Le rayonnement, moteur du climat

Jean-Louis Dufresne
jean-louis.dufresne@lmd.ipsl.fr

Laboratoire de Météorologie Dynamique et Institut Pierre-Simon Laplace

"MathsInFluids" , Lyon, 14 avril 2022

Le rayonnement, moteur du climat

I. Bilan radiatif et température des planètes

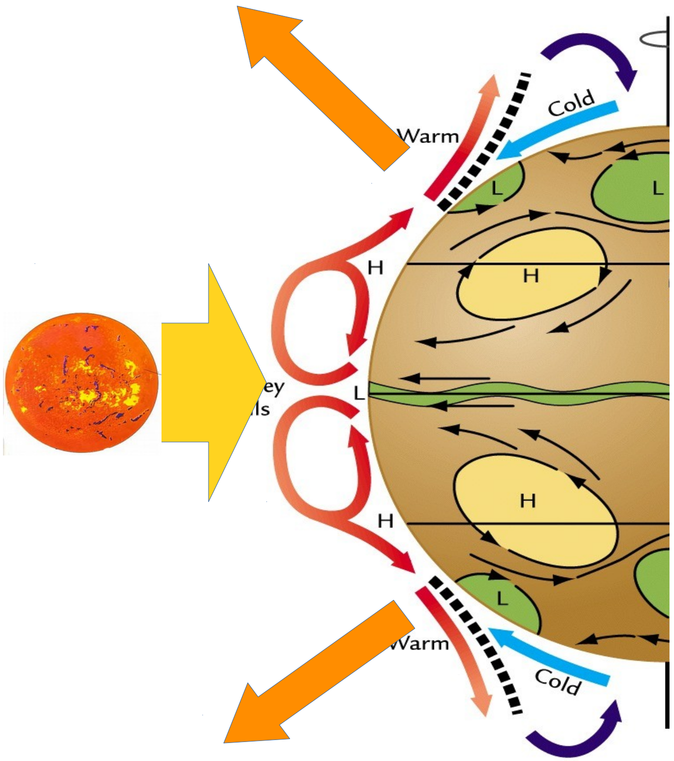
II. Echanges radiatifs dans les atmosphères

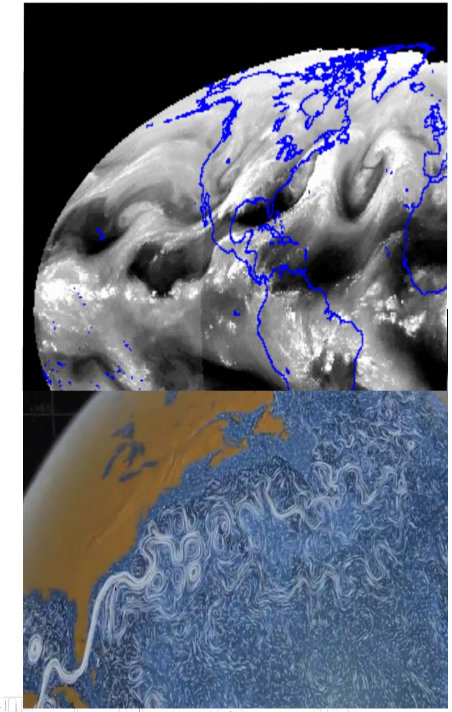
III. Aspects spectraux: intéraction avec les gaz et effet de serre

IV. Accroissement de CO_2 et H_2O à profile de température fixe

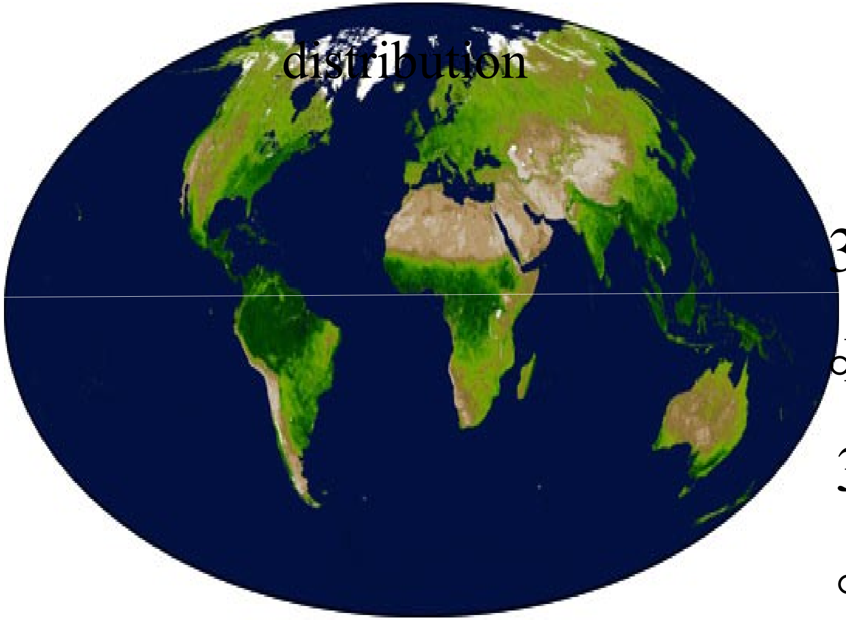
V. Aspects géométriques: intéractions avec les nuages

Climate: radiation, rotation, circulation





That drives, e.g. the vegetation distribution



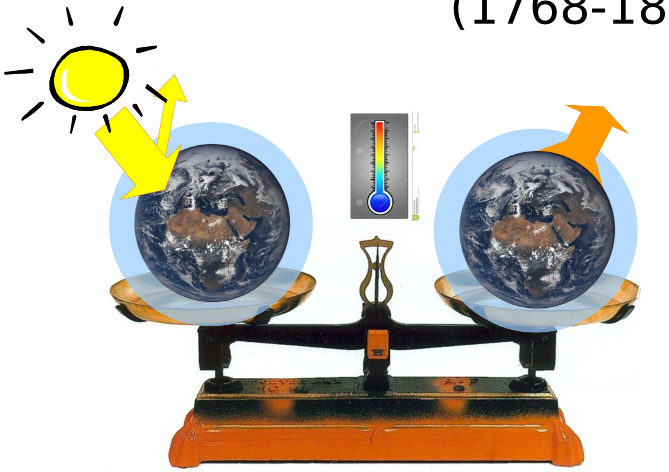
NASA

Emergence of the physics of climate

J. Fourrier: *Mémoire sur les températures du globe terrestre et des espaces planétaires*, 1824

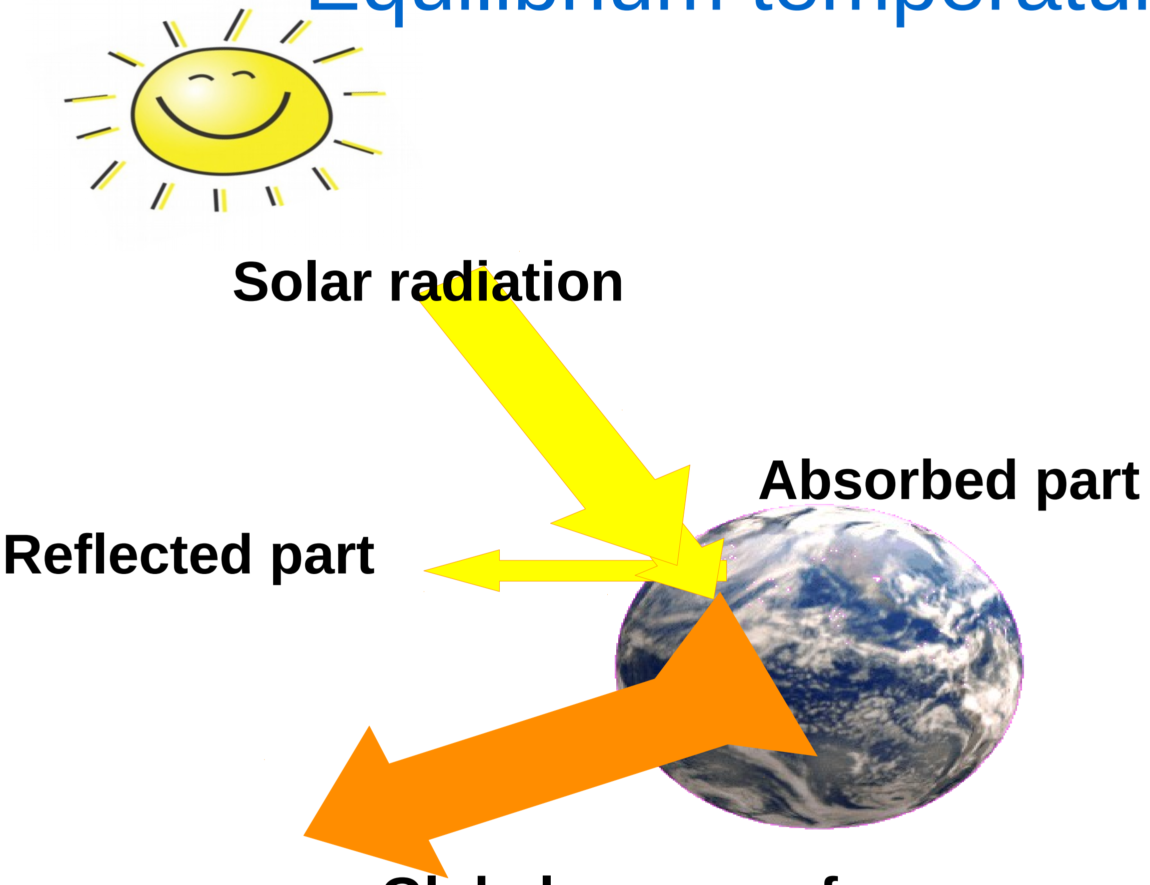
Joseph Fourrier
(1768-1830)

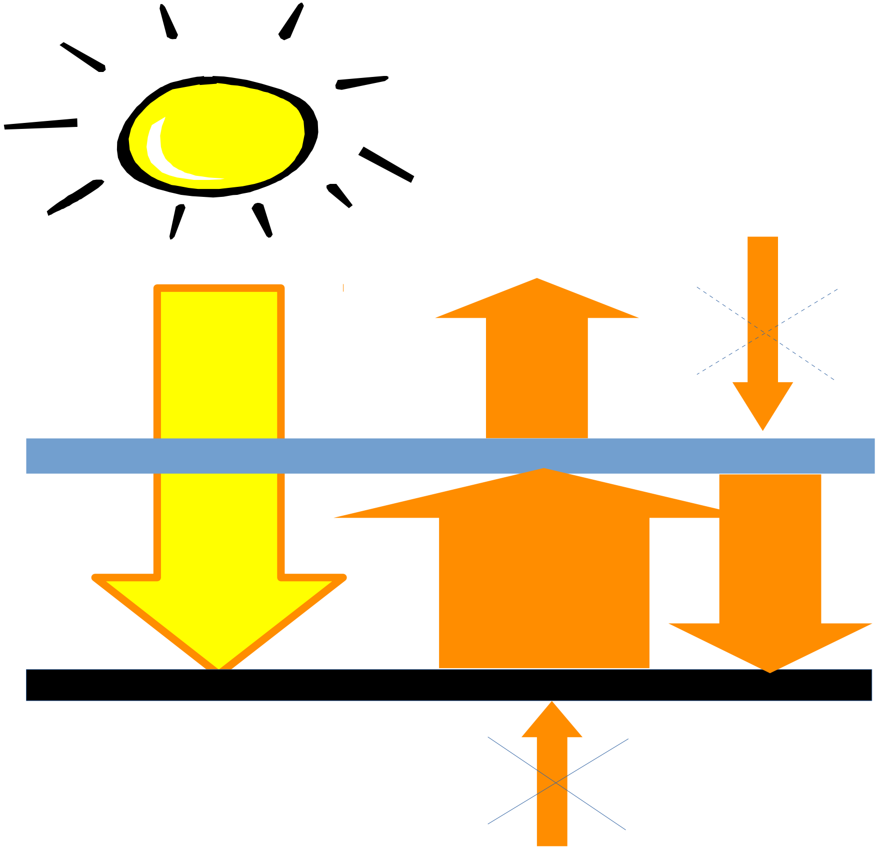
- He consider the Earth like any other planet
- The **energy balance equation** drives the temperature of all the planets
- The major heat transfers are
  1. **Solar radiation**
  2. **Infra-red radiation**
  3. Diffusion with the interior of Earth



- He formulates the principle of the **greenhouse effect**
- He envisages that ***climate may change***

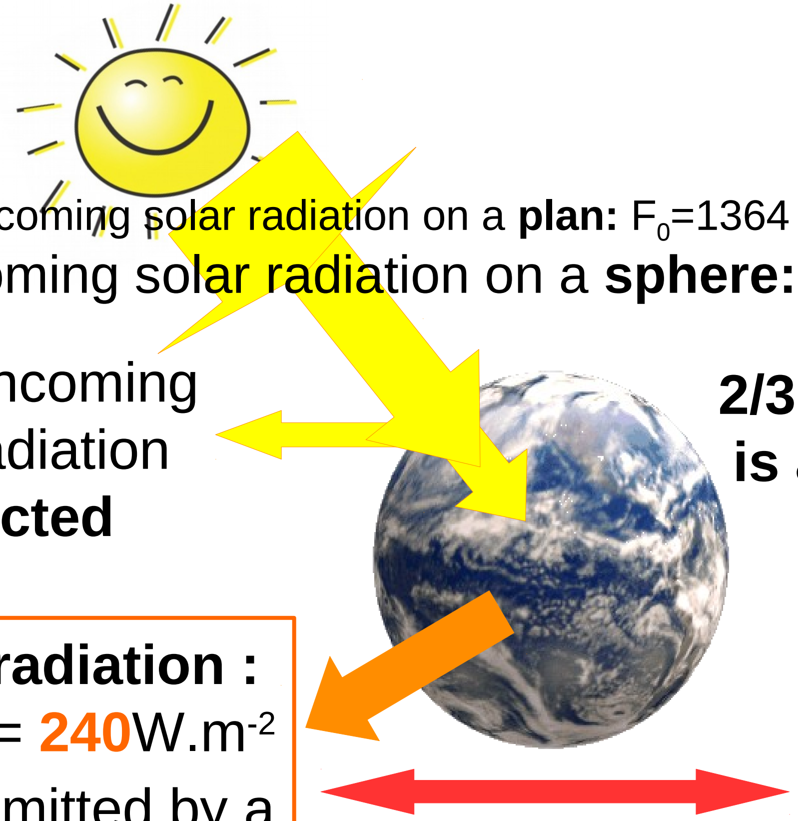
Equilibrium temperature of a planet





Global mean surface temperature is the result of the global energy budget

Equilibrium radiative temperature of Earth



Incoming solar radiation on a **plan:** F_0=1364 W.m^{-2}

Incoming solar radiation on a **sphere:** $F_s=F_0/4$ = 341 W.m^{-2}

1/3 of incoming solar radiation **is reflected**

2/3 of incoming solar radiation **is absorbed :** F_a = 240W.m^{-2}

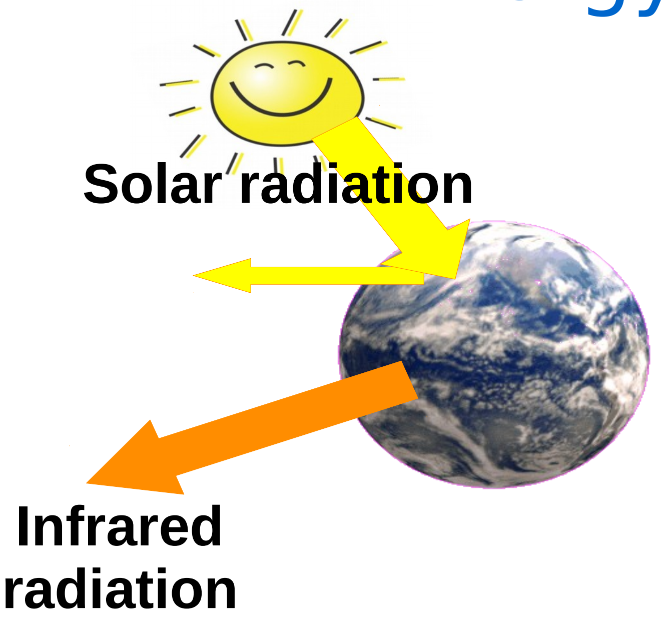
Emitted longwave radiation :
F_e = **240**W.m^{-2}
Same as the flux emitted by a black body at T_s= 255K (-18°C)

Greenhouse effect
$G=F_s-F_e$

Global mean surface temperature is 15°C

Emitted longwave radiation:
F_s = **390**W.m^{-2}

Energy balance model (0D)



Equilibrium solution:

$$T_s = f(I_0, A, \epsilon_a, \ldots)$$

I_0: incoming solar radiation

A: planetary albedo

ϵ_a: planetary emissivity

Response to a perturbation:

$$\Delta T_s = \frac{\Delta N - \Delta Q}{\lambda}$$

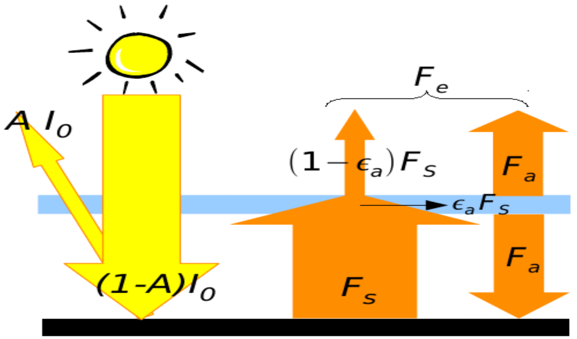
ΔT_s: temperature response

ΔQ: radiative forcing

ΔN: heat imbalance (ocean)

λ: climate feedback parameter

Single layer greenhouse model



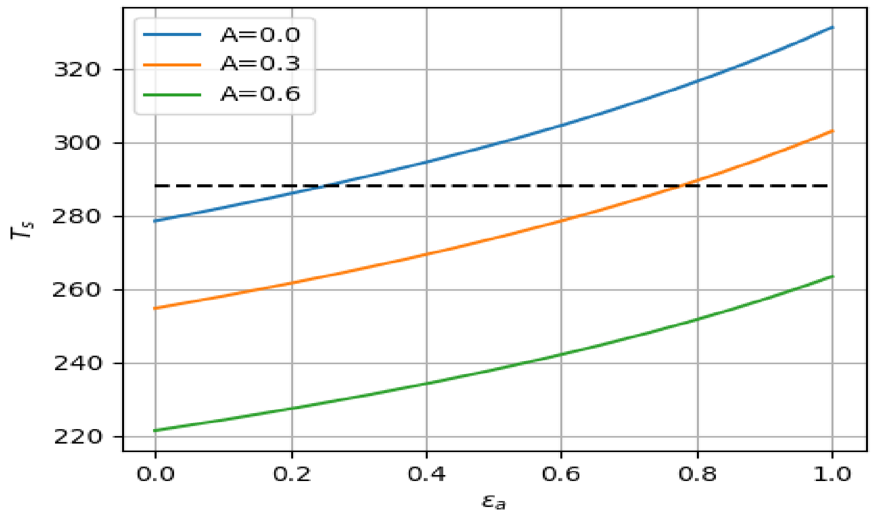
I_0: incoming solar radiation

A: planetary albedo

ε_a: planetary emissivity

- Surface temperature: $$\sigma T_s^4 = \frac{(1-A)I_0}{1-\epsilon_a/2}$$

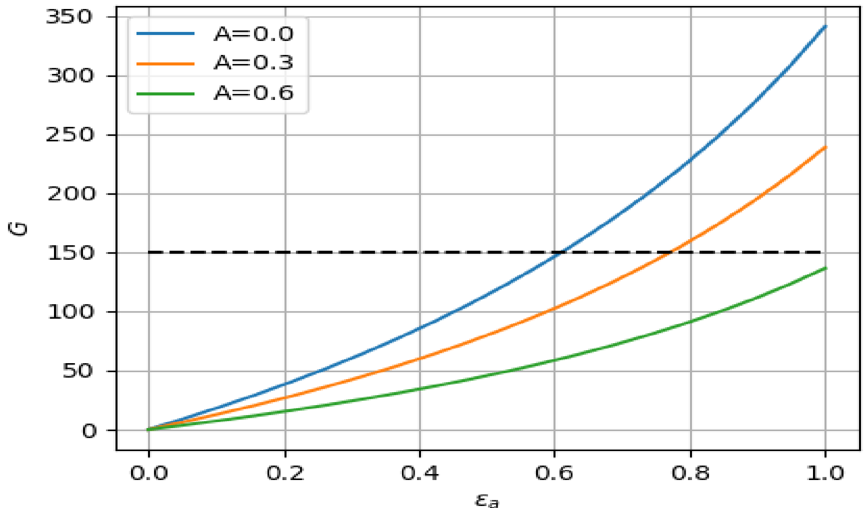
$$T_s = f(\epsilon_a)$$



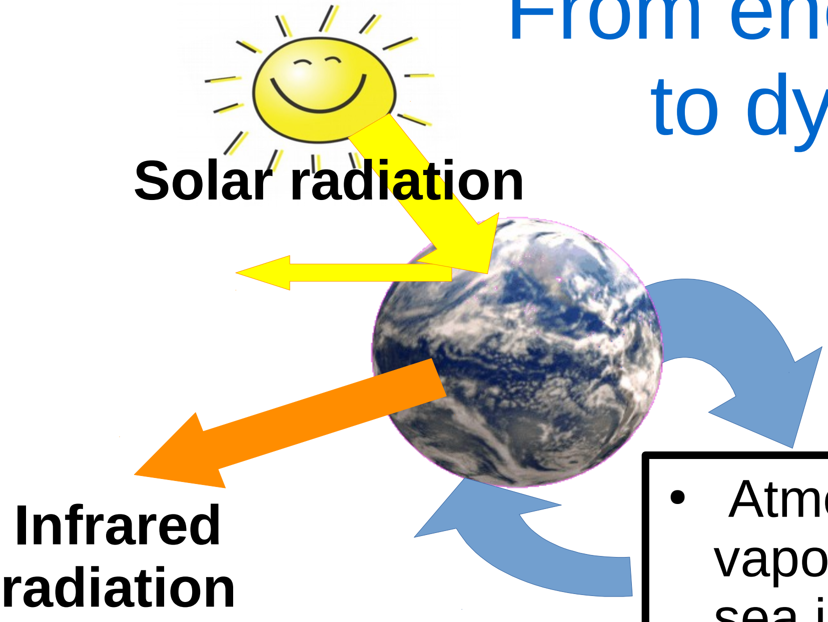
The greenhouse effect

$$G = F_s - F_e = (1-A)I_0\left(\frac{1}{1-\epsilon_a/2} - 1\right)$$

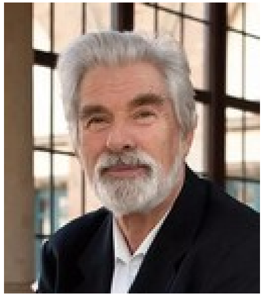
$$G = f(\epsilon_a)$$



From energy balance models to dynamical systems



	Time constant (years)
Atmospheric circulation, water vapor, clouds, snow, surface ocean, sea ice, etc.	10
Same + ocean circulation, CO2 continental biosphere, CH4, etc.	100
Ice sheets, continental CO2	1000
Geological" CO2 (continental erosion, volcanism)	$>10^6$

