The 20-parameter general circulation model

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1 The planetary atmospheric general circulation model

Schematically, atmospheric general circulation models consist of two parts. (1) The "dynamical part" is dedicated to a temporal and spatial integration of the hydrodynamical equations for the explicitly resolved scales (generally of the order from few tens to few hundreds km on the horizontal for classical terrestrial applications). Terrestrial models are generally based on the "primitive equations of meteorology" which are valid for Earth-like planets (solid body covered by a thin atmosphere assumed to remain vertically in hydrostatic balance). This part of the code can thus be used for various planets by just changing some parameters such as the gravity or rotation rate or some boundary conditions such as the specification of the surface height. (2) The "physical part" is dedicated to the diabatic forcing of the atmospheric circulation and to the modeling of sub-grid scale processes. The corresponding parametrizations are generally much less universal and must be significantly modified from one planet to the other. For example, the complete intergation of atmospheric radiative transfer (which includes absorption by complex molecular vibration-rotation bands with complex temperature and pressure dependencies) is by far out of reach of present computational means, since one wants to integrate those equations several time per day and for thousands various locations on the planet. Numerous approximations are therefore used in general circulation models which are generally strongly dependent on the particular planetary conditions. In fact the three planetary versions of the Planetary Atmospheric General Circulation Model developed at LMD essentially differ by the way these radiative transfer calculations are performed.

1.1 Dynamics

The dynamics of the planetary model are taken from the terrestrial general circulation model used at LMD for climate studies Classical *primitive equations* of meteorology (Navier Stokes equations for a perfect gas plus hydrostatic hypothesis and thin layer approximation, see *e. g.* Holton 1979) are integrated using finite differences in space and an explicit centered (or leapfrog) temporal scheme. The original formulation of this model is presented in details by Sadourny and Laval, 1984. We use in fact a more recent formulation by Sadourny, coded in FORTRAN 77 by Le Van, in which latitudes and longitudes of the grid can be chosen arbitrarily, which allows to zoom in a particular region on the sphere. For our experiments, we chose a uniform area grid with points equidistant in longitude λ and sine of latitude ϕ so that all meshes have the same area. Two horizontal resolutions were used, one with 32 longitudes and 24 latitudes and the other one with 64 longitudes and 48 latitudes. The vertical discretization is based on a hybrid $\sigma - p$ coordinate (σ beeing pressure normalized by its surface value $\sigma = p/p_s$). The $\sigma - levels$ are regularly distributed in $\ln \sigma$ except near the lower and upper boundaries.

The discretization scheme conserves both potential enstrophy for barotropic divergent flows [**?**, **?**] and total angular momentum for stationary axi-symmetric flows. The latter property was not included in the original terrestrial version but was introduced in the Martian atmospheric circulation model [**?**] in which, because of the non numerical conservation of angular momentum, spurious prograde zonal winds were produced in equatorial regions. The new conservative formulation has since then been included in the terrestrial climate model and was found to strongly improve the simulation of the tropical trade-winds. Accurate conservation of angular momentum is, of course, crucial for the study of atmospheric superrotation since the budget of this particular quantity plays a central role in the phenomenon. The numerical scheme and conservation properties are presented in details in Appendix **??**.

Non linear interactions between explicitly resolved scales and subgrid-scale processes are parametrized applying an horizontal dissipation operator on potential temperature and wind. This dissipation is based on an iterated Laplacian:

$$\frac{\partial q}{\partial t} = (-1)^{n_{diss}} \frac{1}{\tau_{diss}} (\delta x)^{2n_{diss}} \triangle^{n_{diss}} q \quad (1)$$

where δx is the smallest horizontal distance represented in the model and τ_{diss} the dissipation time of a structure of scale δx, a technique commonly used in the modeling of two-dimensional turbulence and in many general circulation models.

