WAVETRISK adaptive dynamical core coupled to dry physics

G Ching-Johnson[1], NK-R Kevlahan[2], T Dubos[3], and F Hourdin[4]

[1]Department of Meteorology, University of Reading, Reading RG6 7BE, United Kingdom
[2]Department of Mathematics and Statistics, McMaster University, Hamilton ON L8S 4K1, Canada
[3]Laboratoire de Météorologie Dynamique, École Polytechnique, 91128 Palaiseau, France
[4]Laboratoire de Météorologie Dynamique, Institut Pierre Simon Laplace, Paris, France

Correspondence: Gabrielle Ching-Johnson (g.n.ching-johnson@pgr.reading.ac.uk)

0.1 Radiation

Absorption of solar radiation by the atmosphere and surface is the first driver of the atmospheric circulation. Then the atmosphere and surface emits thermal radiation, essentially in the infra-red spectrum, which is then absorbed by the surface and atmosphere, or emitted back to space. The modification of the efficiency of the atmosphere to emit and absorb thermal radiation is at the origin of the on going global warming. As is the case for the dynamical core based on a simplified version of the well established Navier-Stokes equations, radiative codes in GCMs rely on the well established transfer equation for local thermodynamic equilibrium. The integration of this equation in all its spacial and spectral dimensions is however far from practicable in GCMs, and this by orders of magnitudes of computer time. Many classical approximations are thus used in operational codes, which are even simplified further here as detailed bellow.

We could put the general radiative transfer equation here. But not sure it is usefull.

0.1.1 Approximations of the radiative transfer equation

The main approximations used when integrating radiative transfer in a climate or weather forecast model are the following:

The separation of solar radiation and terrestrial radiation is made possible and is not an approximation because the radiative transfer equation is linear with respect to its sources: here the solar emission for solar radiation and the thermal emission by the atmosphere and planetary surface for the "terrestrial" or "thermal" or "long-wave" radiation.

The plan parallel approximation is used in all the GCM codes. It consists in assuming that all the atmospheric properties depends on altitude only on an infinite horizontal plane. The approximation extends to the statistics of clouds (clouds may vary horizontally but there distribution is the same everywhere in this plane). The plan parallel approximation results in radiative transfer models which are purely 1D on the vertical.

The two stream approximation consists in replacing the integral on angles by only two directions, one upward and one downward. It can be seen as a particular case of the discrete ordinate method, which consists in integrating on a discrete number of angles. Here the integration is restricted to two angles. The model computes separately downward and upward fluxes, assuming that the attenuation is done as if all the photons were transported along a unique direction. The solar beam is of course integrated along the real solar zenith angle Θ_S, considering all photons which have not been scattered at the surface or in the atmosphere (correcting often the atmospheric scattering from its forward propagation with the Delta Eddington approximation). In this framework, the solar radiation is split between direct and indirect diffuse radiation. For both the thermal infrared radiation and for the diffuse solar radiation, and for both upward and downward fluxes, the zenith angle of propagation of light Θ_0 is chosen according to the **diffusive approximation**, with $\mu_0 = 1/\cos\Theta_0 = 5/3$.

Approximations for the spectral integration are of various nature. For the atmospheric gas, coefficients for absorption and scattering have a very complex spectral signature made of billions of absorption lines associated with quantum energy transition levels of three-atomic molecules. Line-by-line integrations consisting of taking into account all the individual transition lines is far from practicable in GCMs. Various possibilities of Band models are used in GCMs, summarizing the absorption over a finite spectral intervals. Those models consists either of learning functions based on line-by-line integration, or in k-distribution or k-correlated approaches in which the integration on frequencies within a given band is replaced by an integration over values of the absorption coefficient.

Further simplifications are used here. First scattering is neglected. Scattering couples all the direction in the radiative transfer equation. Even in the two stream approximation, it couples the equations of the direct solar radiation and that of the tho diffuse angle associated with upward and downward solar radiation. It couples as well the downard and upward fluxes for the terrestrial radiation. Neglecting scattering simplifies a lot the computation. For the solar radiation in particular, the downard radiation is computed purely along the solar zenith angle while the upward radiation is only diffuse (as is always the case if not accounting for specular reflection by the surface). The second simplification concerns the spectral integration. The extinction is assumed independant of the frequency. In the pure "grey" approximation, i.e. with no variation of the gaz abosrption coefficient, the transmission τ along zenithal angl Θ readds

The equations of the resulting radiative code are derived bellow.

0.1.2 Integration for the terrestrial or long-wave radiation

The net radiative flux in the longwave (or terrestrial or thermal) radiation reads is the sum of and upward (${F_L}^{\uparrow}$) and downard (${F_L}^{\downarrow}$) flux: $F_L(z) = {F_L}^{\uparrow}(z) - {F_L}^{\downarrow}(z)$.