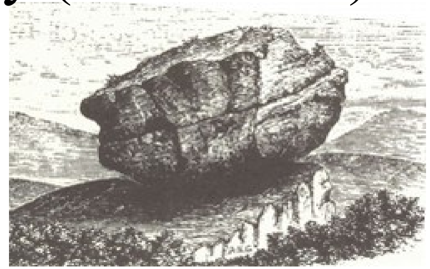


Klaus Hasselmann (1931-)

Nobel prize in Physics 2021

Paleo climate changes

The discovery (1840-1860)

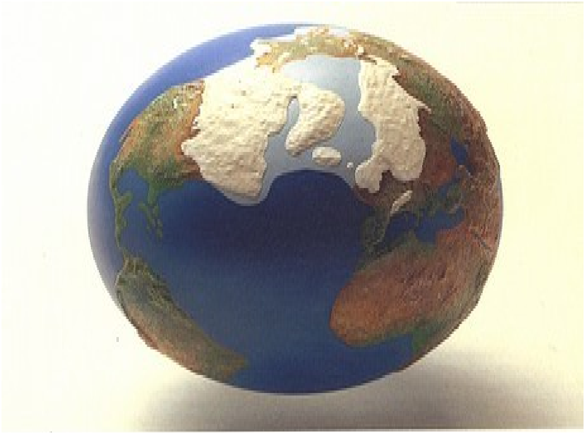


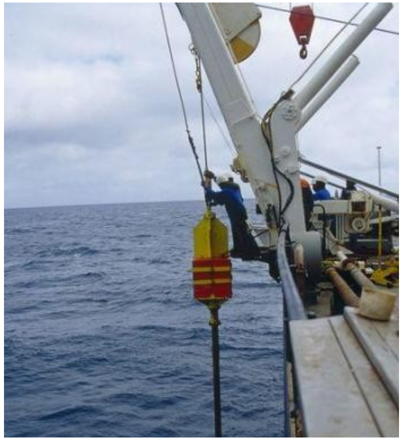
J. de Charpentier

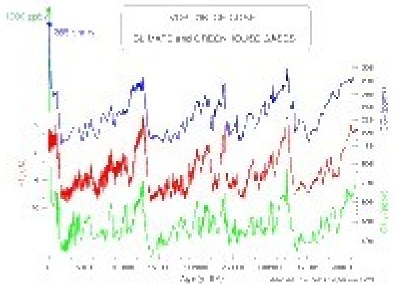
Blocs erratiques

L. Agassiz

The detailed description (1970-)





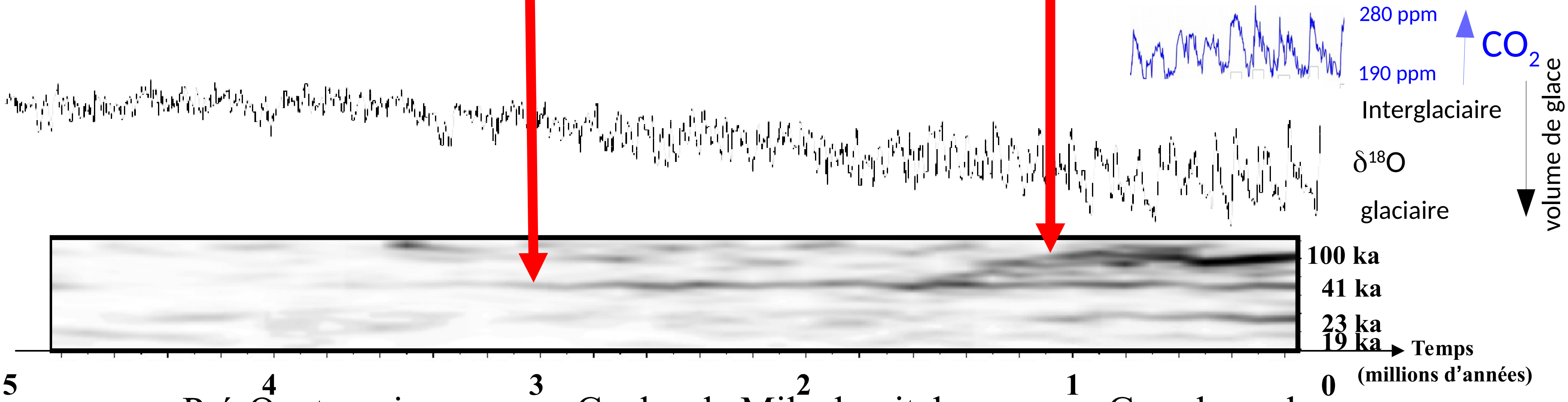


Ice volume change on paleo time scales

Pas de grande calotte dans l'hémisphère Nord

Apparition des cycles de 41 000 ans

Apparition des cycles de 100 000 ans



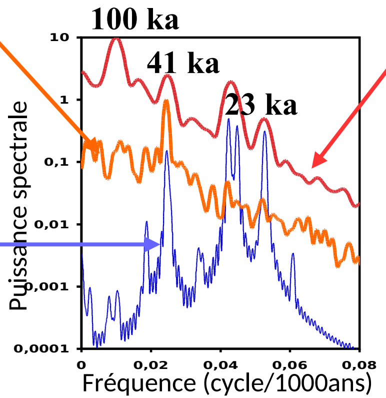
Pré-Quaternaire

Cycles de Milankovitch

1-3 Ma

« Grands cycles »

0-1 Ma



(V19-30, ODP 677/846, Shackleton et al., 1990)
(Dome C, Lüthi et al., 2008)
(Berger, 1978)

Insolation 65°N été

Les cycles de 100 ka ne s'expliquent pas simplement avec la théorie de Milankovitch

By courtesy of Didier Paillard

Le rayonnement, moteur du climat

I. Bilan radiatif et température des planètes

II. Echanges radiatifs dans les atmosphères

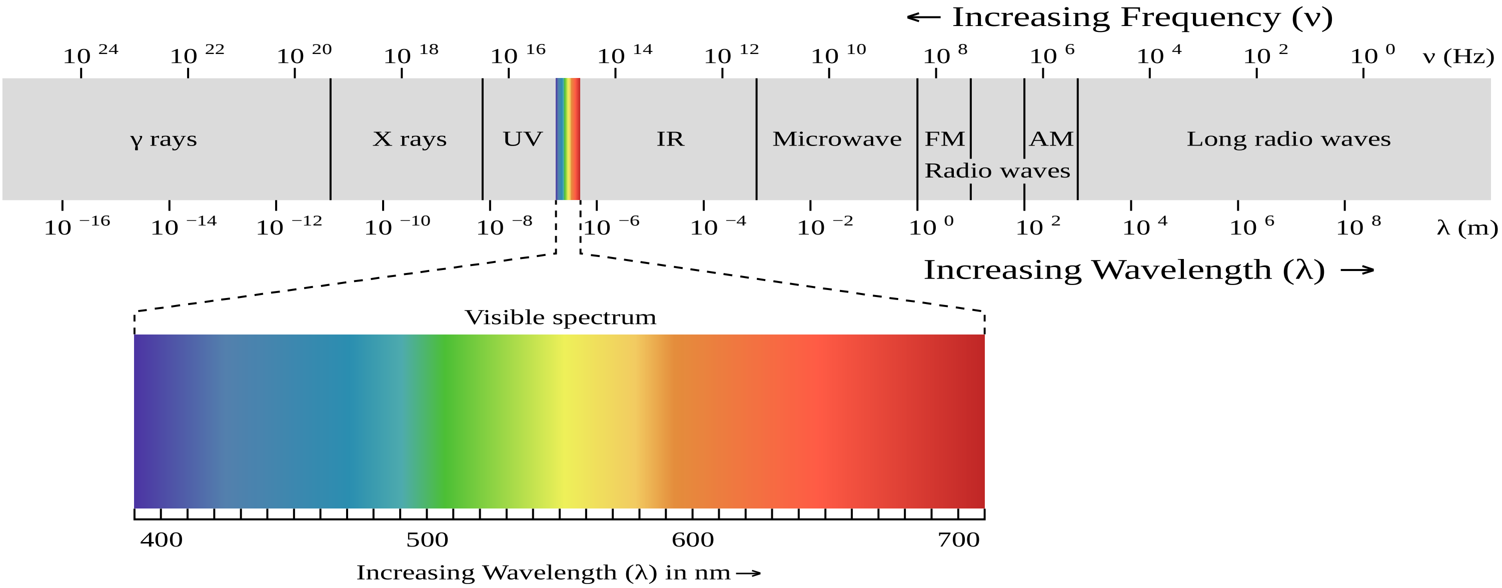
III. Aspects spectraux: intéraction avec les gaz et effet de serre

IV. Accroissement de CO_2 et H_2O à profile de température fixe

V. Aspects géométriques: intéractions avec les nuages

Radiative transfer : basis

Electromagnetic spectrum : electromagnetic waves are characterized by their wavelength λ, frequency $\nu = c/\lambda$ or wavenumber $\bar{\boldsymbol{\nu}} = 1/\lambda$



We will mainly consider two spectral domains:

- Short wave (SW) radiation, or solar radiation (0.4 – 4 µm)
- Long wave (LW) radiation, or (thermal) infrared radiation (4-100 µm)

Radiative transfer : basis

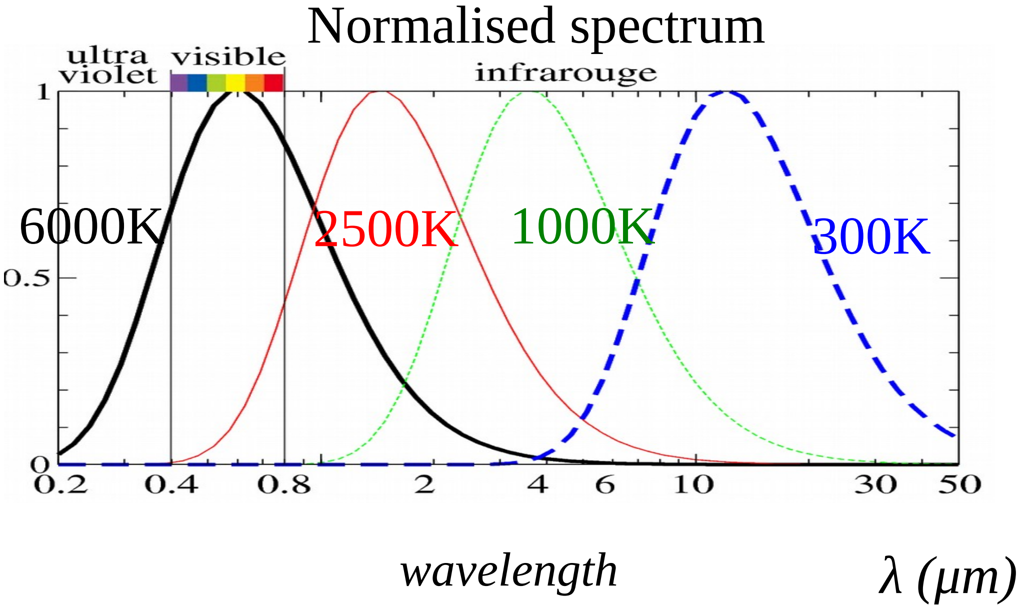
"Black body" emission

Planck law :

$$B_\lambda(T) = \frac{C_1 \lambda^{-5}}{\pi \left(e^{C_2/\lambda T} - 1\right)}$$

B in $W.m^{-2}.\mu m^{-1}.sr^{-1}$

T in K, C_1 et C_2 are constants



Stefan-Boltzmann law (integral of the Planck law over the whole spectrum and over one hemisphere). Power *F* lost by emission of radiation by a body of temperature *T* :

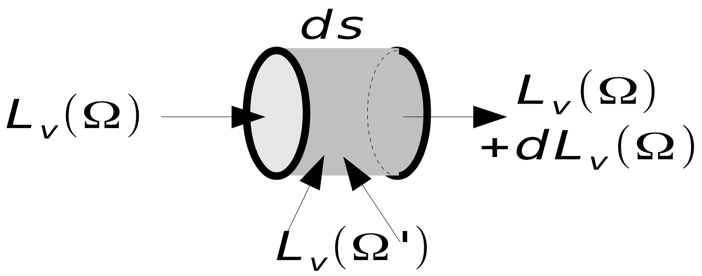
$$F = \epsilon \sigma T^4$$

With ε : emissivity (=1 black body)

σ = 5,67 10^{-8} : Stefan-Boltzmann constant

F in W.m-2, T in K

Radiative transfer equation



$$\frac{dL_v(\Omega)}{ds} = \underbrace{\kappa_{a,v}\left(B_v(T) - L_v(\Omega)\right)}_{\text{emission - absorption}} + \underbrace{\kappa_{s,v}\left(\int_{4\pi} L_v(\Omega')P(\Omega',\Omega)\,d\Omega' - L_v(\Omega)\right)}_{\text{scattering}}$$

L_ν: spectral radiance, at frequency ν and in a solid angle Ω

B_ν(T): black body emission, at frequency ν and temperature (T) (Planck function)

$\kappa_{a,v}$, $\kappa_{s,v}$: absorption and scattering coefficient, at frequency ν

P(Ω,Ω‘): phase function

The RT equation must be integrated over the entire space (Ω,s) and over frequency (v)

$\kappa_{a,v}$***: same coefficient for absorption and emission***. Absorption and emission properties are equals

Radiative transfer equation : ***no-scattering media***

$$\frac{d L_\nu(\Omega)}{d s} = \kappa_{a,\nu}\left(B_\nu(T) - L_\nu(\Omega)\right) + \kappa_{s,\nu}\left(\int_{4\pi} L_\nu(\Omega') P(\Omega', \Omega) d\Omega' - L_\nu(\Omega)\right)$$

Media with a black boundary in s_0:

$$L_\nu(s, \Omega) = B_\nu(T(s_0)) e^{-\tau(s_0, s)} + \int_{s_0}^{s} B_\nu(T(s')) \kappa_{a,\nu} e^{-\tau(s, s')} ds'$$

with: $\tau(s, s') = \int_{s}^{s'} \kappa_{a,\nu} ds''$

τ, optical thickness

emitted by the boundary

Transmitted from the boundary in s_0 to s

emitted by the media

transmitted from s to s'

Radiative transfer equation : ***no-scattering media***

$$\frac{dL_\nu(\Omega)}{ds} = \kappa_{a,\nu}\left(B_\nu(T) - L_\nu(\Omega)\right) + \kappa_{s,\nu}\left(\int_{4\pi} L_\nu(\Omega')P(\Omega',\Omega)\,d\Omega' - L_\nu(\Omega)\right)$$

Medium with a black boundary in s_0:

$$L_\nu(s,\Omega) = B_\nu(T(s_0))e^{-\tau(s_0,s)} + \int_{s_0}^{s} B_\nu(T(s'))\kappa_{a,\nu}e^{-\tau(s,s')}ds'$$

with: $\tau(s,s') = \int_{s}^{s'} \kappa_{a,\nu}\,ds''$ τ, optical thickness

$$L_\nu(s,\Omega) = B_\nu(T(s_0))\Gamma_\nu(s_0,s) + \int_{s_0}^{s} B_\nu(T(s'))\frac{d\Gamma_\nu(s,s')}{ds}ds'$$

with: $\Gamma_\nu(s,s') = e^{-\tau_\nu(s,s')}$ transmissivity

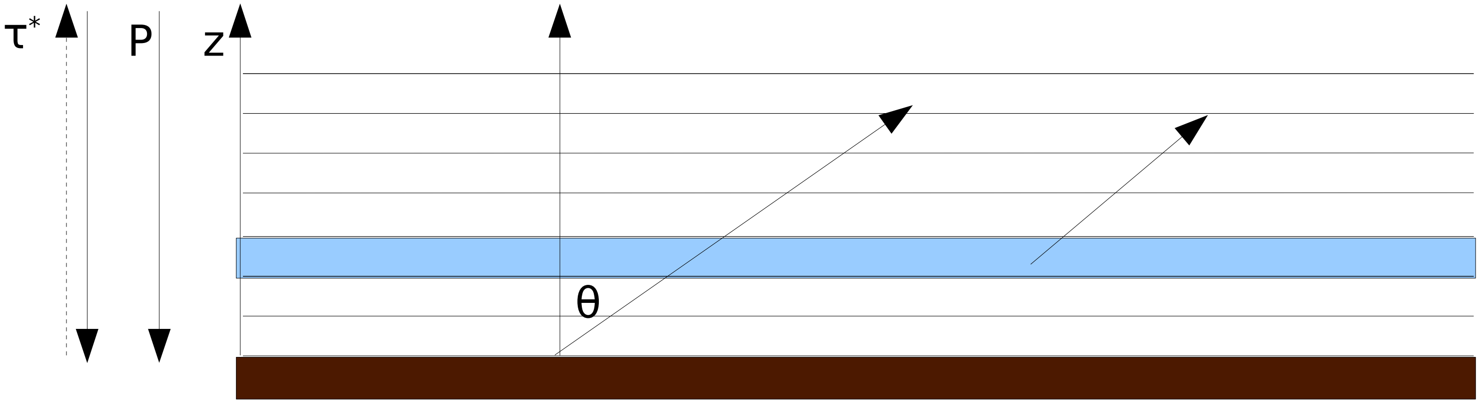
Isotherm semi-infinite medium: $L_\nu(s,\Omega) = B_\nu(T_0)$ ≡ black body emission

Semi infinite medium:

- If some absorption:
  - perfect absorbent
  - black body emission
- If some scattering:
  - perfect reflector
  - “white” reflection

Radiation in a stratified atmosphere

- Plan parallel approximation: horizontally infinite and homogeneous atmosphere
- It is common to separate the radiative flux into upward and downward flux



Pressure: $P(z)=\int_0^z \rho(z)\, g\, dz$

Massic extinction coef.