The iteration is introduced to be more selective in smaller scales, not acting directly on the explicitly resolved flow (in the present simulations, $n_{diss} = 2$ and $\tau_{diss} = 6 \times 10^4$ s). Too strong a dissipation could produce some efficient process to transport angular momentum equatorward which would artificially reinforce superrotation. This was responsible for some peculiar results in the past: a strong superrotation was produced in some terrestrial simulations [**?**] which were due to a strong latitudinal filter on the zonal wind, acting as a very efficient horizontal dissipation.

1.2 Physical parametrizations

The main problem for the determination of an appropriate set of physical parametrizations was the development of a realistic but simple radiative scheme. Many fundamental numerical studies of the atmospheric general circulation are based on the use of a so-called Newtonian forcing in which the temperature field is simply relaxed toward a prescribed "radiative equilibrium" state [**?**, **?**, **?**]. This approach has the advantage of simplicity and is characterized by a time constant which can easily be compared to the other time constants of the circulation, but it presents important drawbacks: it is physically incorrect and moreover, the equilibrium temperature field is prescribed and not related to the relevant external parameters of the planetary environment. In such experiments the zonal wind field is also indirectly relaxed to that in thermal wind balance with the equilibrium temperature field whereas, in real atmospheres, the coupling between temperature and wind is also important to determine the equilibrium temperature field itself.

For the present model, a complete but very simple radiative transfer code was developed, based on some kind of grey absorption approximation. The parametrization depends upon only two independent and external parameters: the total atmospheric transmission for the visible and thermal parts of the spectrum (τ_{IR} and τ_{Vis} respectively). Finally, the parametrization accounts for the effect of temperature variations of the Planck function (which is fundamental in order to obtain a radiative equilibrium) but not for the main complexity of atmospheric opacities. We also introduced (as is usually done in terrestrial models) parametrizations for the vertical turbulent diffusion, convective adjustment (in order to prevent supper-adiabatic vertical temperature profiles) and thermal conduction under the surface. All those parametrizations are presented in Appendix **??**. In the present study, it was not attempted to introduce any kind of hydrological cycle or phase change as occur on most known planetary atmospheres with significant impact on the atmospheric circulation.

1.3 The planetary parameters

In this simple planetary atmospheric general circulation model, the description of the planet only depends upon 19 independent parameters:

- 6 astronomical parameters: the radius a and gravity g of the planet; its rotation rate Ω; the length of year yr, obliquity δ and solar constant F_0 (the orbit is assumed to be circular).

- 4 parameters for the surface properties: emissivity ϵ_s, albedo A_s, drag coefficient C_D and thermal inertia I, assumed to be constant over the whole surface of the planet.
- 3 parameters to describe the atmosphere: mean surface pressure p_s, molecular mass μ_{gas} (which enters in the definition of the gas constant $R = \mu R^*$, where R^* is the universal constant for perfect gas) and the adiabatic coefficient $\kappa = R/C_p$.
- the 2 transmissions for the visible and thermal part of the spectrum (τ_{IR} and τ_{Vis}).
- 4 coefficients for the sub-grid scale processes: a mixing length l_{mix} and a minimum turbulent kinetic energy e_{mix} for the vertical mixing and the two coefficients n_{diss} and τ_{diss} of the horizontal dissipation operator.

The choice of this particular set of parameters is of course rather arbitrary. For instance, other astronomical parameters must be introduced to study the effects of strong excentricity as arise on Mars. Some choices are also strongly model dependent such as those parameters which are related to sub-grid scale processes. Finally, for the purpose of general investigation of the possible atmospheric circulation regimes, it will be useful to reduce those parameters through dimensional analysis, which has not been attempted here.

A Physical parametrizations

The physical parametrizations are presented in details below.