Let us first consider just the upward thermal radiation, averaged over the zenith angles in a half-space. Without scattering and in a two stream approach, the monochromatic equation of radiative transfer integrated on the above half-space reads

$${F_L}^{\uparrow}(\nu,z) = {F_L}^{\uparrow}(\nu,0)\tau_\nu(0,z) + \int_0^z \pi B_\nu\left[T(z')\right] K_\nu(z')\tau_\nu(\mu_0,z',z)\,\mu_0 \rho dz' \tag{1}$$

where $B_\nu\left[T(z')\right]$ is Planck's function:

$$B_\nu\left[T(z')\right] = \frac{2h\nu^3}{c^2(e^{h\nu/kT(z')} - 1)}, \tag{2}$$

where h is Planck's constant. Planck's function describes the radiation emitted by a black body in thermal equilibrium at a given temperature $T(z')$. $K_\nu(z)$ is the absorption coefficient per unit mass of air and

$$\tau_\nu(\mu_0, z, z') = \exp\left[-\int_z^{z'} K_\nu(z")\mu_0 \rho dz"\right] \tag{3}$$

is the transmission function, i.e. the fraction of incident radiation, at altitude z and frequency ν, which is received at altitude z'. Eq. 1 expresses the upward flux at altitude z as the sum of the surface emission after atenuation between the surface and altitude z, and of the sum of the emissions (after atenuation between z' and z) from all the altitudes z' bellow ($\pi B_\nu\left[T(z')\right]\mu_0\rho dz'$ for a layer of thickness dz'). The coefficient μ_0 accounts for the fact that the atmospheric optical depth used in emission (Eq 1) and absorption (Eq 3) is computed along the mean zenith angle Θ_0.

Noticing that the derivative of the transmission function appears in the integral, and assuming that the surfgace emits as a black body (emissivty equal to unity), this equation can be rewritten as:

$$F_+(\nu, z) = \pi B_\nu\left[T_s\right]\tau_\nu(0, z) + \int_0^{z'} \pi B_\nu\left[T(z')\right]\frac{\partial \tau_\nu(z', z)}{\partial z'}dz' \tag{4}$$

If the transmission fonction is independant of ν, the equation can be integrated on the full spectrum and reads:

$$F_+(z) = \sigma {T_s}^4 \tau(0, z) + \int_0^{z'} \sigma T^4(z')\frac{\partial \tau(z', z)}{\partial z'}dz' \tag{5}$$

using the Stefan–Boltzmann law (Pierrehumbert, 2010) where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant.

This equation is integrated here with a straitforward finite differences, conservative scheme, as

$$F_L{}^{\uparrow}(l+\frac{1}{2}) = \sigma {T_s}^4 \tau(0, l+\frac{1}{2}) + \sum_{k \le l}\left[\tau(k+\frac{1}{2}, l+\frac{1}{2}) - \tau(k-\frac{1}{2}, l+\frac{1}{2})\right]\sigma T(k)^4, \tag{6}$$

A very similar form is used for the downward flux:

$$F_L{}^{\downarrow}(l+\frac{1}{2}) = \sum_{k>l}\left[\tau_L(l, k-\frac{1}{2}) - \tau_L(l, k+\frac{1}{2})\right]\sigma T(k)^4 \tag{7}$$

J'ai pris les notations avec des 1/2 pour coller avec ce qui est fait plus loin sur la couche limite, mais on peut changer ça

0.1.3 Integration for the solar radiation

The shortwave net flux is the net result of the upward and downward fluxes evaluated for shortwave radiation,

$$F_S(z) = F_{S+}(z) - F_{S-}(z). \tag{8}$$

Since the unique source accounted for in this part of the code is the emission by sun, and because scattering is neglected, the integration for the solar radiation, reads (after integration over the full spectrum):

$$F_{S-}(z) = \tau_S(z, \infty)\mu S \tag{9}$$

$$F_{S+}(z) = \tau_S(0, z)F_{S+}(0) \tag{10}$$

where S is the downward flux at the top of the atmosphere, which in the case of shortwave radiation is the solar flux and μ is the cosine of the zenith angle. **Look in the code, but the angle should not be the same for the direct downward radiation and diffuse upward radiation**

The boundary upward flux of radiation, $F_{S+}(0)$, is the portion of downward shortwave radiation that is reflected by the ground,

$$F_{S+}(0) = -(1-\alpha)F_{S-}(0), \tag{11}$$

where α is the albedo, which depends on the reflective properties of the ground ($\alpha \approx 0.8$ for fresh snow, $\alpha \approx 0.1$ for oceans or forests). The albedo $\alpha = 0.112$ for our simulations, independent of position, although the physics package allows location-dependent albedo (e.g., different for land and ocean regions).

0.1.4 Transmission function

In the most general case, the absorption coefficient K_ν entering in the defintion of the transmission function τ_ν (Eq. 3), depends upon the absorbers (i.e. gas) in the atmosphere, $K_\nu(z) = \sum_i \rho_i \kappa_i(p,T,\nu)$, where $\kappa_i(p,T,\nu)$ is the absorption coefficient of gas i and ρ_i is the density of gas i. Each coefficient κ_i is the sum of "lines" of absorption, each line being a function of p, T and ν.