Optical thickness: $\tau_\nu(z_1, z_2, \mu)=\left|\int_{z_1}^{z_2} k_\nu \rho(z) \frac{dz}{\mu}\right|$ $\quad \mu=\cos(\theta)$ $\quad \kappa_{a,\nu}=k_{a,\nu}\rho$

$\tau_\nu(P_1, P_2, \mu)=\left|\int_{P_1}^{P_2} \frac{k_\nu}{g\mu} dP\right|$ $\quad \tau_\nu^*(P)=\tau_\nu(0, P, 1)$

Radiation in a stratified atmosphere

Radiative transfer equation without scattering:

$$\frac{d L_v(\Omega)}{d s} = \kappa_{a,v}\left(B_v(T) - L_v(\Omega)\right)$$

$$\frac{d L_\nu^\uparrow(\mu)}{d P} = -\frac{k_{a,\nu}}{g\mu}\left(B_\nu(T) - L_v(\mu)\right)$$

Hydrostatic equilibrium:

$$dP = -\rho g\, dz = -\rho g \mu\, ds$$

Massic extinction coef.

$$\kappa_{a,v} = k_{a,v}\rho$$

Transmission and optical thickness :

$$\Gamma_\nu = e^{-\tau_\nu}$$

$$\tau_\nu(s_1, s_2) = \int_{s_1}^{s_2} \kappa_{a,\nu}(s)\, ds$$

$$\tau_\nu(P_1, P_2, \mu) = \left|\int_{P_1}^{P_2} \frac{k_{a,\nu}(P)}{g\mu}\, dP\right|$$

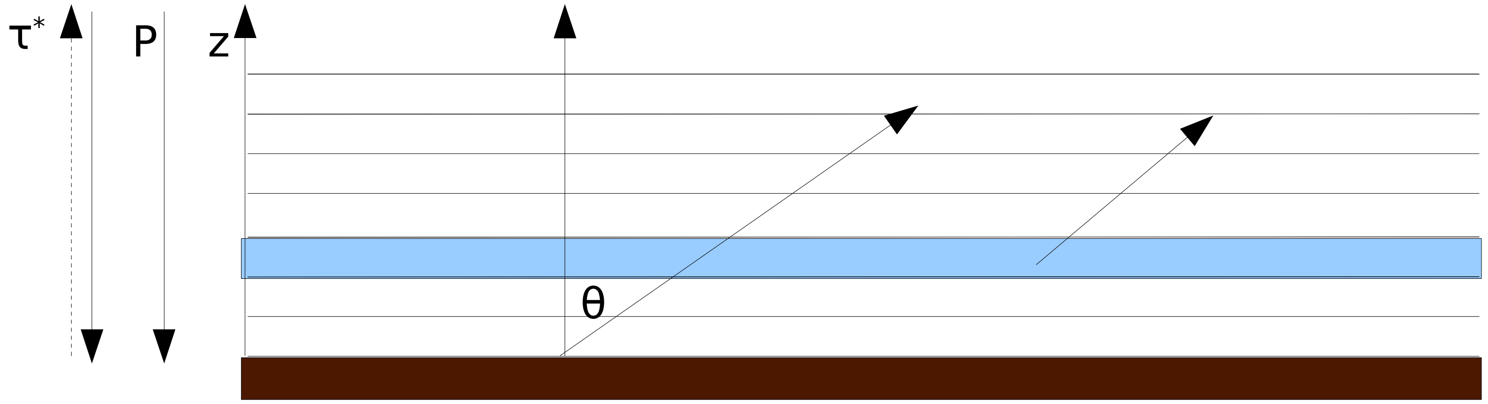
$$d\Gamma_\nu = -\Gamma_\nu \frac{k_{a,\nu}}{g\mu}\, d\tau_\nu$$

$$\frac{d L_\nu^\uparrow(\mu)}{d P} = -\frac{\partial \Gamma_\nu(P, P, \mu)}{\partial P}\left(B_\nu(T) - L_\nu^\uparrow(\mu)\right)$$

$$\frac{d L_\nu^\downarrow(\mu)}{d P} = -\frac{\partial \Gamma_\nu(P, P, \mu)}{\partial P}\left(B_\nu(T) - L_\nu^\downarrow(\mu)\right)$$

Flux: $F_\nu^\uparrow(P) = 2\pi \int_0^1 L_\nu^\uparrow(P, \mu)\, d\mu$

Radiation in a stratified atmosphere



$L_\nu^\uparrow(P,\mu), F_\nu^\uparrow(P)$

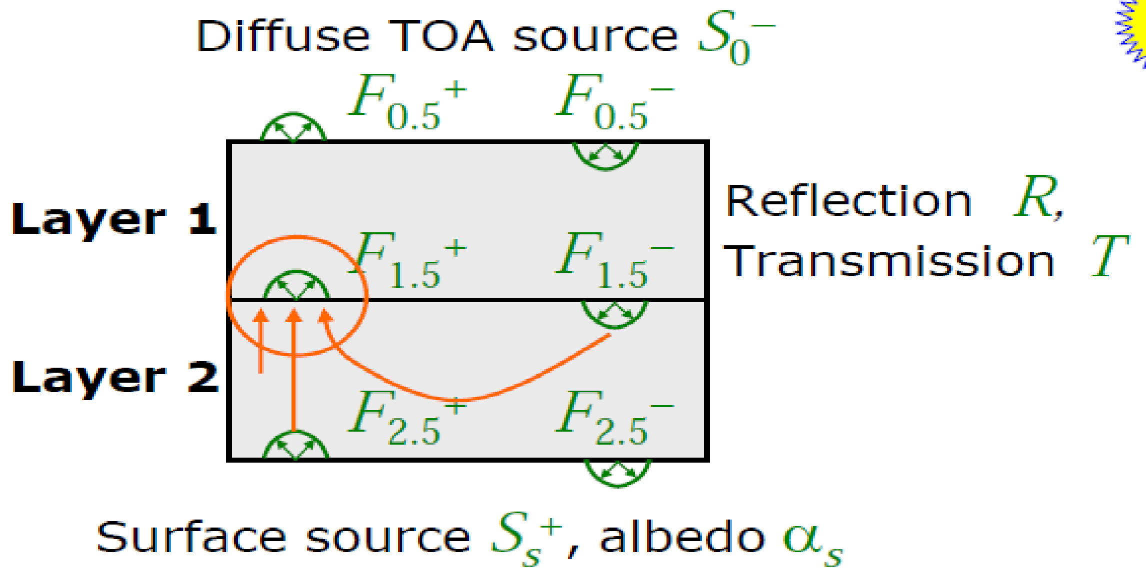
Radiance: $$L_\nu^\uparrow(P,\mu)=B_\nu(T_s)\Gamma_\nu(P_s,P,\mu)-\int_{Ps}^{P} B_\nu(T(P'))\frac{\partial\Gamma_\nu(P,P',\mu)}{\partial P'}dP'$$

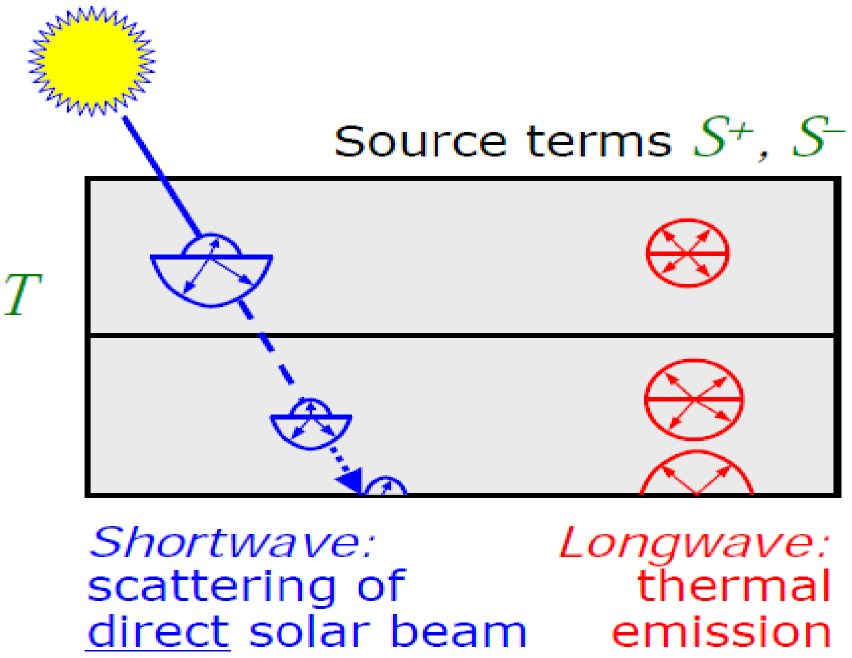
Flux: $$F_\nu^\uparrow(P)=\pi B_\nu(T_s)\mathring{\Gamma}_\nu(P_s,P)-\int_{Ps}^{P} \pi B_\nu(T(P'))\frac{\partial\mathring{\Gamma}_\nu(P,P')}{\partial P'}dP'$$

With the slab transmittance: $$\mathring{\Gamma}_\nu(P_1,P_2)=2\int_0^1 \Gamma_\nu(P_1,P_2,\mu)\mu\, d\mu$$

$E_3(x)$: Exponential integral $$=2\int_0^1 e^{-\tau(P_1,P_2,0)/\mu}\mu\, d\mu=E_3(\tau(P_1,P_2,0))$$

Discretized two-stream scheme





- Equations relating diffuse fluxes between levels take the form:

$$F^+_{i-0.5} = T_i F^+_{i+0.5} + R_i F^-_{i-0.5} + S^+_i$$

- Terms T, R and S given by Meador and Weaver (1980)

[R. Hogan lecture]

Le rayonnement, moteur du climat

I. Bilan radiatif et température des planètes

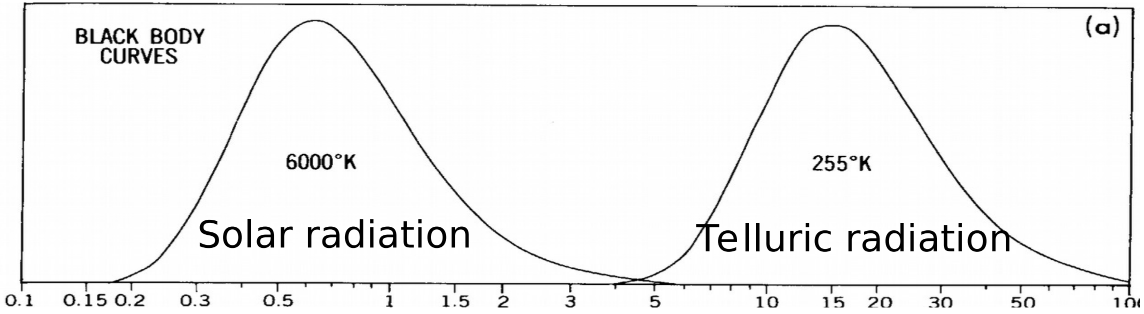
II. Echanges radiatifs dans les atmosphères

III. Aspects spectraux: intéraction avec les gaz et effet de serre

IV. Accroissement de CO_2 et H_2O à profile de température fixe

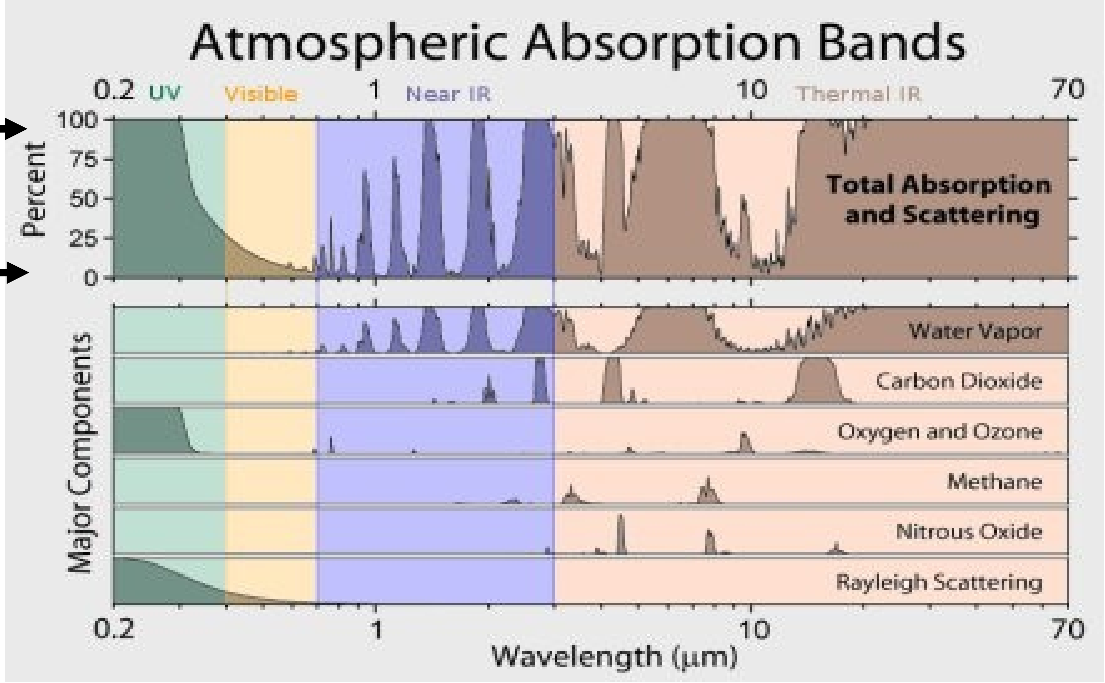
V. Aspects géométriques: intéractions avec les nuages

Absorption by gases

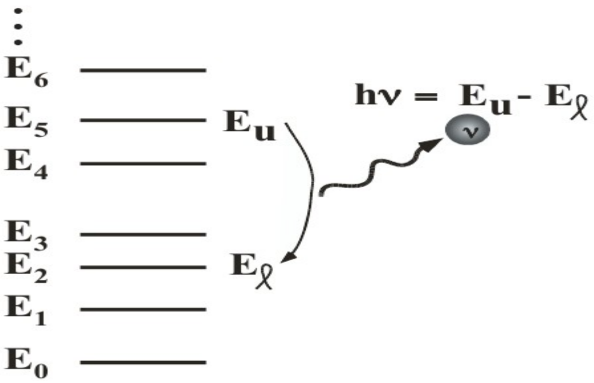


Opaque

Transparent

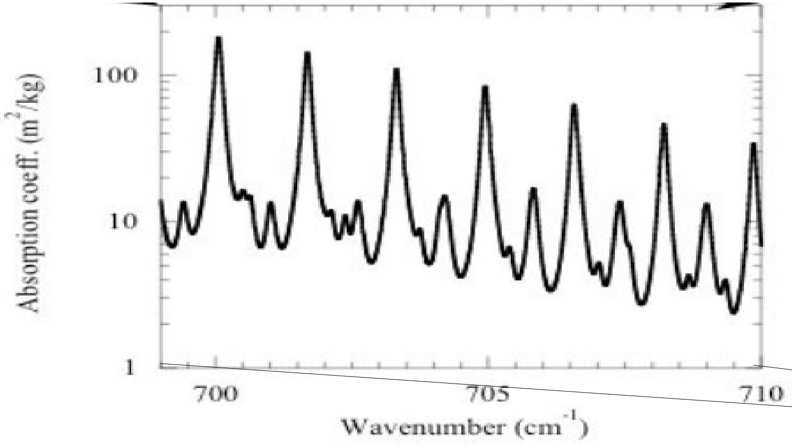


Absorption by gases

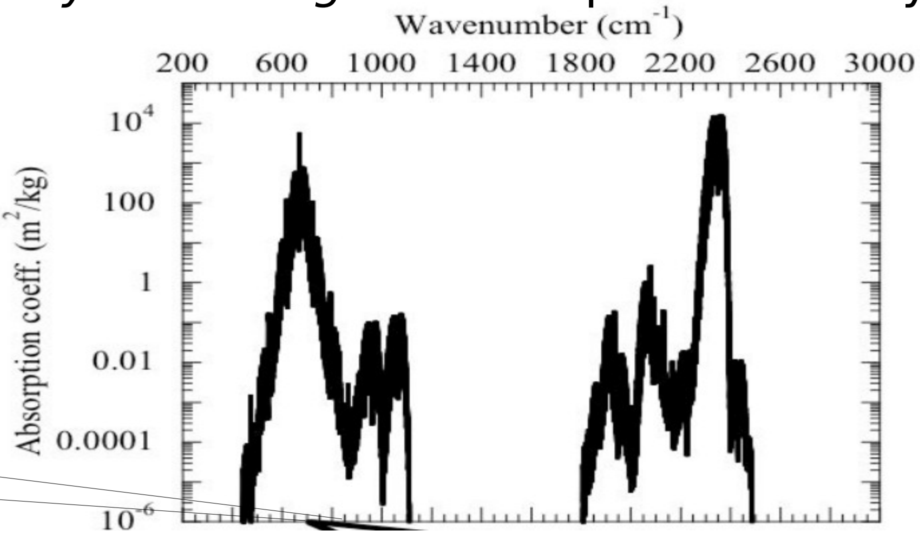


- **Line emission and absorption** by transition between states of different energy
- In the infra-red: level of energy correspond to change in vibration and rotation of molecules
- Complex molecules are better absorbent
- Line broadening (collision (Lorentz), Doppler)

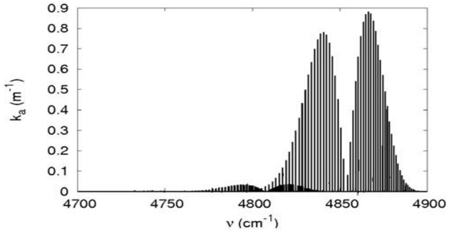
Large number of absorption (emission) lines (≈10^6)



Very wide range of absorption intensity

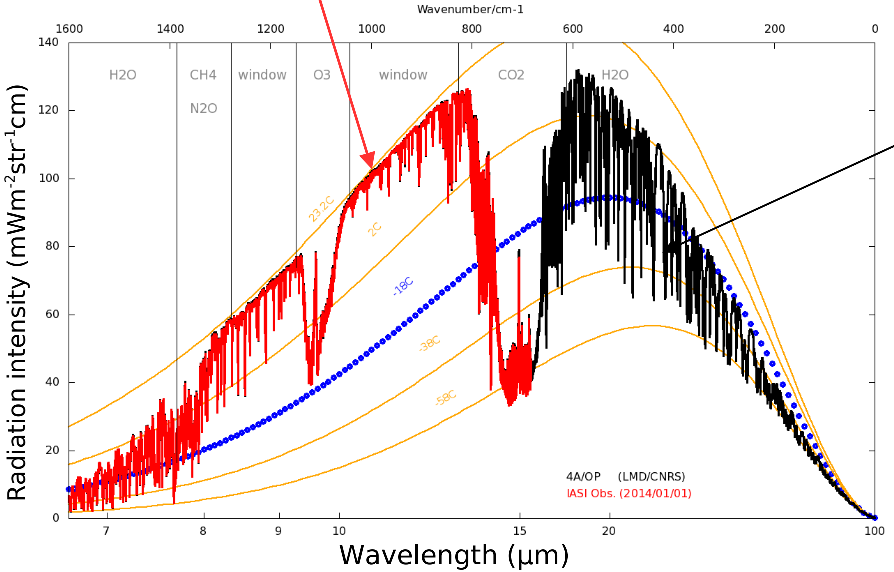


Line-by-line radiative transfer model



Gas radiative properties

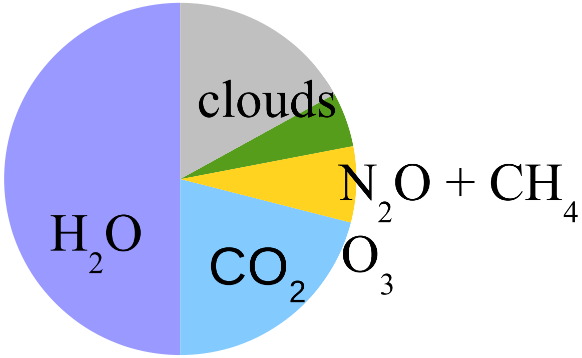
Atmospheric characteristics



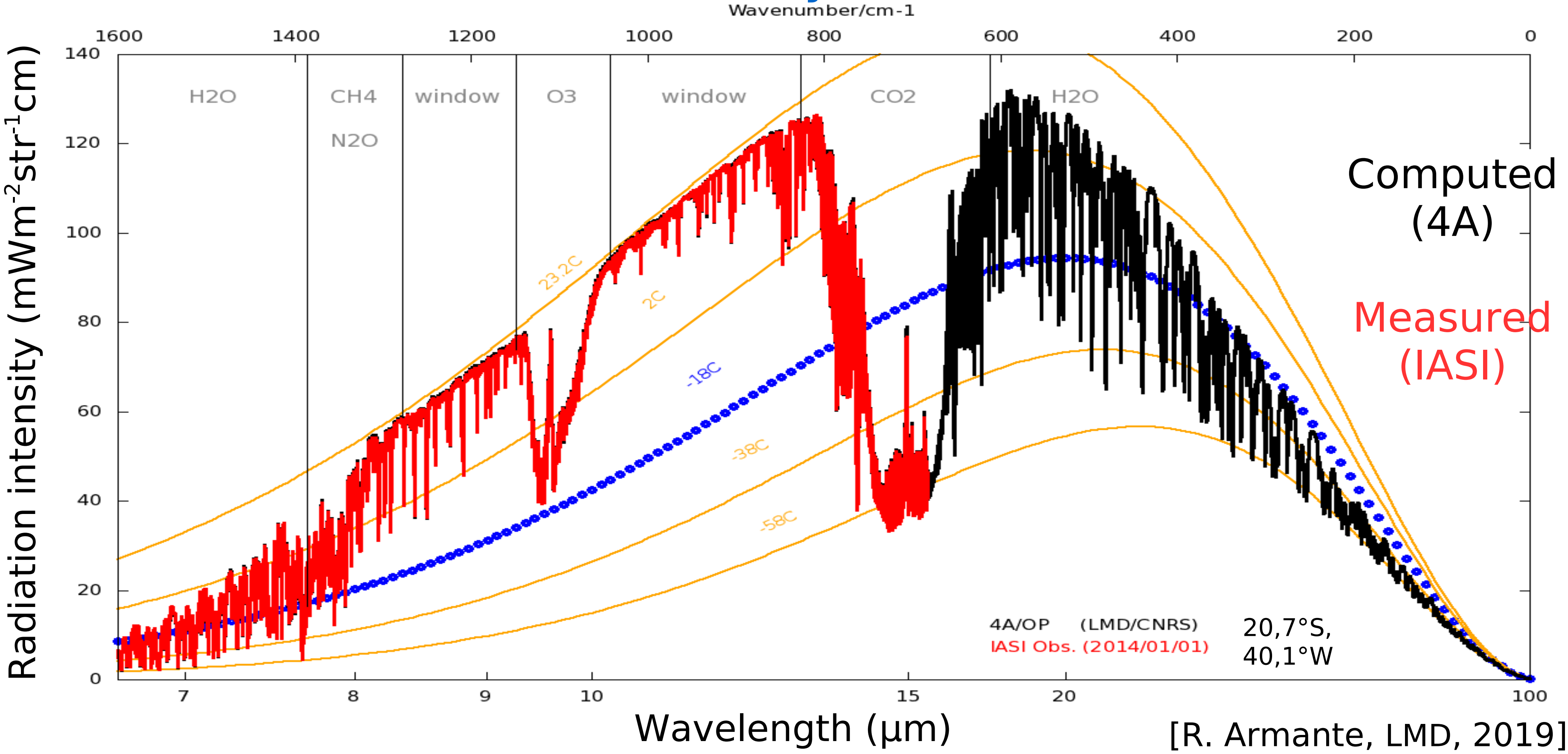
Contributions to current greenhouse effect

Greenhouse effect

$G=F_s-F_e$



Spectrum of the radiation emitted by the Earth as measured by satellites



[R. Armante, LMD, 2019]