A.0.1 Radiative transfer

The time evolution of the temperature ($T_{k+\frac{1}{2}}$) at the middle of a layer (or s level), due to absorption and emission of infra-red radiations, is computed as

$$\left(\frac{\partial T_{k+\frac{1}{2}}}{\partial t}\right)_{IR} = \frac{g}{C_p}\frac{F_k^{\uparrow} - F_k^{\downarrow} - F_{k+1}^{\uparrow} + F_{k+1}^{\downarrow}}{p_k - p_{k+1}} \tag{2}$$

where the upward $F_k^{\uparrow}$ and downward $F_k^{\downarrow}$ fluxes are computed using a spectrally averaged transmissivity $\tau_,$ in the following form

$$F_k^{\uparrow} = \epsilon_s \sigma {T_s}^4 \tau_{0,k} + \sum_{l=0}^{k} \sigma T^4_{l+\frac{1}{2}} \left[\tau_{l+1,k} - \tau_{l,k}\right] \tag{3}$$

$$F_k^{\downarrow} = \sum_{l=k}^{K} \sigma T^4_{l+\frac{1}{2}} \left[\tau_{k,l} - \tau_{k,l+1}\right] \tag{4}$$

(where ϵ_s is the surface emissivity). Various simple forms can be chosen for the transmissivity. First tests, performed with a grey absorption approximation, led to strong temperature discontinuities at the surface (between the ground temperature and the air above the surface), as already found by other authors at the beginning of climate numerical modeling [**?**].

Finally, the transmissivities were computed in the frame of the strong absorption approximation, better adapted to the tropospheric conditions where most of the efficient molecular bands are strongly saturated. In this case, assuming that lines are randomly distributed over the spectrum, and neglecting temperature variations of the absorbing coefficients along the optical path, the transmissivity of the whole band, between two layers k and l, can be computed, assuming a completely isotropic radiation, in the form (for those who are not familiar with such models see Goody, 1964):

$$\tau_{k,l} = \exp\left(-k_{IR}\overline{up}\right) \tag{5}$$

where $\overline{up}$ is a pressure weighted absorber amount

$$\overline{up} = \int p\rho dz = (p_k - p_l)\,\frac{(p_k + p_l)}{2} \tag{6}$$

and $k_{_{IR}}$ a constant describing the efficiency of the atmospheric absorption.

This formula produces again some temperature discontinuity at the surface but less than 10 K. The transmissivity depends on one free parameter only, for example the total optical transmission of the atmosphere $\tau_{_{IR}} = \exp\left(-k_{_{IR}} p_s^2/2\right)$ (fraction of the surface thermal radiation directly emitted to space). The model accounts for the fundamental effect of temperature variations linked to the Planck function, essential for the thermal equilibrium and radiative forcing of the circulation, but not for the much more complex pressure and temperature dependencies linked to the molecular absorbing coefficients.

A classical grey absorption approximation is used for the absorption of solar radiations. For reason of simplicity, no attempt was made to model some kind of scattering by molecules or aerosols. In such conditions, the downward solar radiation consists of a uniq collimated beam and the grey transmission takes the simple form

$$\tau_{k,l} = \exp\left(-\frac{k_{_{Vis}}}{\cos\theta}\,|p_k - p_l|\right) \tag{7}$$

where θ is the local solar zenith angle. As for the infra-red radiative transfer, the model depends on one free parameter, for instance the total atmospheric transmission $\tau_{_{Vis}}$, computed for a mean incidence $\cos\theta = 1/2$ (fraction of the global incoming solar radiation reaching the surface), which is the fraction of the solar flux reaching the surface of the planet. It influences greatly the vertical stability of the atmosphere, an important dynamical parameter.

The reflected radiation, which depends on a surface albedo A_s, is assumed to be totally isotropic in the upward direction and is computed for a mean zenith angle θ_0 with $1/\cos\theta_0 = 1.66$.

The radiative forcing of the atmosphere also depends on the incoming solar flux at the top of the atmosphere F_0 and on those of the orbital parameters which determine the spatial and temporal variations of insolation (length of day, obliquity and length of year).