The most drastic approximation of this absorption spectrum is the so-called grey gas approximation which consists in assuming that we have a single gas with constant absorption $K_{\nu 0}$, in which case the transmission function simply reads:

$$\tau(\mu,z,z') = \exp\left(-\mu K_{\nu 0}\int_{z}^{z'} \rho dz"\right) \tag{12}$$

or

$$\tau(\mu,z,z') = \exp\left(-\frac{\mu K_{\nu 0}}{g}|p(p) - p(z')|\right) \tag{13}$$

for the transmission between pressure level p and p', assuming that the atmosphere is in hydrostatic balance. With this approximation, the transmisivity remains multiplicative ($\tau(\mu,z_0,z_1) = \tau(\mu,z_0,z_1)\tau(\mu,z_1,z_2)$), a fundamental property of monochromatic raditive transfer.

This form of the transmission function is valid when the atmosphere is optically thin. It produces however vertical profiles that are much more stable than those generally observed in the tropospheres of terrestrial planets, where the upper layers of the atmosphere are directly heated by surface radiation. In the tropospheres of terrestrial planets, the significant molecular bands are generally saturated very quickly, and exchanges between distant layers are possible only within certain windows that generally play a minor role in the radiative budget.

When integrating radiation for a single absorption line, two extreme approximations can be considered: a thin layer approximation where $\ln\tau$ is linear with respect to the mass of absorber encountered and scales with Δp, and a thick layer approximation, where the line is saturated in its center, so that the effective absorption increases as the width of the saturated band. In this case, the transmission function scales with $\sqrt{\Delta p^2}$. With this dependency, a larger fraction is absorbed at short distance. (Pierrehumbert, 2010).

Here the strong approximation is used for the thermal terrestrial radiation and the thin approximation is used for the solar radiation.

The transmission function for the solar radiation is finally computed assuming a thin approximation as

$$\tau_S(\mu_S, z, z') = \exp\left(-c_S|\psi(z) - \psi(z')|\right) \tag{14}$$

where $\psi(z) = p(z)/p_{\mathrm{rad}}$ and μ_S is the solar zenith angle. The reference pressure, p_{rad} ==1000 hPa, is introduced so that that the transmissivity of the atmosphere computed from the surface $p \simeq p_{rad}$ to the top of the atmosphere ($z = \infty$ and $p = 0$) is $\tau_S(\mu, 0, \infty) = \exp(-cs)$. We choose $c_S = 0.005$ is the attenuation of shortwave radiation at pressure height p_{rad}.

Regarder cette histoire d'angle direct et diffus.

For the long-wave transmission, we use the strong absorption approximation

$$\tau(\mu, z, z') = exp\left(-c_L\sqrt{|\psi(z') - \psi(z)|}\right) \tag{15}$$

$$\psi(z) = \left(\frac{p(z)}{p_{\mathrm{rad}}}\right)^2, \tag{16}$$

with $C_L = 2.53\ \mathrm{m}^{-1}$, so that the transmission of the whole atmospheric columns is $\tau(\mu 0, 0, \infty) = \exp(-C_L)$=0.08.

Vérifier les valeurs

0.1.5 Integration in the climate model

The warming or cooling of a layer is affected by the net change in radiation with height, and is derived from the conservation of energy equation. This gives an expression for the temperature tendency,

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_p}\frac{\partial F_{rad}}{\partial z}, \tag{17}$$

where $c_p = 1004\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific heat capacity of air (Arya, 1988) and $F_{rad} = F_L(z) + F_S(z)$

Finally, the change in net radiative surface boundary flux with respect to the surface temperature is

$$\frac{\partial F_{rad}(l)}{\partial T_s} = 4\epsilon\sigma T_s^3 \tau_L(0, l). \tag{18}$$

This relation is used for the implicit coupling at the boundary.

The radiative transfer model is forced by solar radiation, which is time-dependent and set by the planet's astronomical parameters (e.g., obliquity, which determines seasons) and the diurnal (day/night) cycle. In this paper we set these parameters to their Earth values, which are listed in Table 1. The physics package includes a flag that activates diurnal radiation forcing.

Parameter	**Value**
Solar constant	$1370\,\mathrm{W\,m^{-2}}$
Perihelion distance	$150 \times 10^{6}\,\mathrm{km}$
Ellipticity	0
Orbital period	1 year
Planet radius	$6371\,\mathrm{km}$
Gravitational acceleration g	$9.8\,\mathrm{m\,s^{-2}}$
Reference temperature T_0	$285\,\mathrm{K}$
Obliquity	23.5°
Rotation Ω	$7.292 \times 10^{-5}\,\mathrm{rad\,s^{-1}}$
Day length	$24\,\mathrm{h}$

Table 1. Astronomical parameters used for the simple physics package (based on Earth values).

References

Arya, S. P.: Chapter 4 Soil Temperatures and Heat Transfer, in: Introduction to Micrometeorology, vol. 42 of *International Geophysics*, pp. 37–48, Academic Press, https://doi.org/https://doi.org/10.1016/S0074-6142(08)60419-2, 1988.

Pierrehumbert, R. T.: Principles of Planetary Climate, Cambridge University Press, https://doi.org/10.1017/CBO9780511780783, 2010.