Spectral integration: k-distribution model

Spectral radiance: $$L_\nu^{\uparrow}(P,\mu)=B_\nu(T_s)\Gamma_\nu(P_s,P,\mu)-\int_{Ps}^{P} B_\nu(T(P'))\frac{\partial \Gamma_\nu(P,P',\mu)}{\partial P'}dP'$$

$$\Gamma_\nu(P_1,P_2,\mu)=e^{-\tau_\nu(P_1,P_2,\mu)}\qquad \tau_\nu(P_1,P_2,\mu)=\left|\int_{P_1}^{P_2}\frac{k_{a,\nu}(P)}{g\mu}dP\right|$$

If $k_{a,v}$ is constant: $$\tau_\nu(P_1,P_2,\mu)=\left|k_{a,\nu}\frac{\Delta P}{g\mu}\right|$$

Frequency integral over Δν:

$$\bar{\Gamma}_{\Delta\nu}(P_1,P_2,\mu)=\int_{\Delta\nu} e^{-\tau_\nu(P_1,P_2,\mu)}\frac{d\nu}{\Delta\nu}=\int_{\Delta\nu} e^{-k_{a,\nu}\frac{\Delta P}{g\mu}}\frac{d\nu}{\Delta\nu}$$

$$=\int_0^\infty e^{-k\frac{\Delta P}{g\mu}}f(k)dk$$

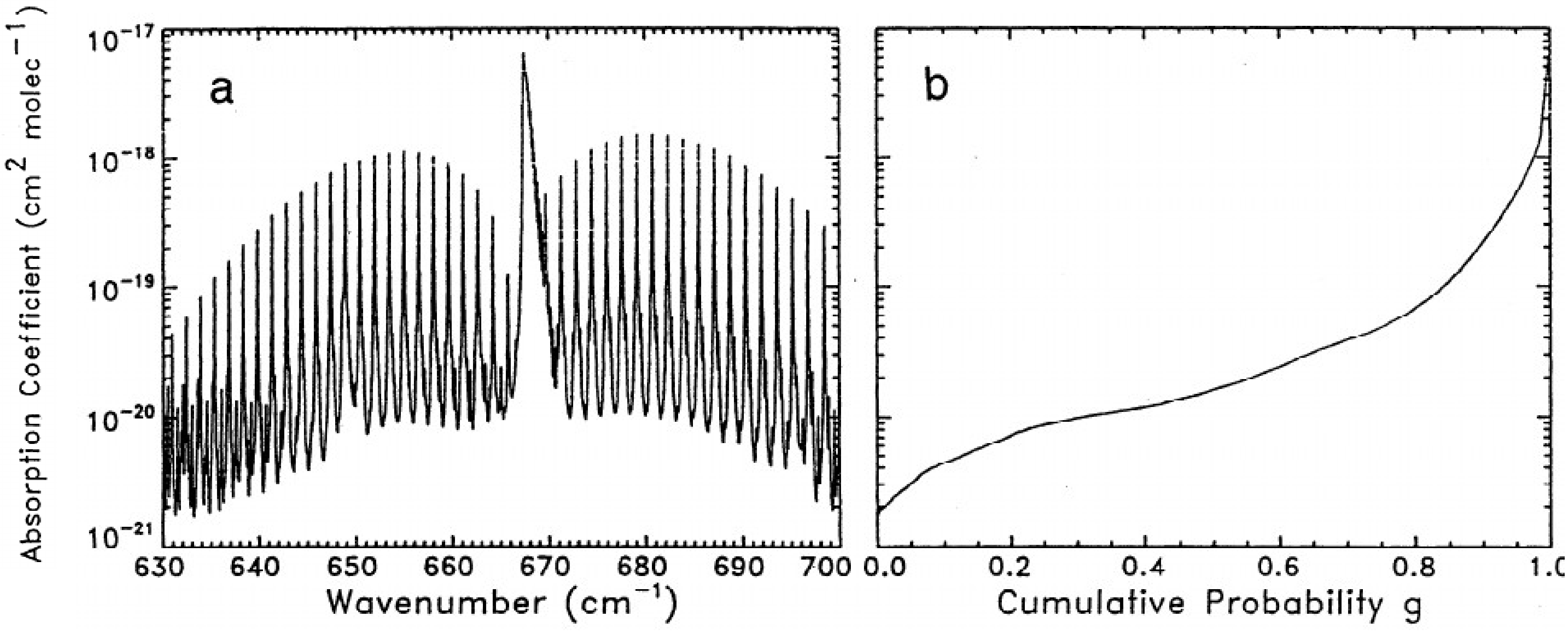
f(k): probability dens. funct. of *k* $$\int_0^\infty f(k)dk=1$$

$$=\int_0^1 e^{-k(h)\frac{\Delta P}{g\mu}}dh$$

h(k): cumulative dens. funct. of *k* $$h(k)=\int_0^k f(k)dk$$

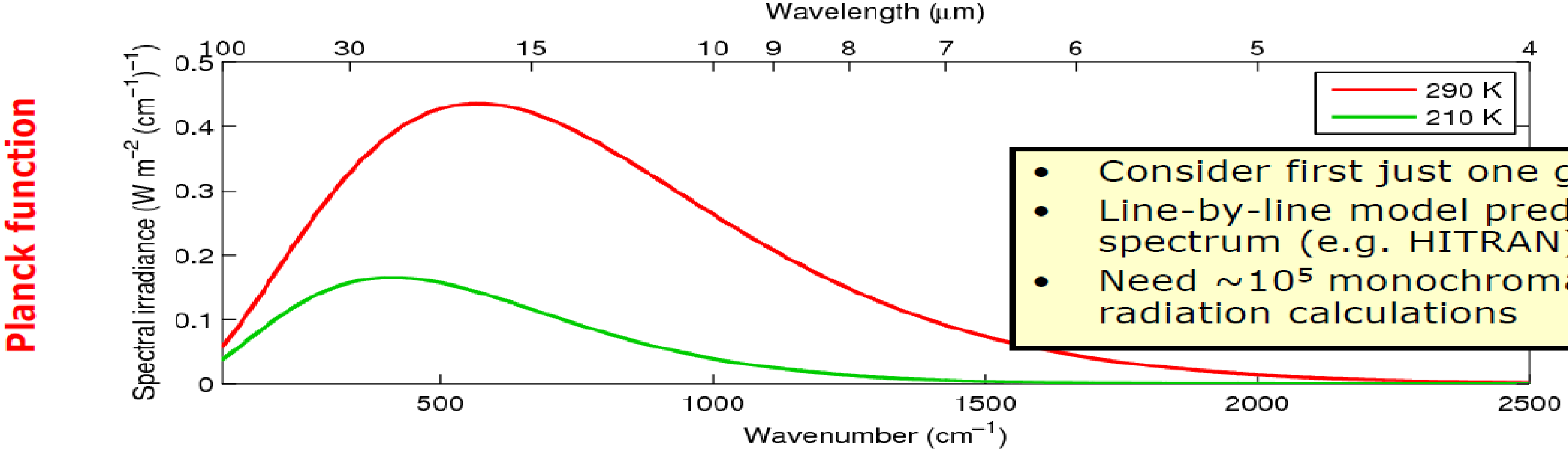
$$\approx\sum_{j=1}^{N} e^{-k(h_j)\frac{\Delta P}{g\mu}}\Delta h_i$$

k-distribution model

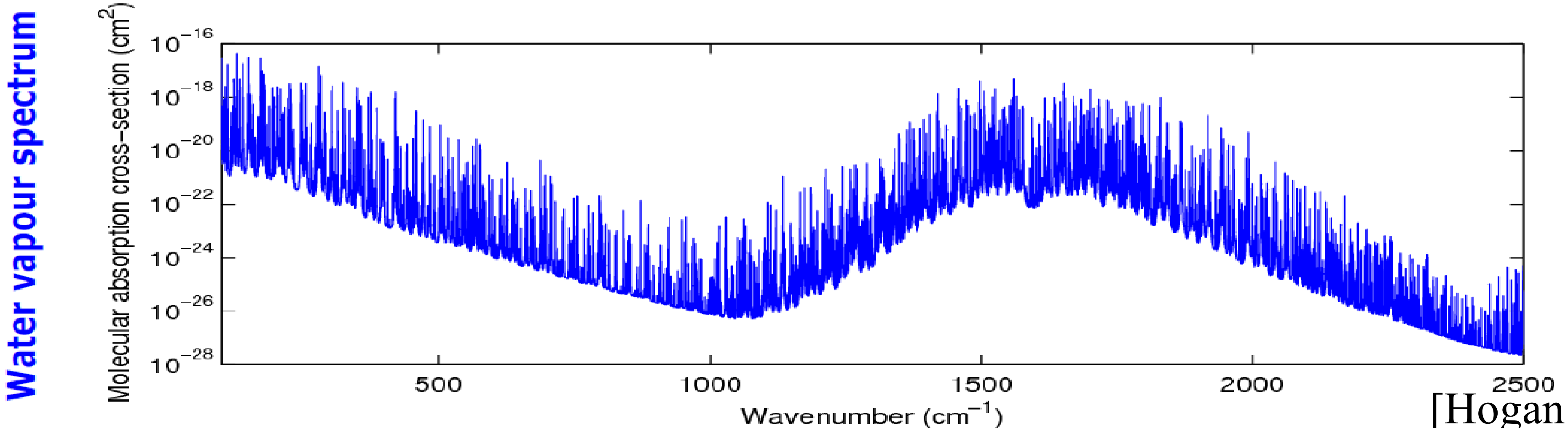


[Mlawer e.a.,1997]

How do we integrate across the spectrum?

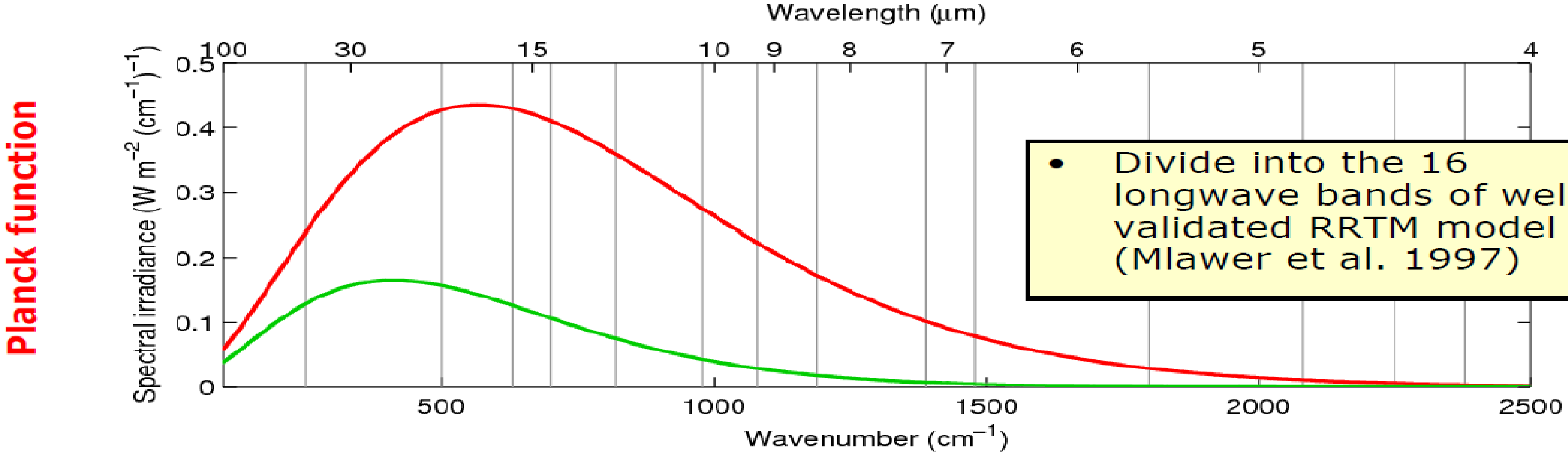


- Consider first just one gas
- Line-by-line model predicts spectrum (e.g. HITRAN)
- Need ~10^5 monochromatic radiation calculations

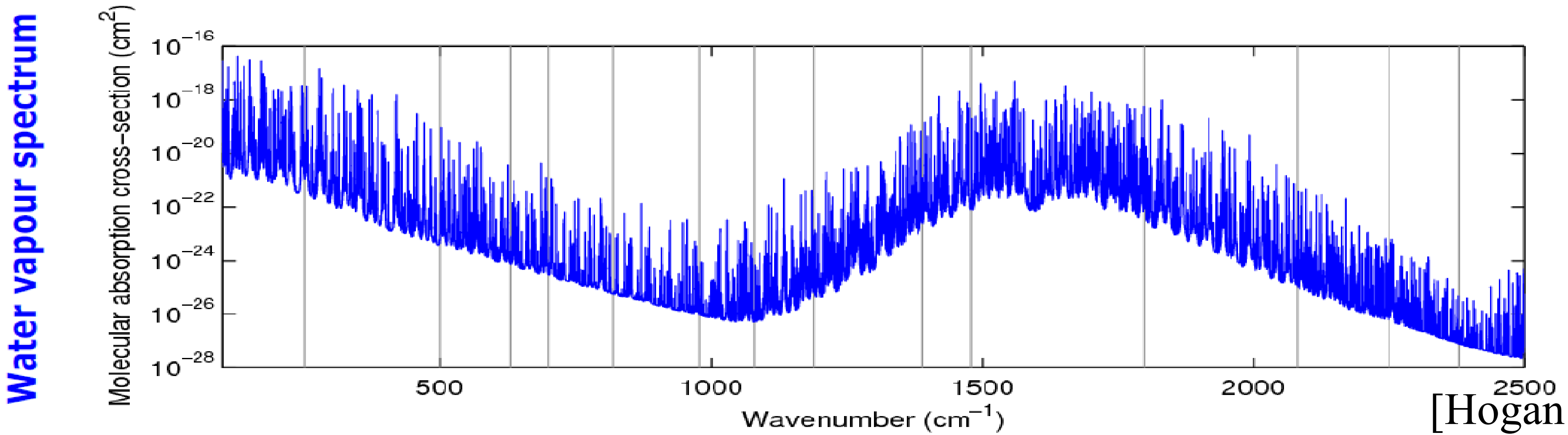


[Hogan lecture]

Divide into bands

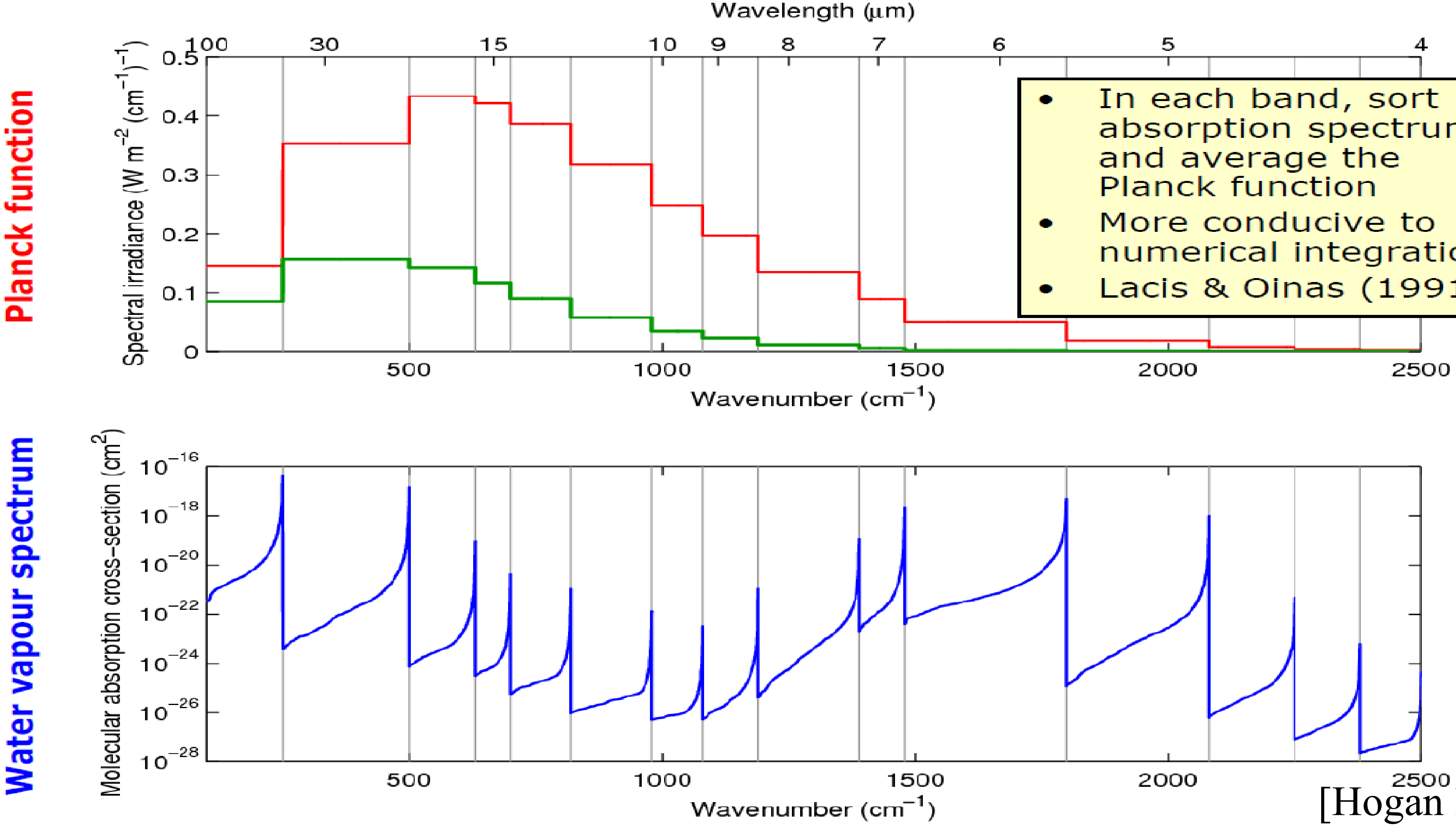


- Divide into the 16 longwave bands of well validated RRTM model (Mlawer et al. 1997)



[Hogan lecture]

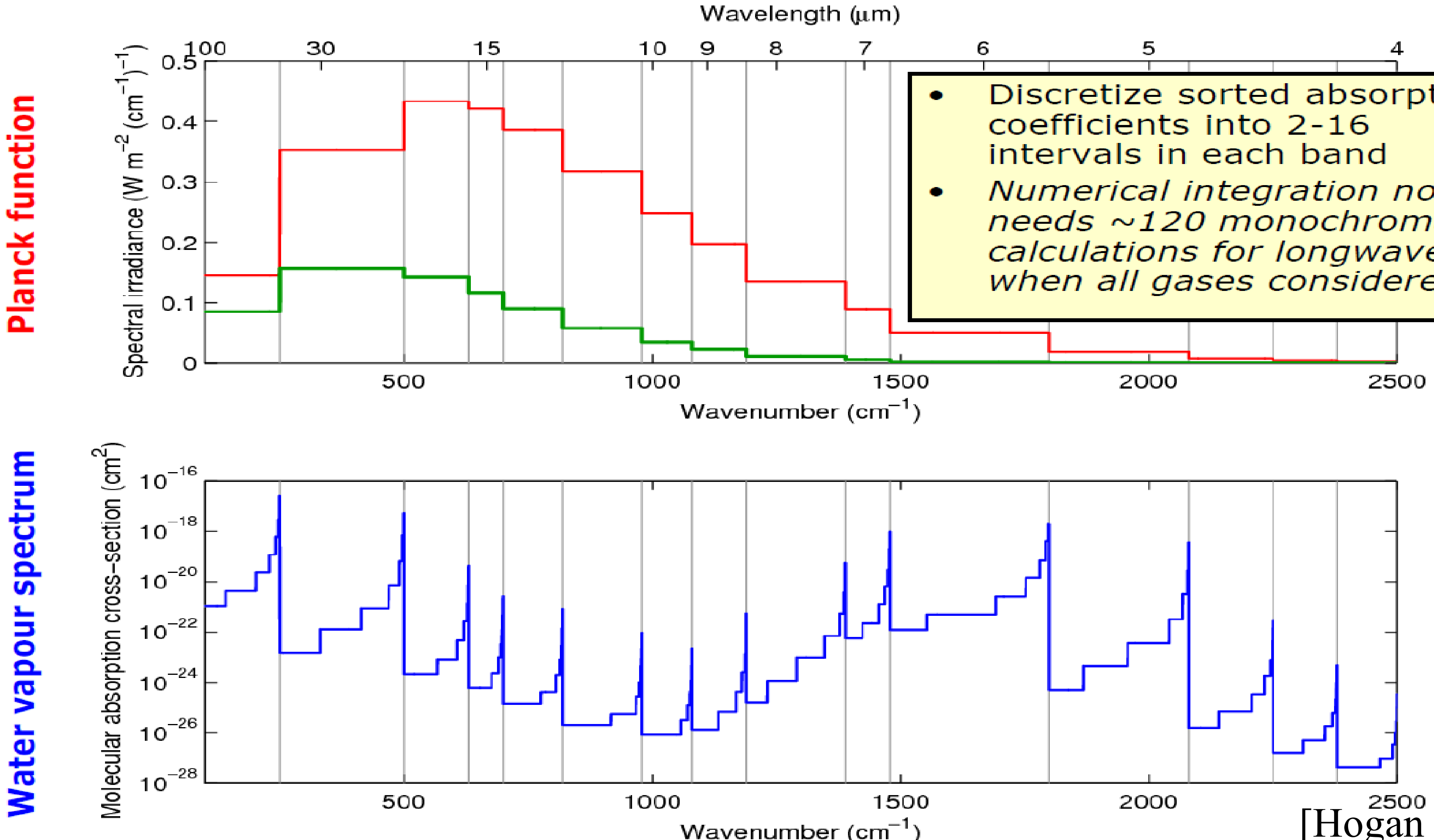
The correlated k-distribution (CKD) method



- In each band, sort absorption spectrum and average the Planck function
- More conducive to numerical integration
- Lacis & Oinas (1991)

[Hogan lecture]