A.0.2 Vertical turbulent mixing

The formulation of the vertical turbulent mixing is taken almost directly from a very simple scheme used in the LMD climate model. The effect of turbulent mixing on both potential temperature and momentum is computed as

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho}\frac{\partial F_q}{\partial z} \tag{8}$$

in term of the vertical divergence of a turbulent flux

$$F_q = -\rho K_z \frac{\partial q}{\partial z} \tag{9}$$

The turbulent mixing coefficient K_z is computed as

$$K_z = l e^{1/2} \tag{10}$$

in term of mixing length l and a diagnostic estimate of the turbulent kinetic energy

$$e = Max\left\{ l^2 \left[\left(\frac{\partial \vec{V}}{\partial z}\right)^2 - 2.5\frac{g}{\theta}\frac{\partial\theta}{\partial z} \right], e_{min} \right\} \tag{11}$$

where the minimum value of the kinetic energy was set to $e_{min} = 1 \times 10^{-6} m^2 s^{-2}$. In the case where $e > e_{min}$, K_z,expressed as a function of the Richardson number

$$Ri = \frac{g\partial\theta/\partial z}{\theta\left(\partial\vec{V}/\partial z\right)^2}, \tag{12}$$

is equal to

$$K_z = l^2 \left\| \frac{\partial \vec{V}}{\partial z} \right\| \sqrt{1 - 2.5Ri} \tag{13}$$

The surface flux F_{q_s} is simply computed as

$$F_{q_s} = -C_{D0} \left[V_0 + \|\vec{V_1}\| \right] (q_1 - q_s) \tag{14}$$

where q_1 (resp. q_s) is the value of q in the first atmospheric layer (resp. at the surface), $V_1 = \sqrt{u_1^2 + v_1^2}$ and V_0 is a minimum wind value fixed to 1 m s^{-1}.

A.0.3 Convective adjustment

The occurrence of vertical unstable temperature profiles

$$\frac{\partial \theta}{\partial z} < 0 \tag{15}$$

is prevented using a simple dry convective adjustment. If such an unstable profile is produced by the model, an adiabatic profile is immediately restored with a simple energy conserving scheme. If the resulting temperature profile is unstable at its upper or lower limit this mechanism is instantaneously extended in such a way that the final profile is entirely stable.

This convective adjustment is in fact achieved in a real atmosphere by parcel exchange through vertical convective motions. These motions not only transport potential temperature but also momentum. The momentum exchange can only be estimated from a diagnostic of the vertical mass flux which can not be obtained from such an unphysical parametrization. Nevertheless, an estimate of the degree of instability of the vertical profile can be obtained from the relative enthalpy exchange necessary to restore the diabatic profile $\bar{\theta}$ from the original profile θ

$$\alpha = \int \left| \theta - \bar{\theta} \right| \rho dz / \int \theta \rho dz \tag{16}$$

When $\alpha < 1$ (this condition is always satisfied in the simulations) the momentum is entirely mixed on a fraction α of the mesh. This arbitrary choice is qualitatively acceptable in the sense that a greater instability will produce a greater momentum mixing.

A.0.4 Surface processes

Temperature at the surface of a planet is governed by the balance between incoming fluxes (direct solar insolation, thermal radiation from the atmosphere and the surface itself and turbulent heat fluxes) and thermal conduction in the soil. The time evolution of the temperature under the surface is given by a classical conduction equation

$$\frac{\partial T}{\partial t} = -\frac{1}{C} \frac{\partial F_c}{\partial z} \tag{17}$$

where the conductive flux F_c is given by

$$F_c = -\lambda \frac{\partial T}{\partial z} \tag{18}$$

and where λ and C are the soil conductivity and volumetric heat, respectively. In the simple case of a vertically homogeneous soil (which is assumed in the model) it can be shown that, as far as the evaluation of surface temperatures and fluxes is concerned, the model depends on only one free parameter: the so-called thermal inertia $I = \sqrt{\lambda C}$.

The conduction in the soil is computed using a finite difference scheme analogous to that developed by Jacobsen and Heise, 1982 . Some complementary information on the numerical scheme is given by Hourdin *et al.*, 1993.

