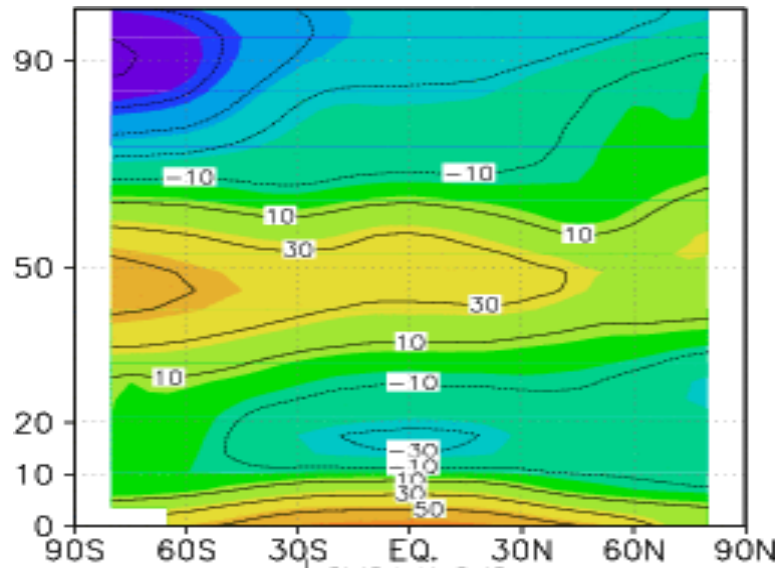


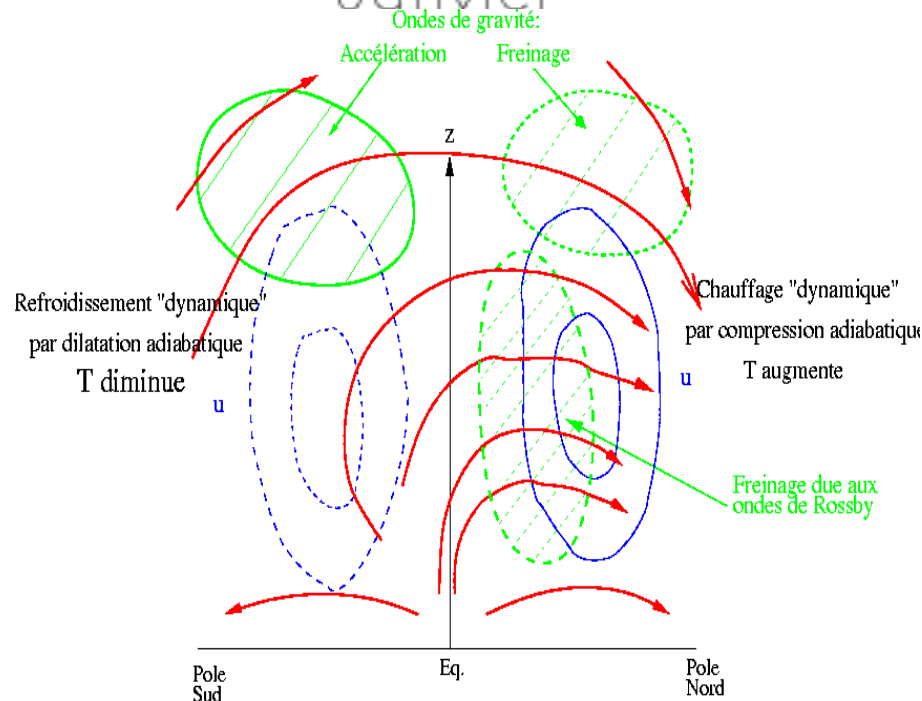
Fig. 7.1. Zonal gradient wind u_z that is in thermal-wind balance with the temperature field T_z of Fig. 1.2 and equals the observed climatological zonal wind at 100 mb. (a) Northern Hemisphere (winter), (b) Southern Hemisphere (summer). (Courtesy of Dr. S. B. Fels.)

c) The middle atmosphere Brewer-Dobson circulation

The middle atmosphere is not in radiative equilibrium
January Temperature (CIRA)



- Temperature is far from the radiative equilibrium seen before:
- They are warmer in at the winter pole, colder at the summer pole.
- During solstices:
 - In the upper mesosphere (70-90km) T increases from the winter pole to the summer pole !!!
 - At the mesopause (90km) over the summer pole is the coldest region of the neutral atmosphere!



Interpretation via an essentially mechanically driven circulation: the Brewer Dobson circulation.