The correlated k-distribution (CKD) method

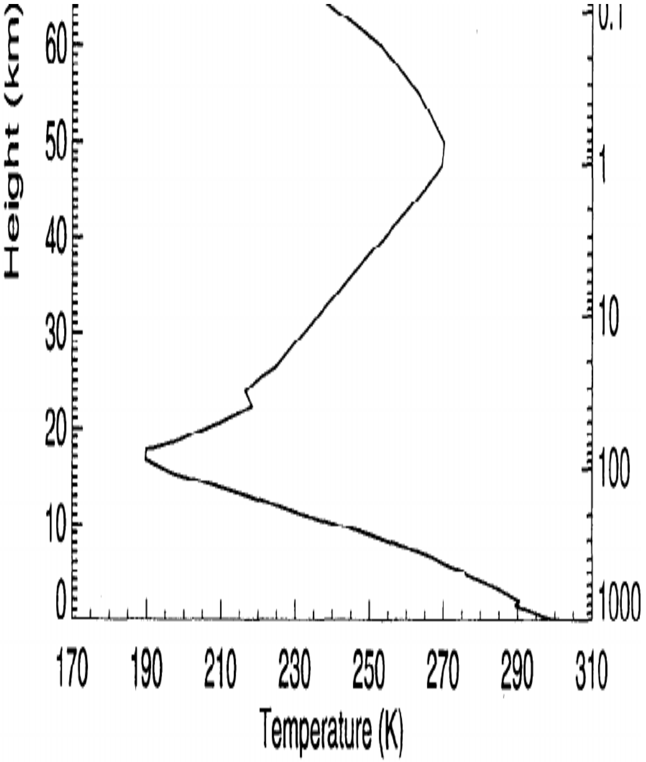


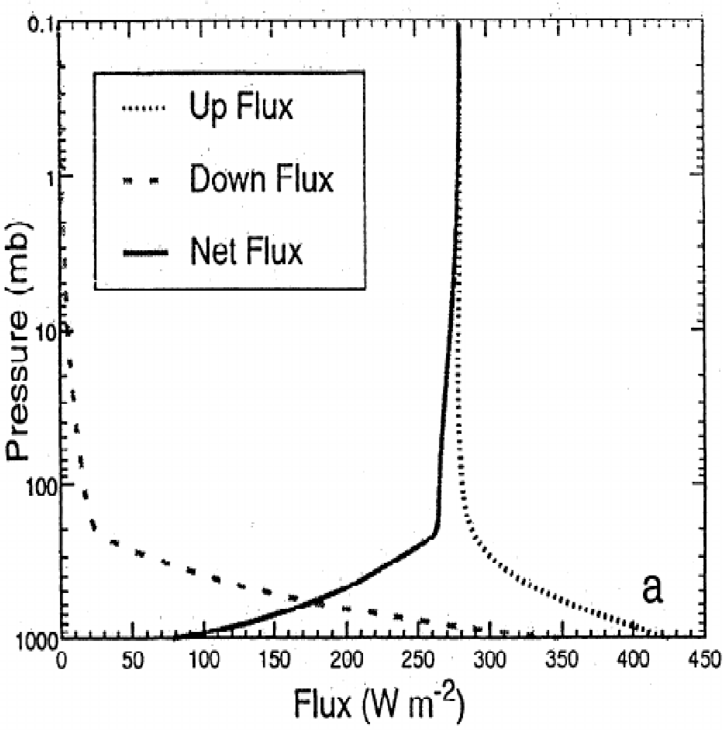
- Discretize sorted absorption coefficients into 2-16 intervals in each band
- *Numerical integration now needs ~120 monochromatic calculations for longwave when all gases considered*

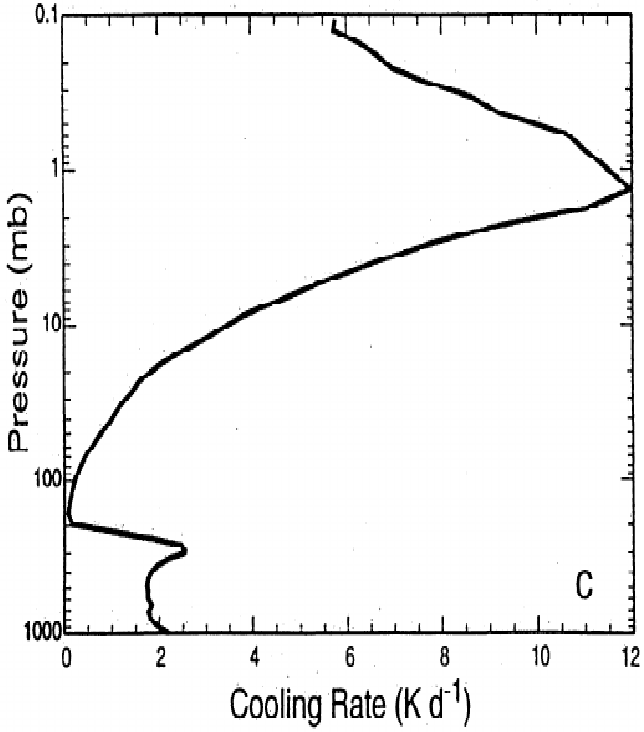
[Hogan lecture]

Typical LW flux and heating rate profile

$$Q=\frac{\partial T}{\partial t}=\frac{1}{\rho C_p}\frac{\partial F(z)}{\partial z}$$

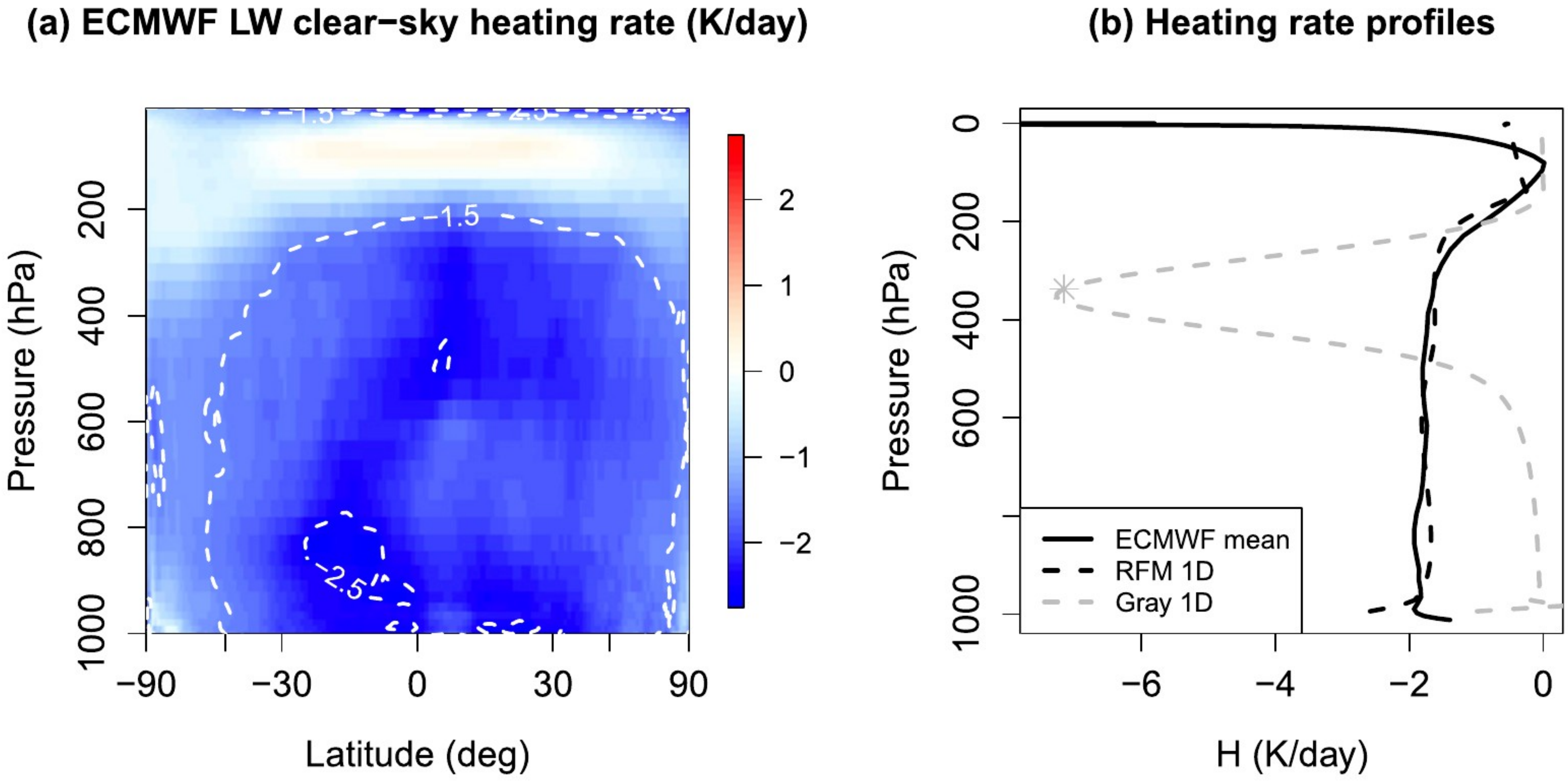






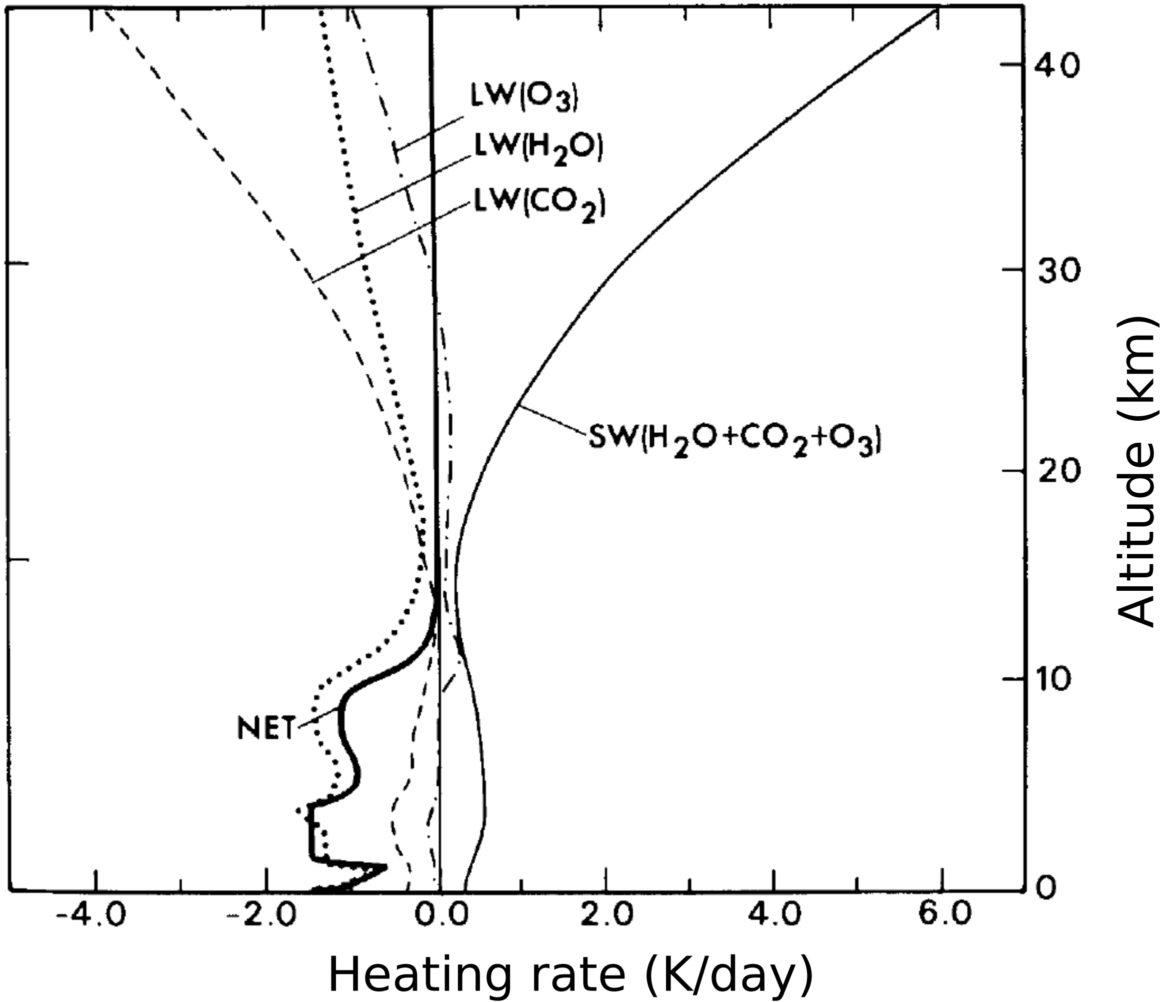
[Mlawer e.a.,1997]

LW heating rate profiles



[Jeevanjee & Fueglistaler, 2020]

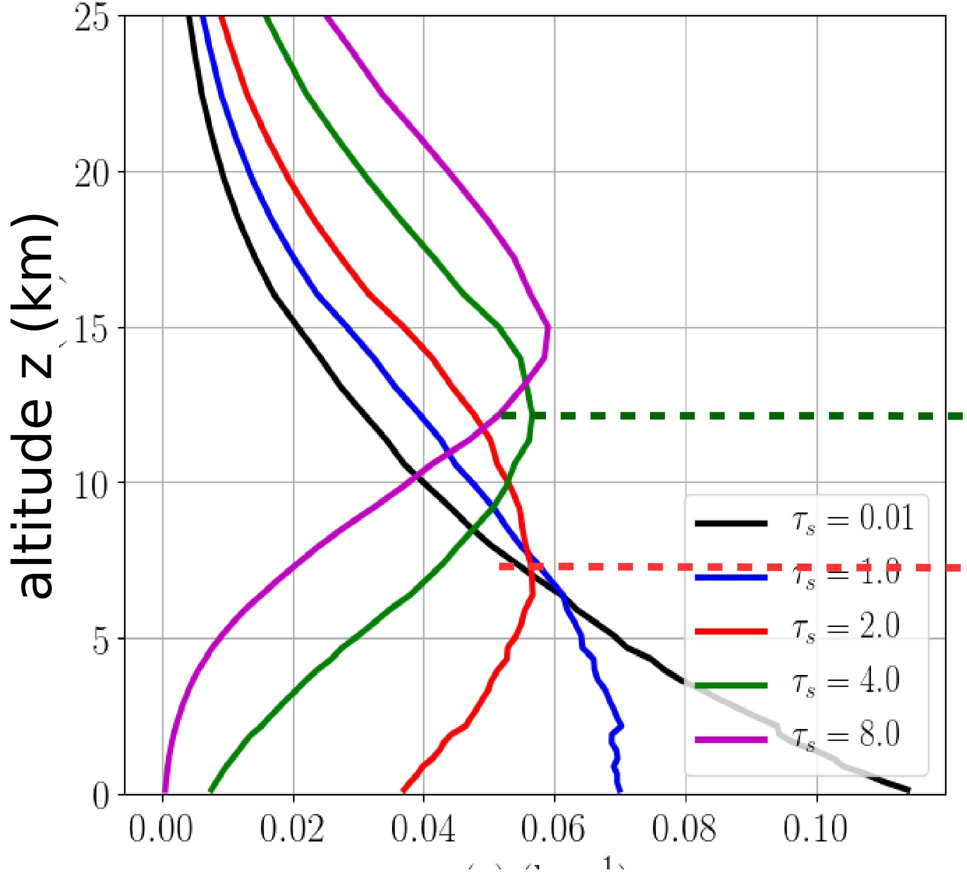
Typical heating rate profile



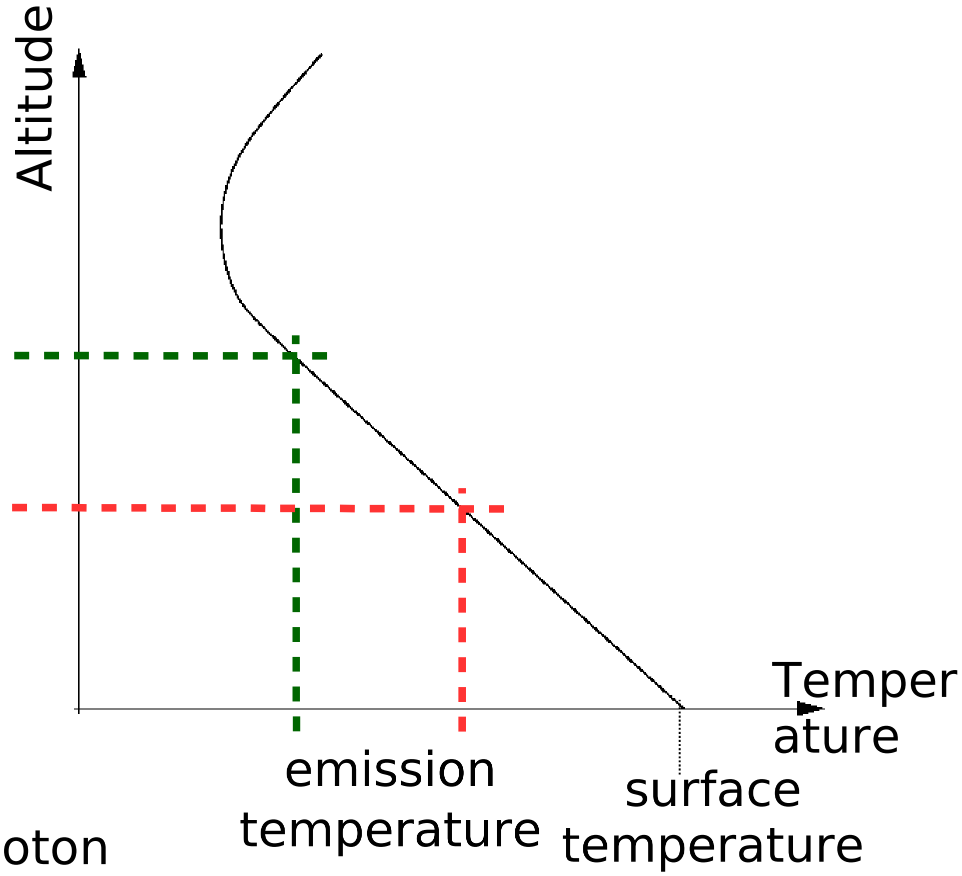
$$Q=\frac{\partial T}{\partial t}=\frac{1}{\rho C_p}\frac{\partial F(z)}{\partial z}$$

[Peixoto, 1992]

The concept of emission height

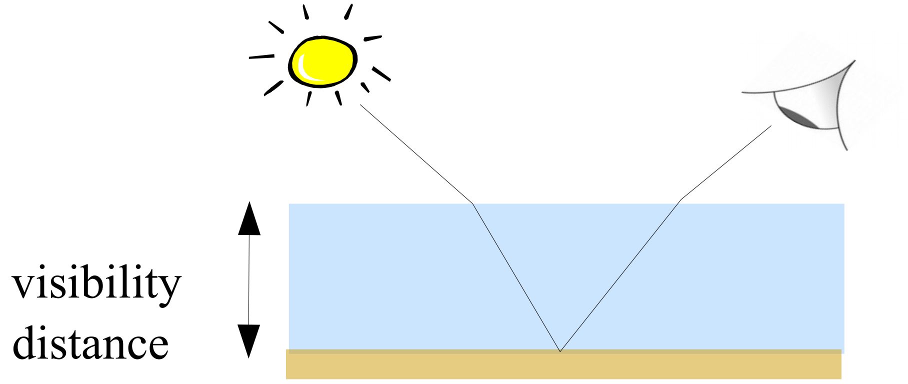


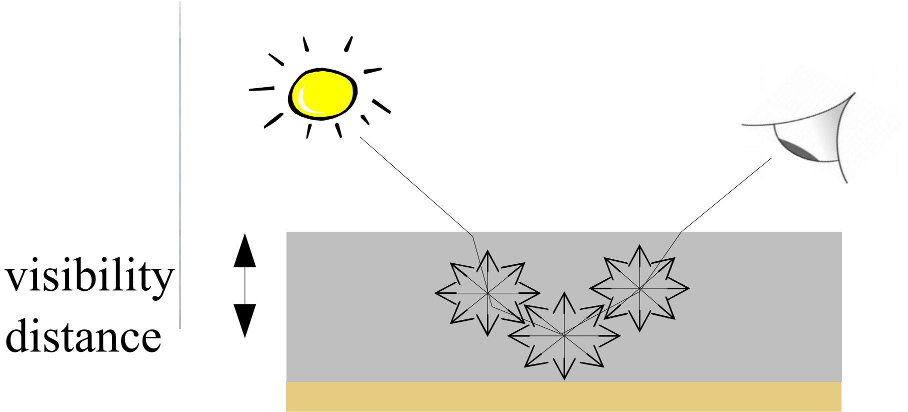
Probability density function (km^{-1}) that a photon reaching space has been emitted at altitude z for different optical thicknesses of the atmosphere



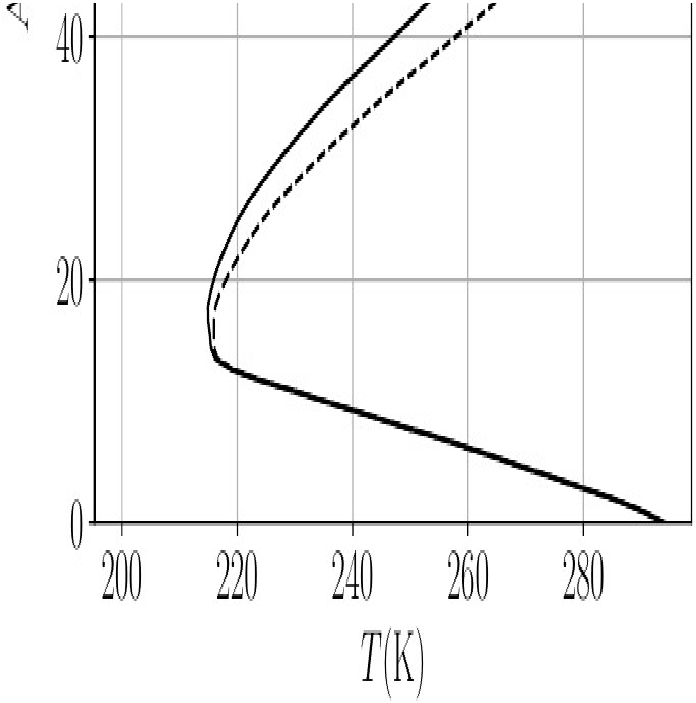
[Dufresne et al., 2020]