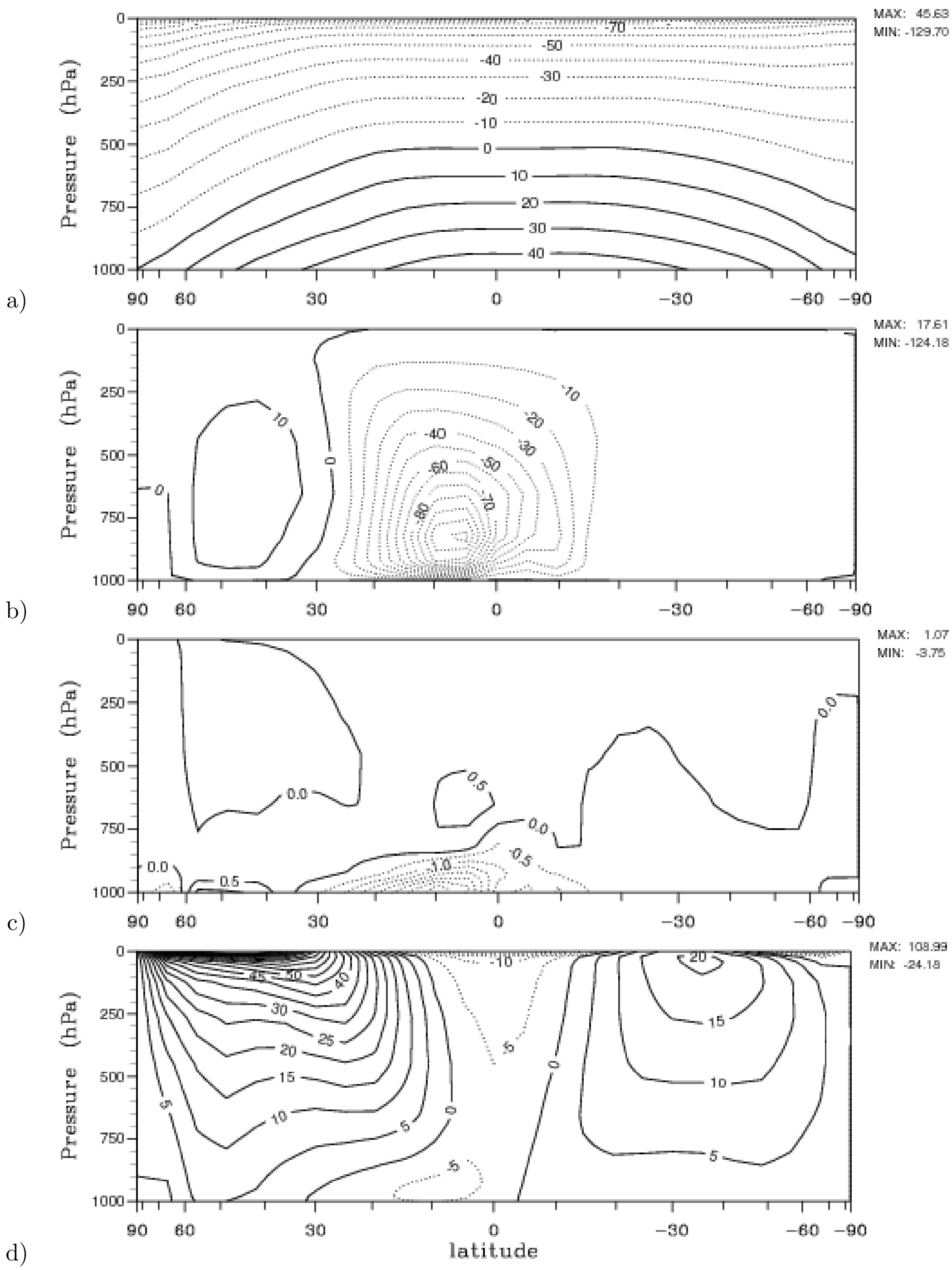

Figure 1: Results for a pseudo dry Earth for December-January-February
(a) Temperature in ^{o}C (upper panel), **(b)** stream function of the mean meridional circulation (10^9 kg s^{-1}), **(c)** meridional wind in m s^{-1} (central panel) and **(d)** zonal wind in m s^{-1} (lower panel) have been average in time over three months and in longitudes.