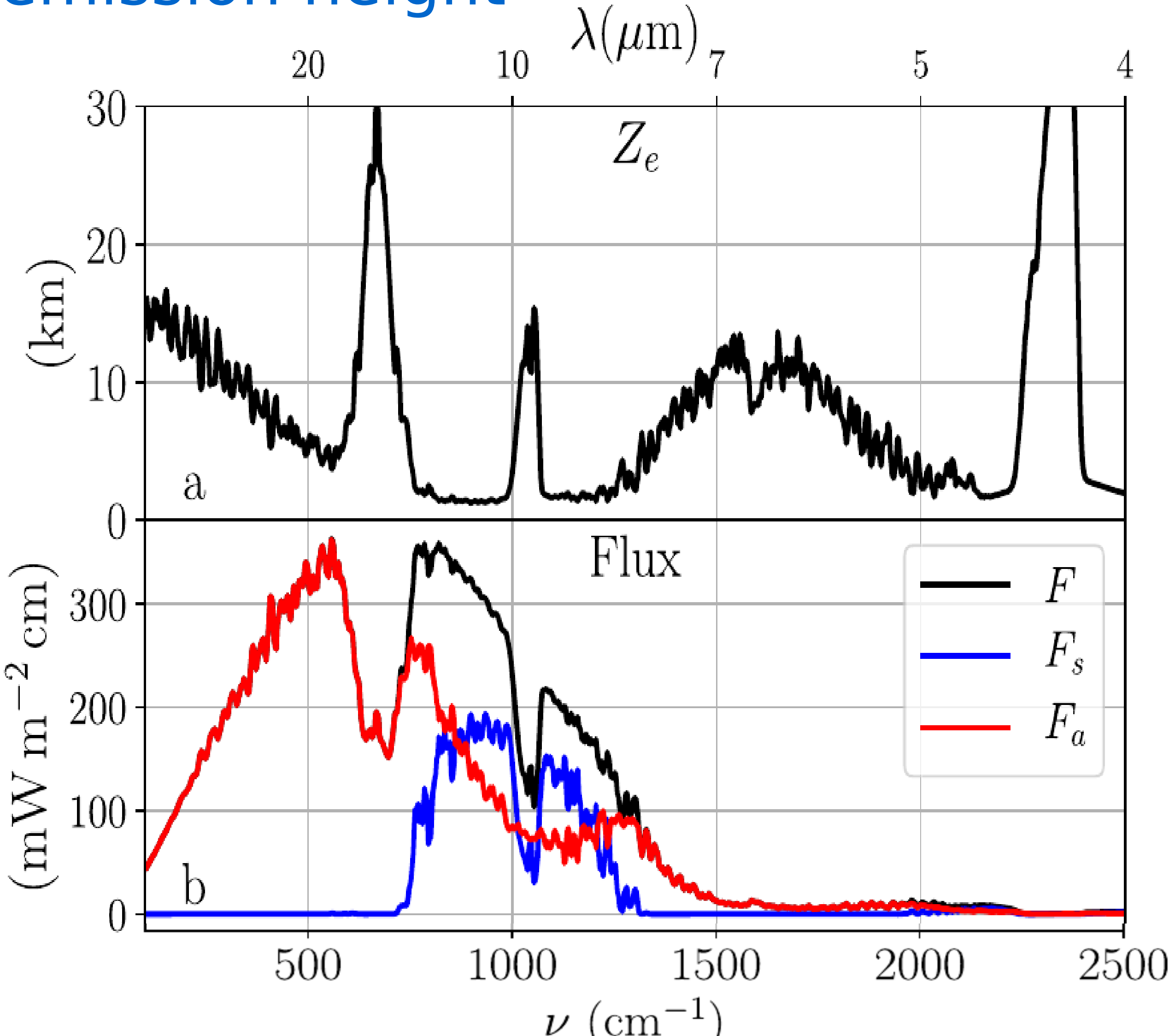
Analogy between emission height and visibility distance





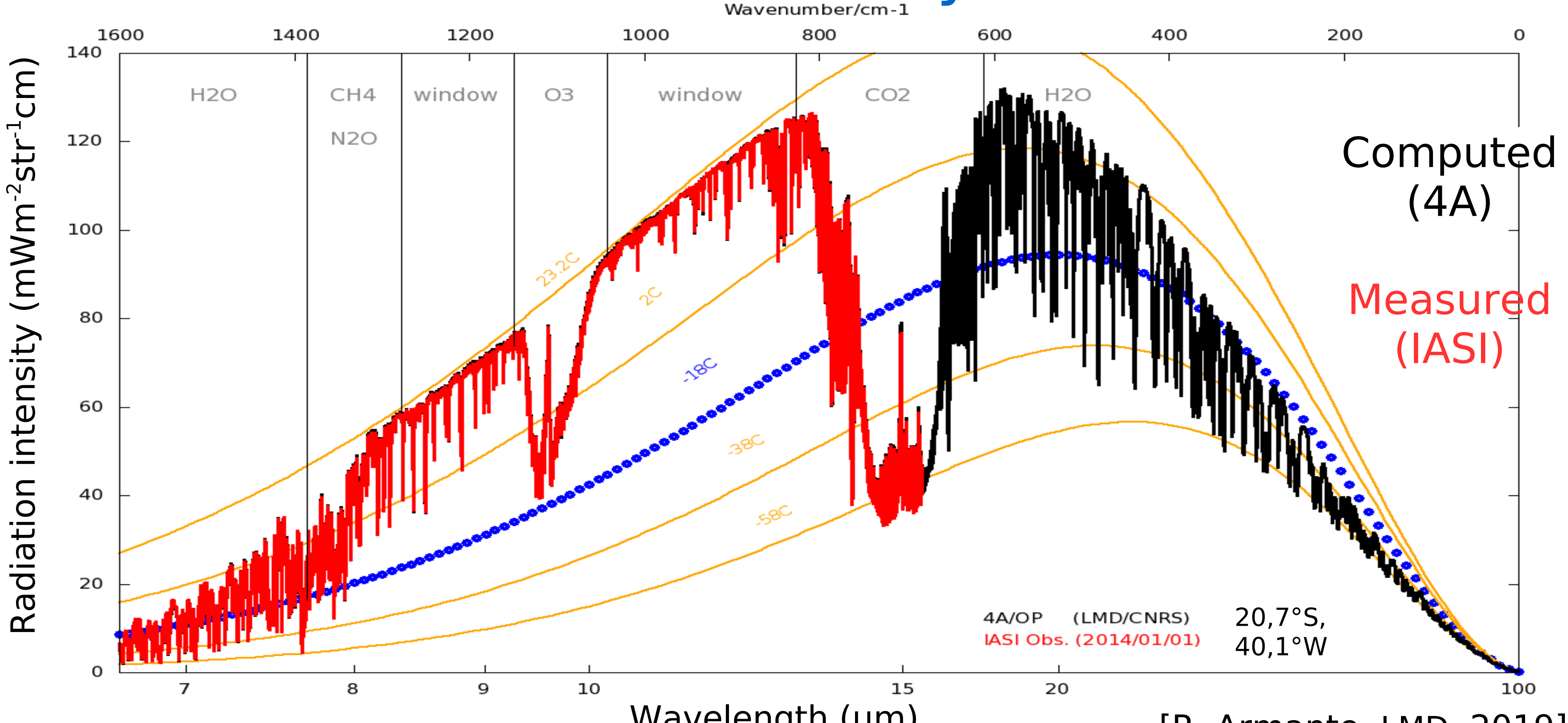
emission height





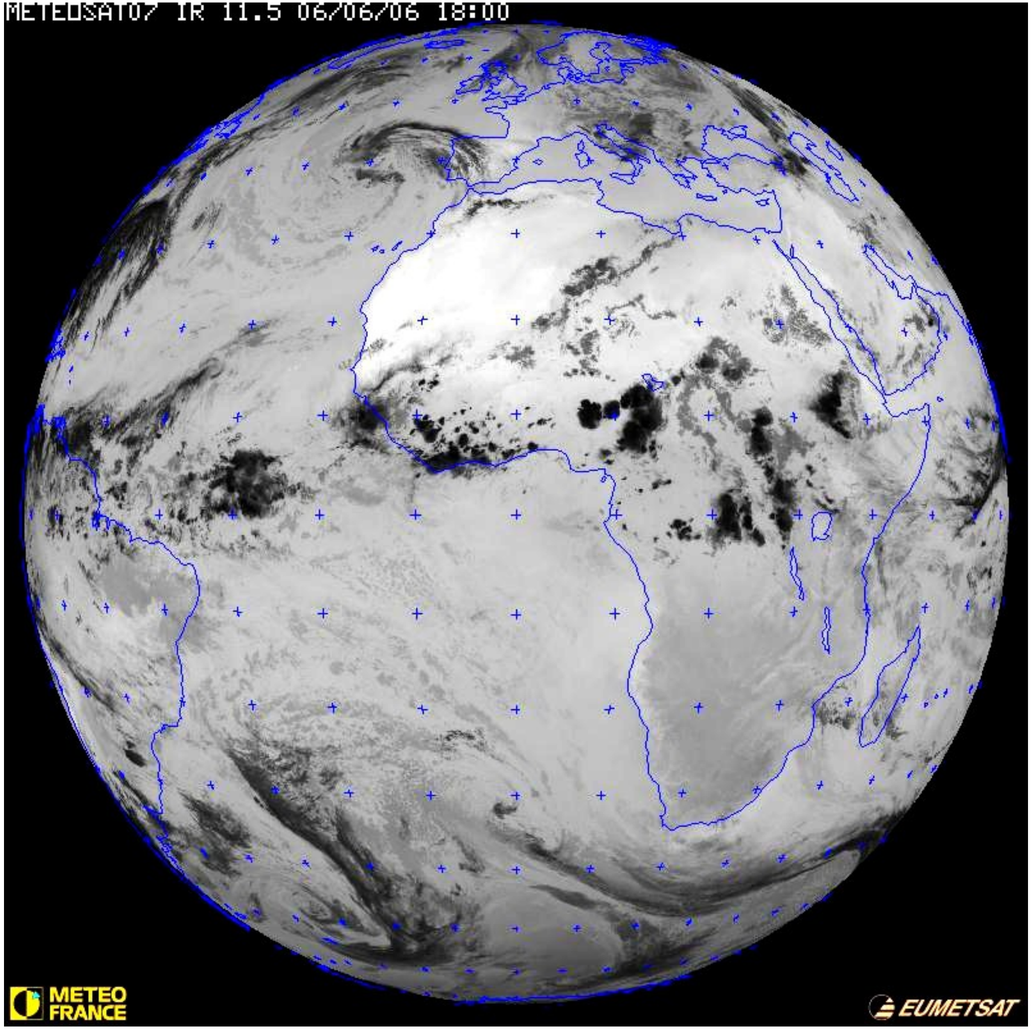
[Dufresne et al., 2020]

Spectrum of the radiation emitted by the Earth as measured by satellites

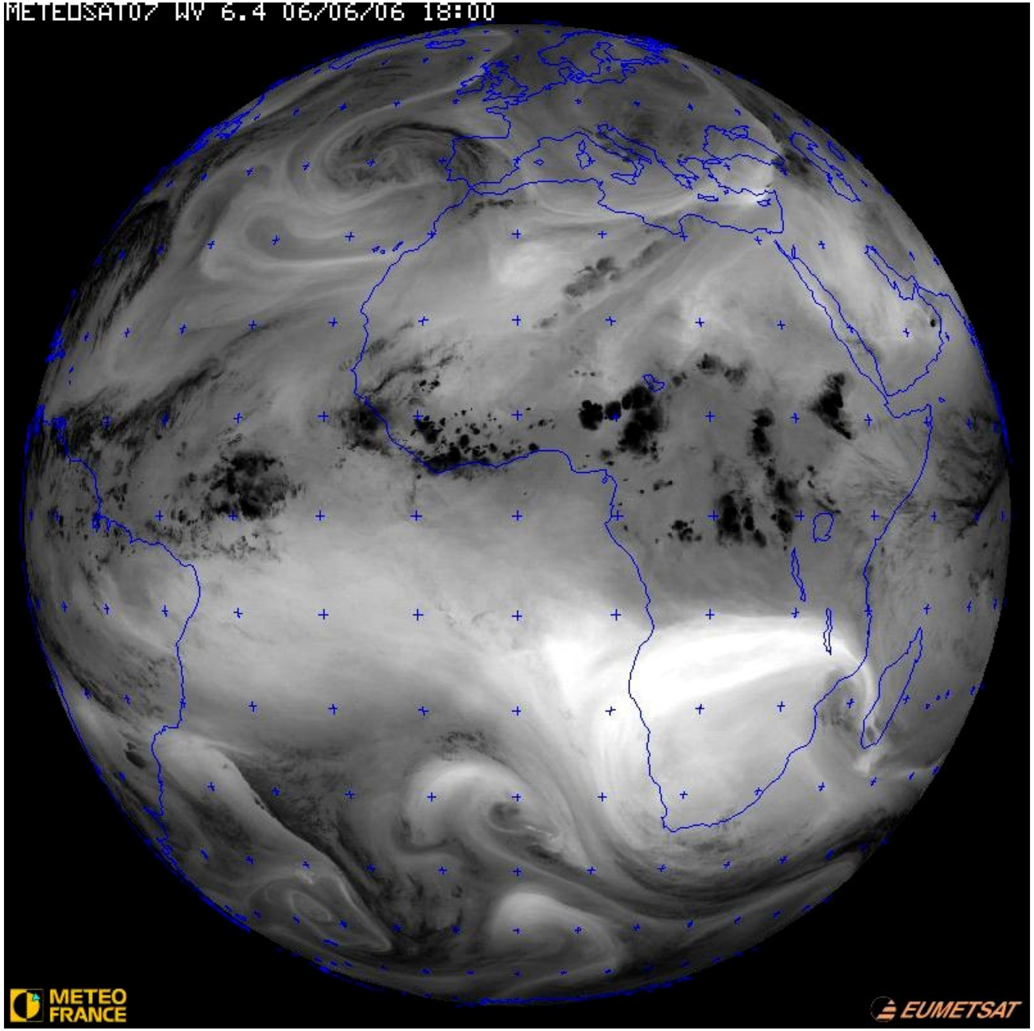


[R. Armante, LMD, 2019]

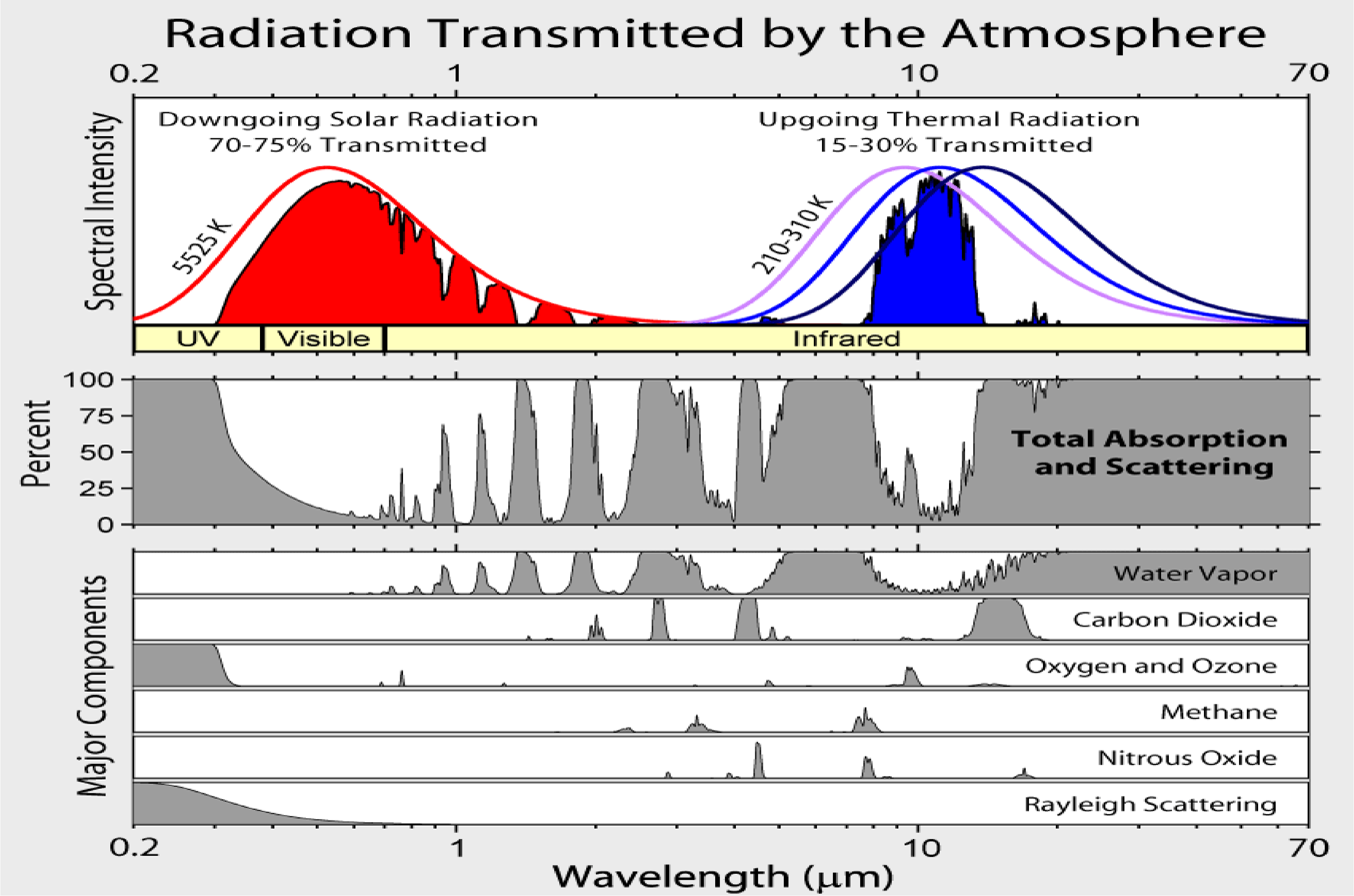
Atmospheric window



Water vapor channel



Brightness temperature: White: warm. Black: cold


Radiation Transmitted by the Atmosphere
0.2
1
10
70
Spectral Intensity
Downgoing Solar Radiation
70-75% Transmitted
Upgoing Thermal Radiation
15-30% Transmitted
5525 K
210-310 K
UV
Visible
Infrared
Percent
100
75
50
25
0
Total Absorption
and Scattering
Major Components
Water Vapor
Carbon Dioxide
Oxygen and Ozone
Methane
Nitrous Oxide
Rayleigh Scattering
Wavelength (μm)

Le rayonnement, moteur du climat

I. Bilan radiatif et température des planètes

II. Echanges radiatifs dans les atmosphères

III. Aspects spectraux: intéraction avec les gaz et effet de serre

IV. Accroissement de CO_2 et H_2O à profile de température fixe

V. Aspects géométriques: intéractions avec les nuages

Change in net radiative flux in response to change in GHG concentration

	$2xCO_2$		$1.2xH_2O$	
	LW		LW	
TOA	2.80		3.78	
200hPa	5.48		4.57	
Surf	1.64		11.52	

Net flux: positive downward

[Collins et al. 2006]

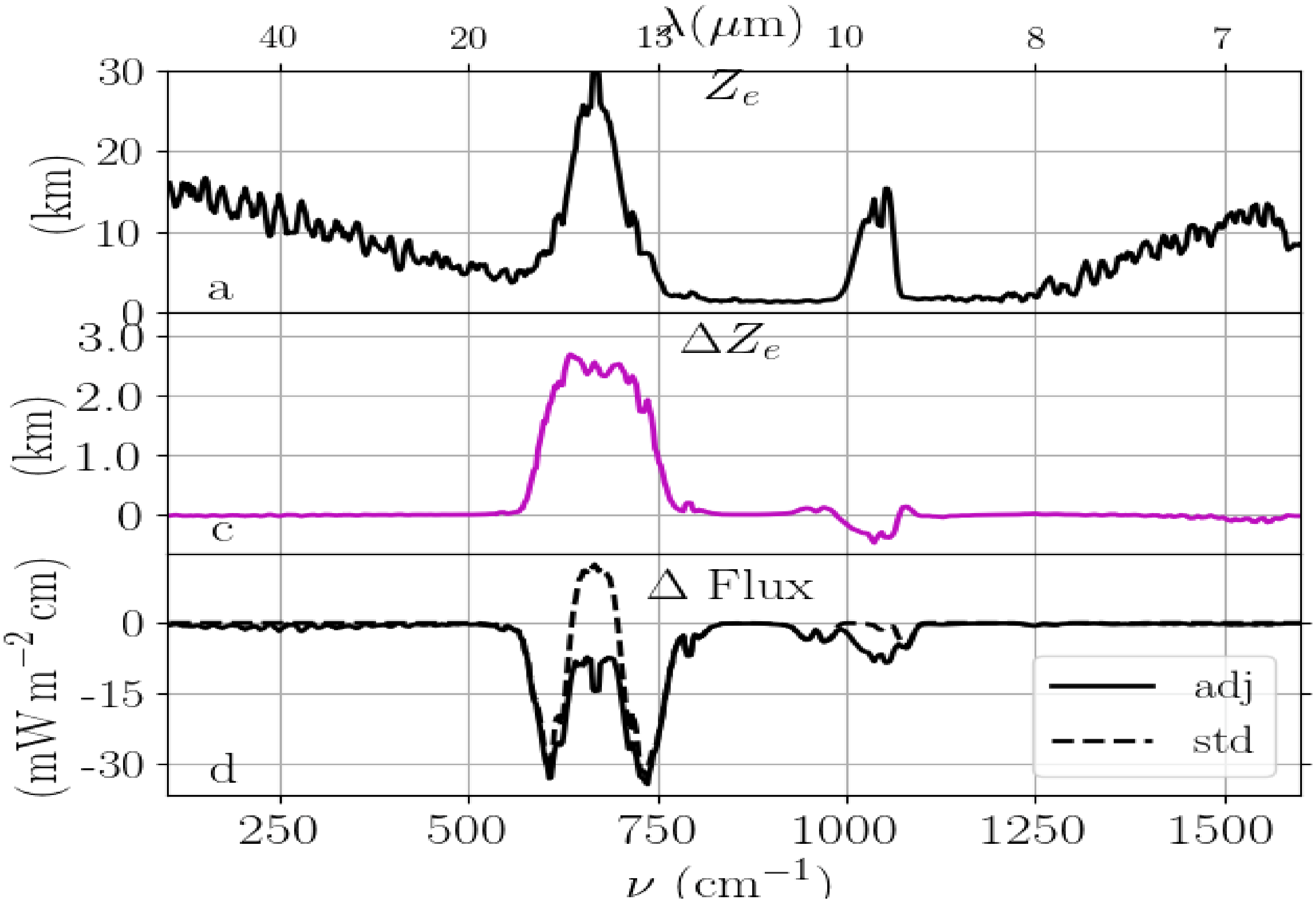
Change in net radiative flux in response to change in GHG concentration

	$2xCO_2$		$1.2xH_2O$	
	LW	SW	LW	SW
TOA	2.80	0.12	3.78	0.75
200hPa	5.48	-0.77	4.57	0.51
Surf	1.64	-0.96	11.52	-5.87

Change in net flux (Wm^{-2}), positive downward

[Collins et al. 2006]

Change in flux and emission height: 2xC0_2



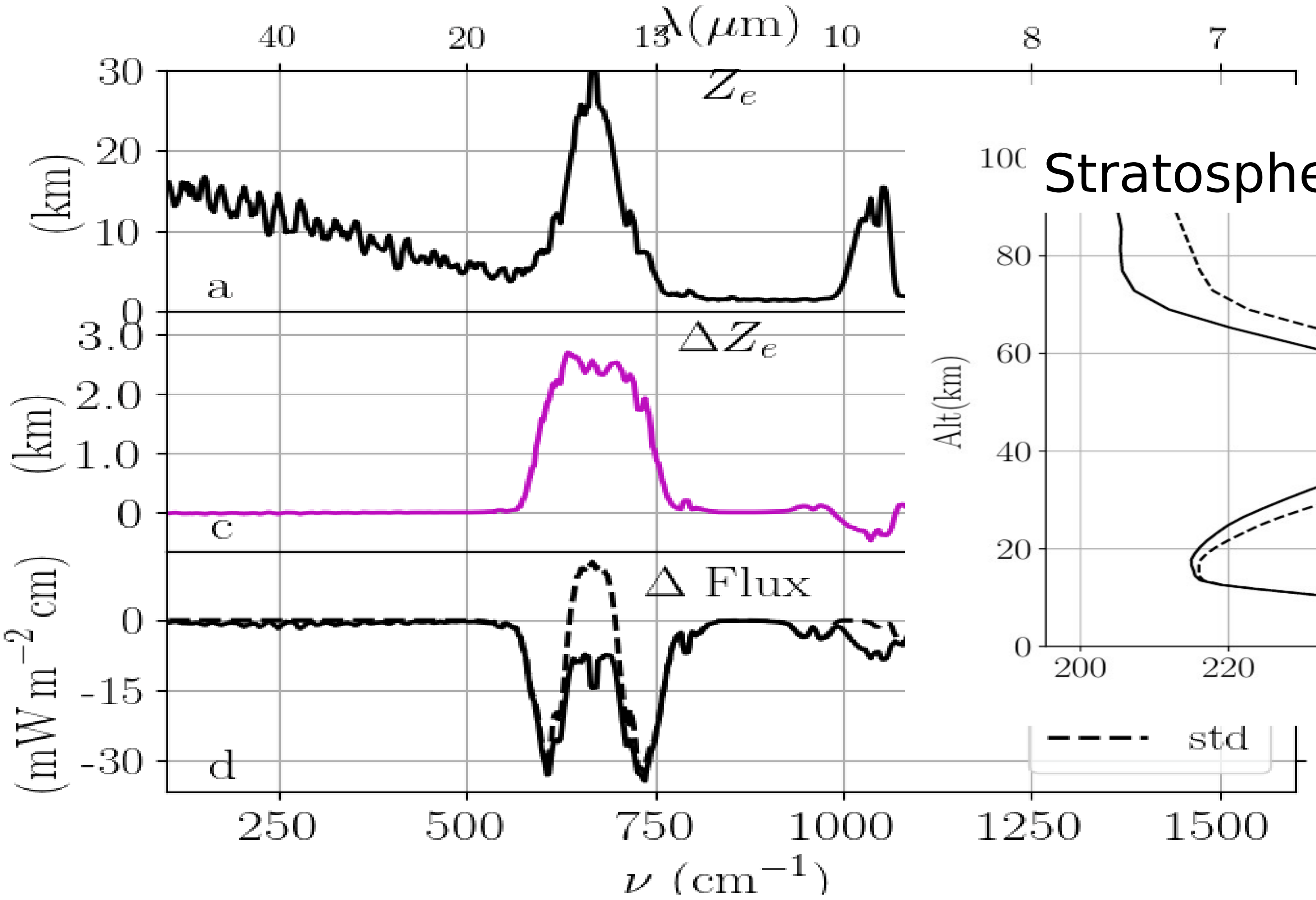
Emission height

Change in emission height for a doubling of CO_2 concentration

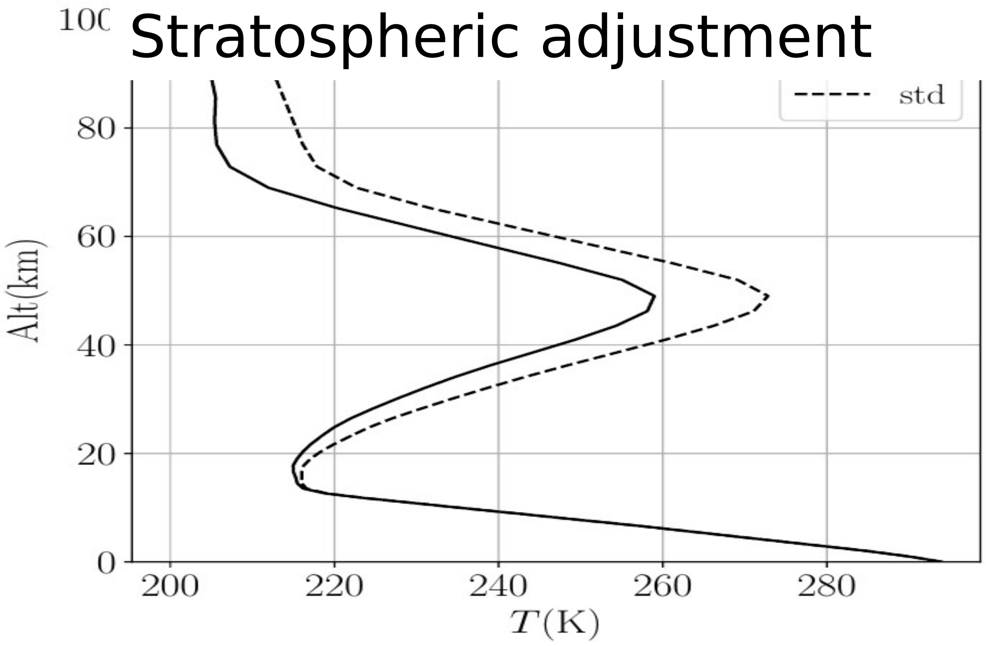
Change in radiative flux at the top of the atmosphere

[Dufresne et al. 2020]

Change in flux and emission height: 2xC0$_2$

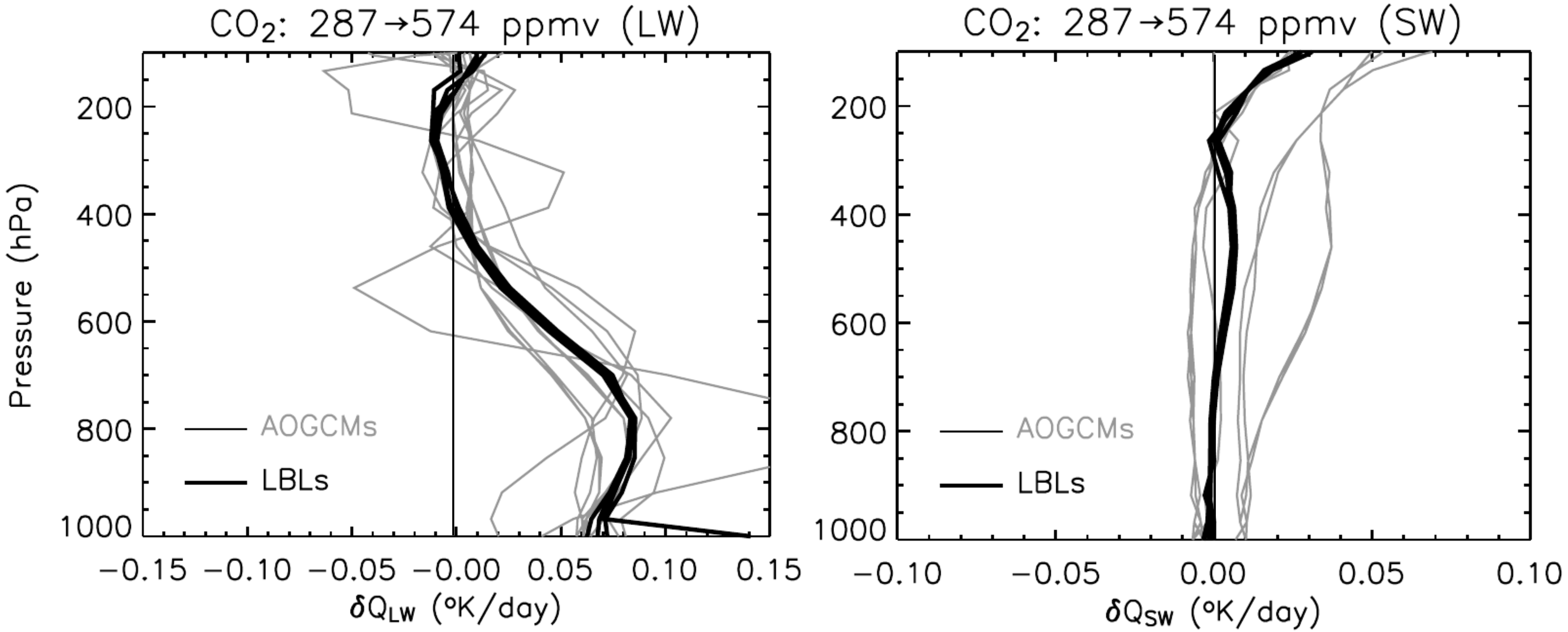


Stratospheric adjustment



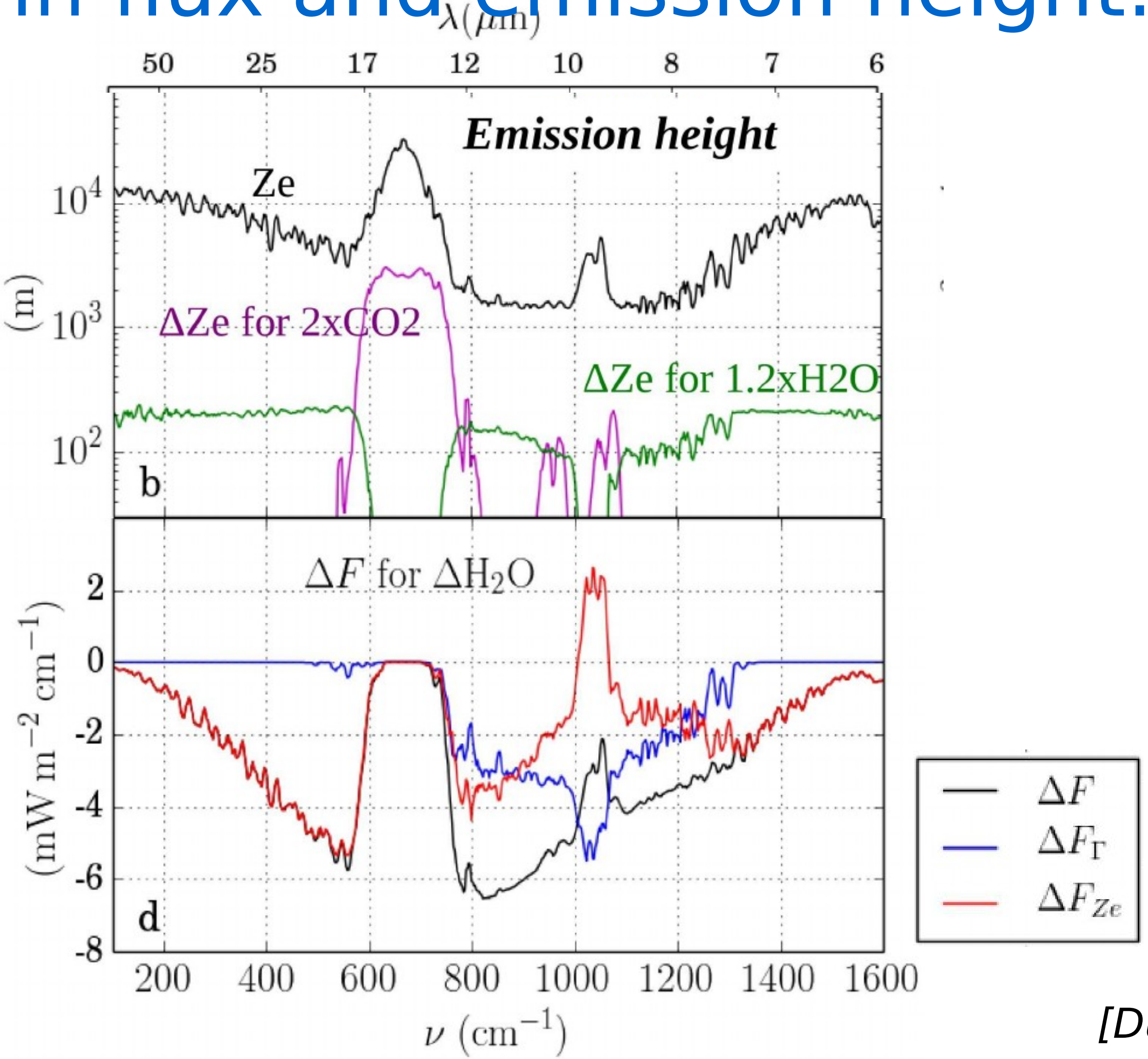
[Dufresne et al. 2020]

Change in heating rate profile $2xCO_2$



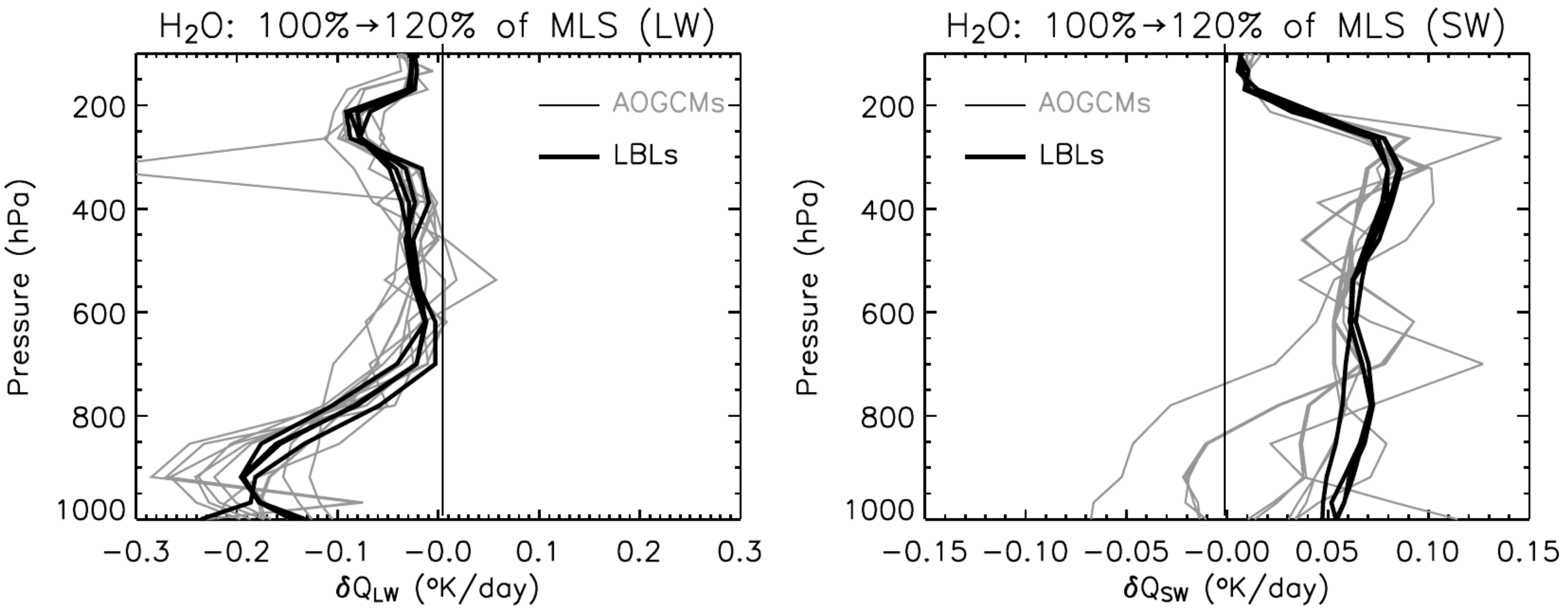
[Collins et al. 2006]

Change in flux and emission height: 1.2x$H_2$0



[Dufresne et al. 2020]

Change in heating rate profile
1.2xH_2O



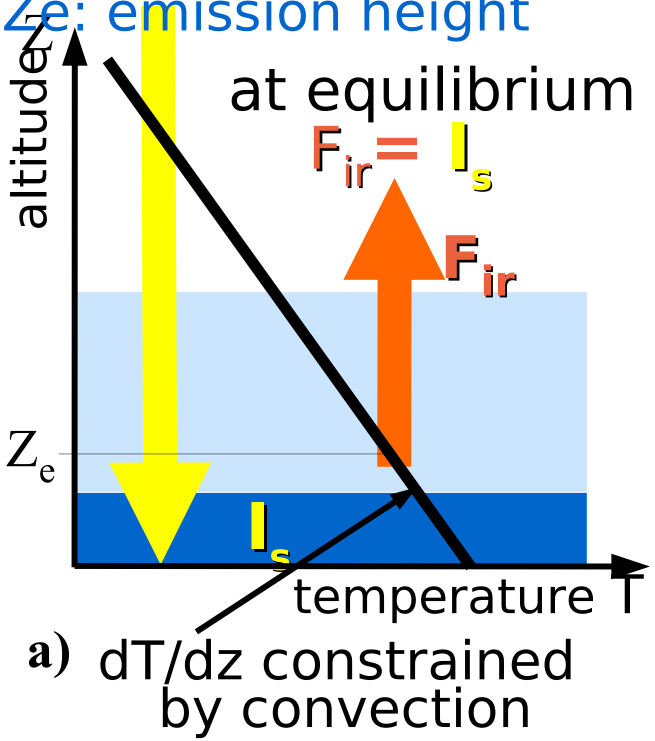
[Collins et al. 2006]

Greenhouse effect in a ***stratified*** atmosphere

I_s: solar radiation (SW)

F_{ir} : outgoing infrared (LW) radiation

Ze: emission height



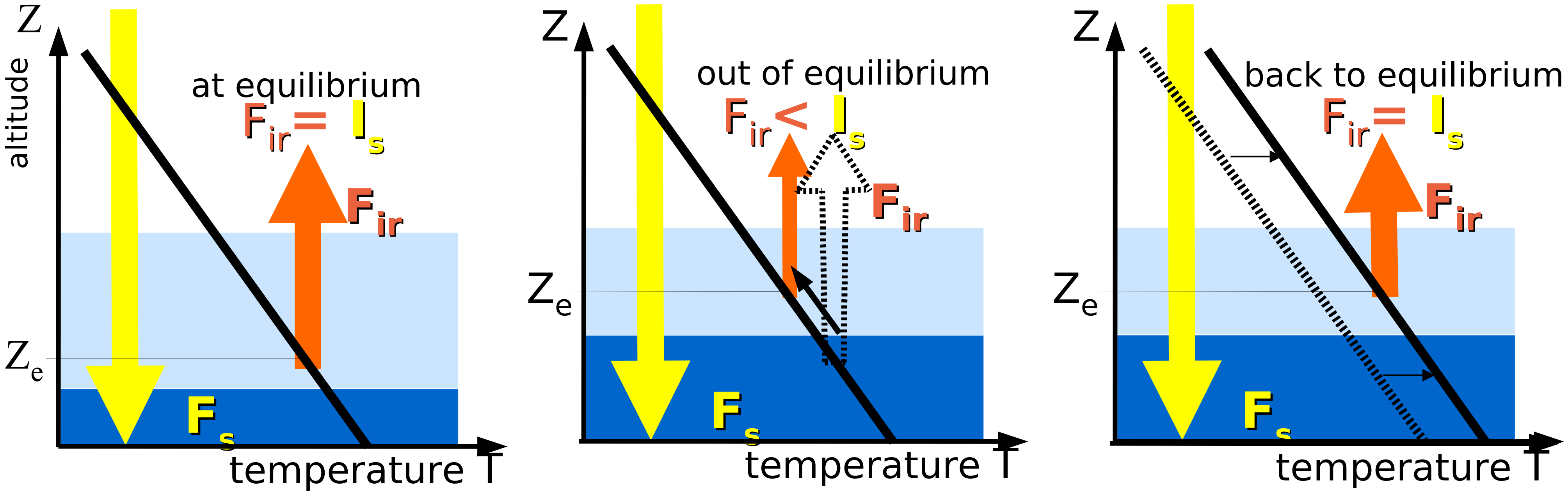
The concentration of greenhouse gases is vertically uniform.

Visible zone (photons emitted upwards reach the space)

Hidden zone (photons emitted upwards are absorbed and do not reach the space)

Greenhouse effect in a ***stratified*** atmosphere

Vertically uniform increase of the GHG concentration



a) Reference value of the CO_2 concentration

b) CO_2 increases, Z_e increases, T_e decreases, **Fir** decreases => radiative **forcing**

c) Ts and T_e increases, **Fir** increases => **response** to forcing

Le rayonnement, moteur du climat

I. Bilan radiatif et température des planètes

II. Echanges radiatifs dans les atmosphères

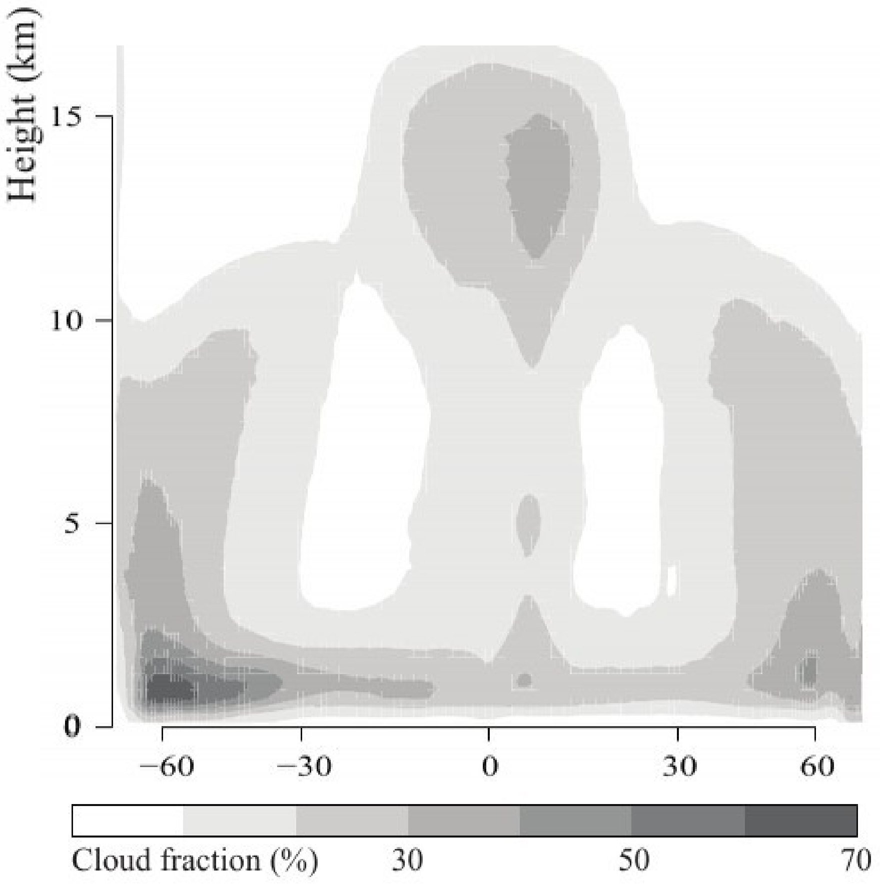
III. Aspects spectraux: intéraction avec les gaz et effet de serre

IV. Accroissement de CO_2 et H_2O à profile de température fixe

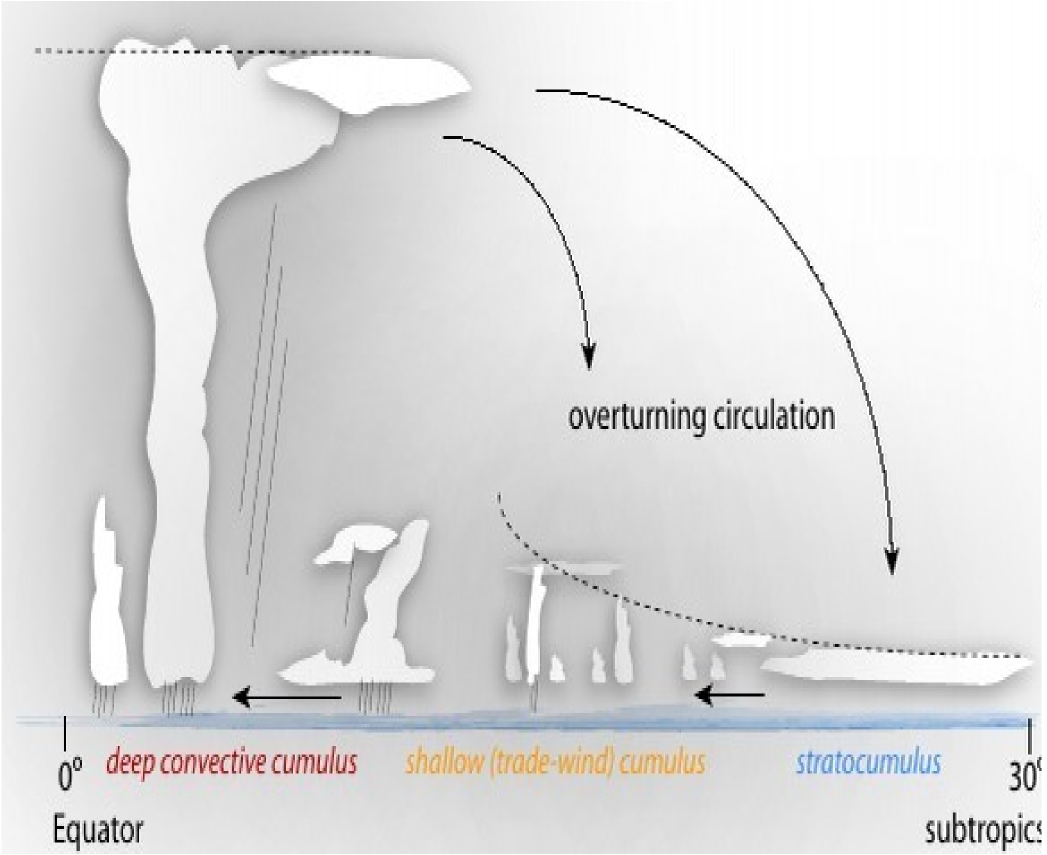
V. Aspects géométriques: intéractions avec les nuages




Clouds are the signature of the atmospheric circulation



[Pincus & Chepfer, 2020]



[Nuijens & Jacob, 2020]

Geometrical aspects: interactions with clouds

Spectral variation of the radiative properties of particles much less abrupt than for gases

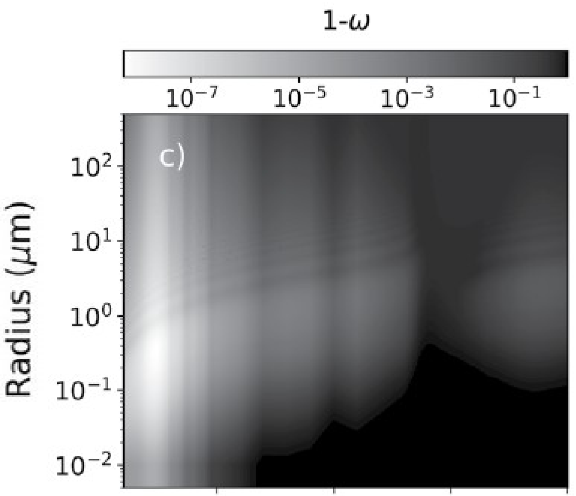
Difficulties come from:

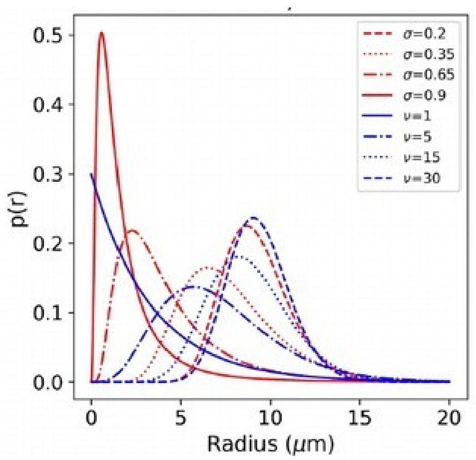
- Radiative properties of cloud particles
- Clouds geometry
- Scattering

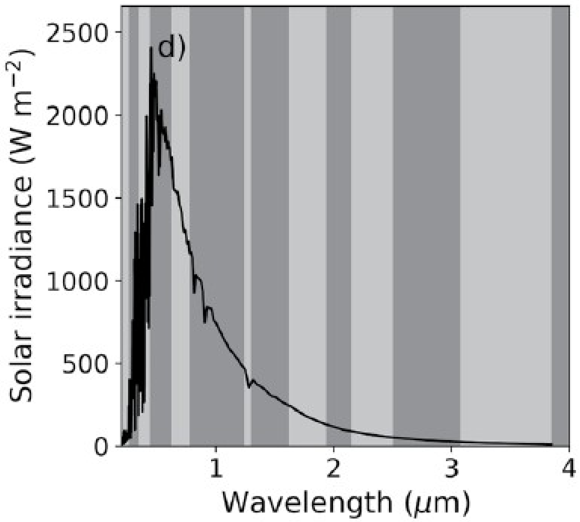
Radiative properties of cloud particles:

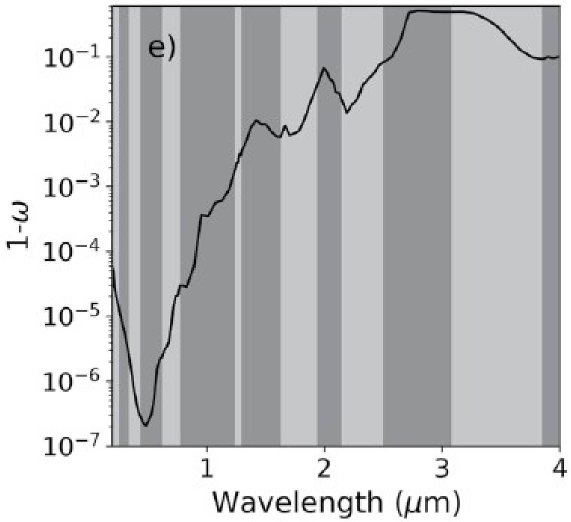
liquid water droplets

Resolution of Maxwell's equations:
Spheres: Mie theory

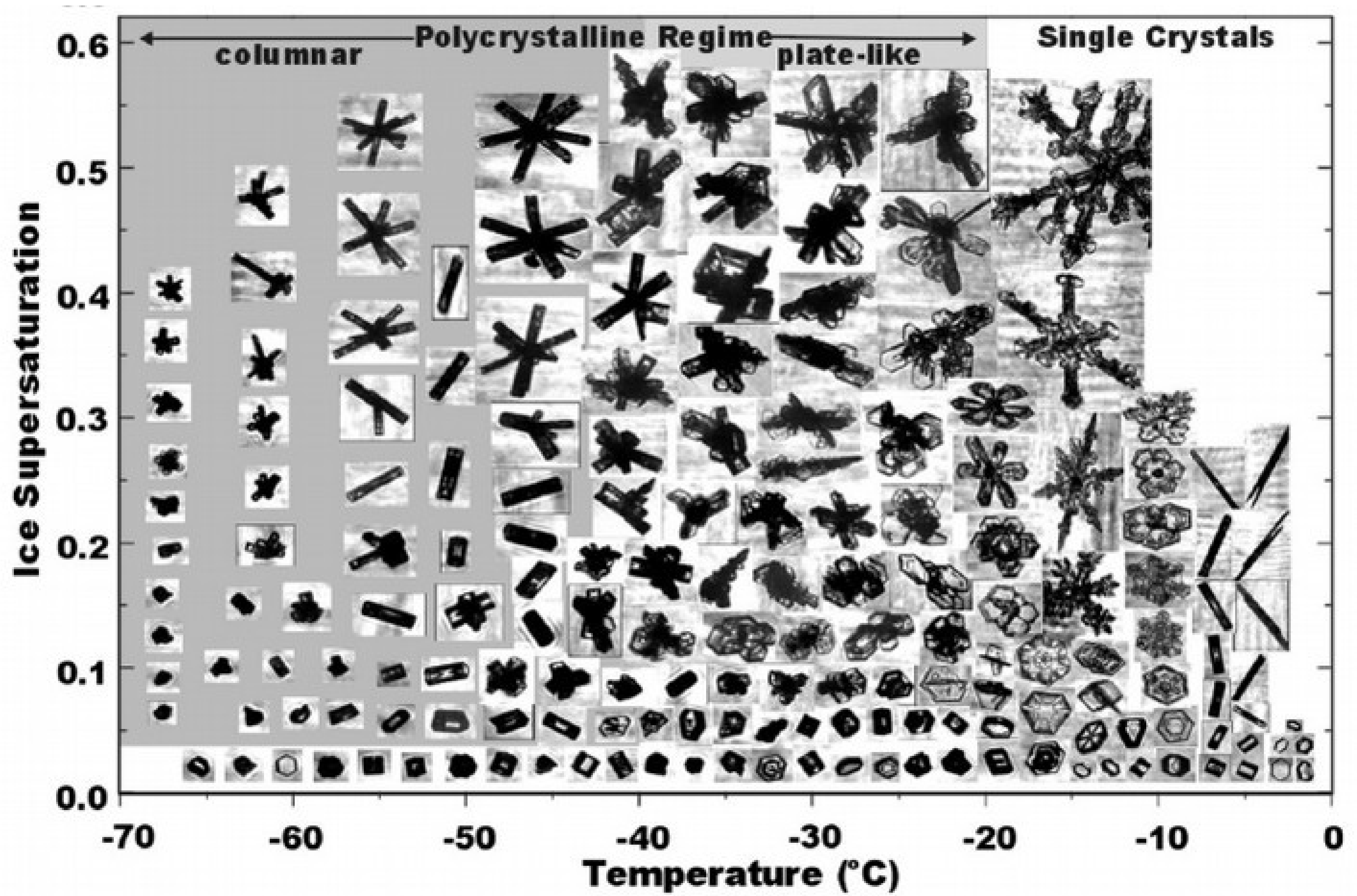








Shape of the ice crystals

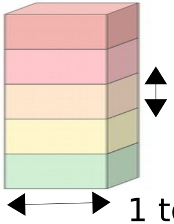


Habit diagram of Bailey and Hallett (2009) from laboratory observations.

[Pawlowska & Shipway, 2020]

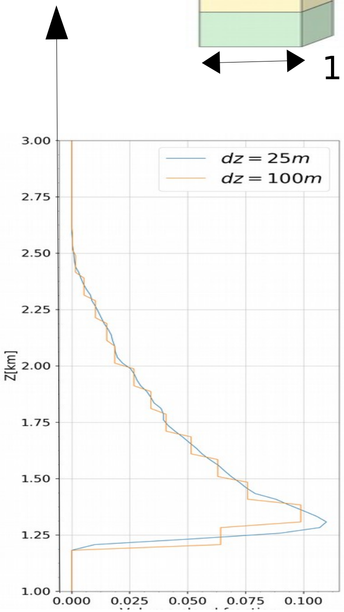
Vertical structure reconstruction of cloudy scenes
from averaged quantities

Single column with a
coarse vertical grid grids

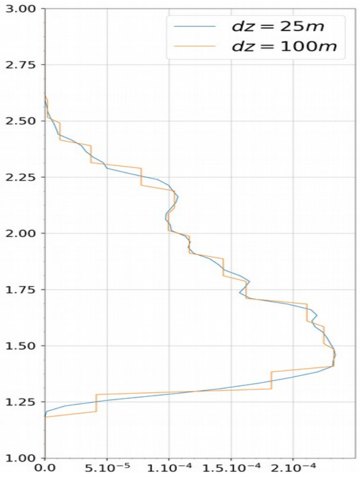


10 m to 1 km

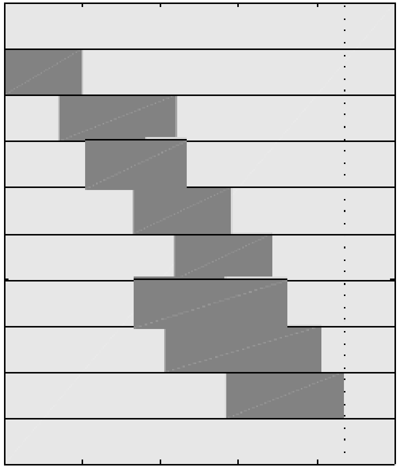
1 to100 km



Volume cloud fraction

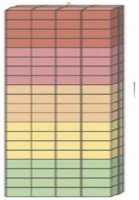


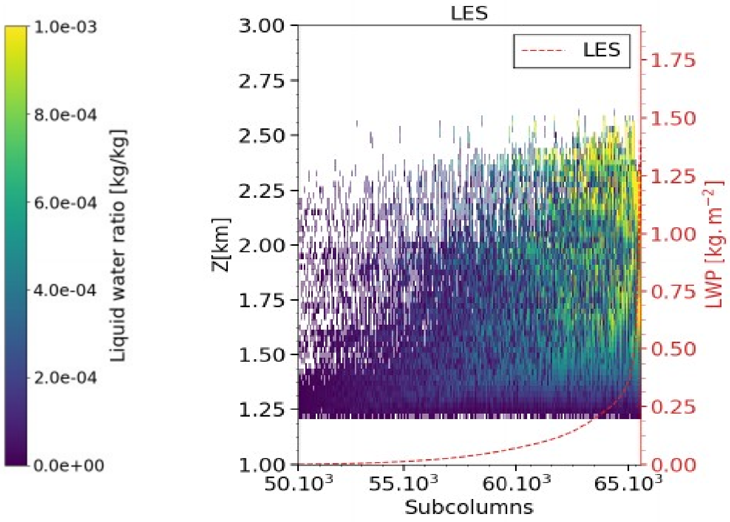
Liquid water content (kg/kg)



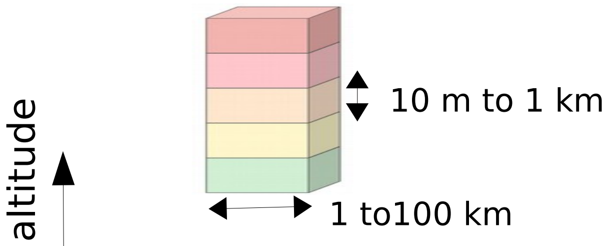
Vertical structure reconstruction of cloudy scenes
from averaged quantities

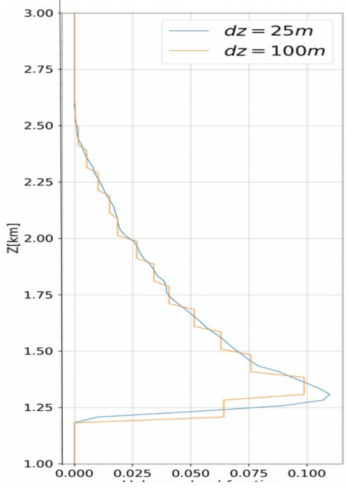
LES output on high resolution vertical and horizontal grids



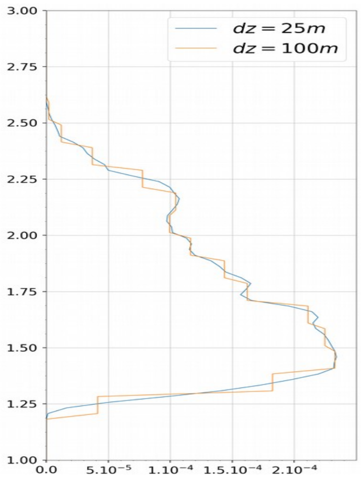


Single column with a coarse vertical grid grids



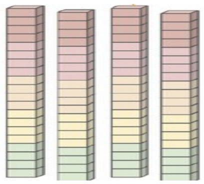


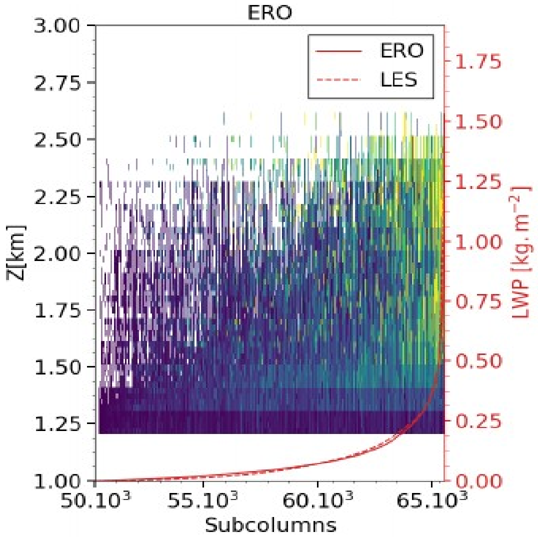
Volume cloud fraction



Liquid water content (kg/kg)

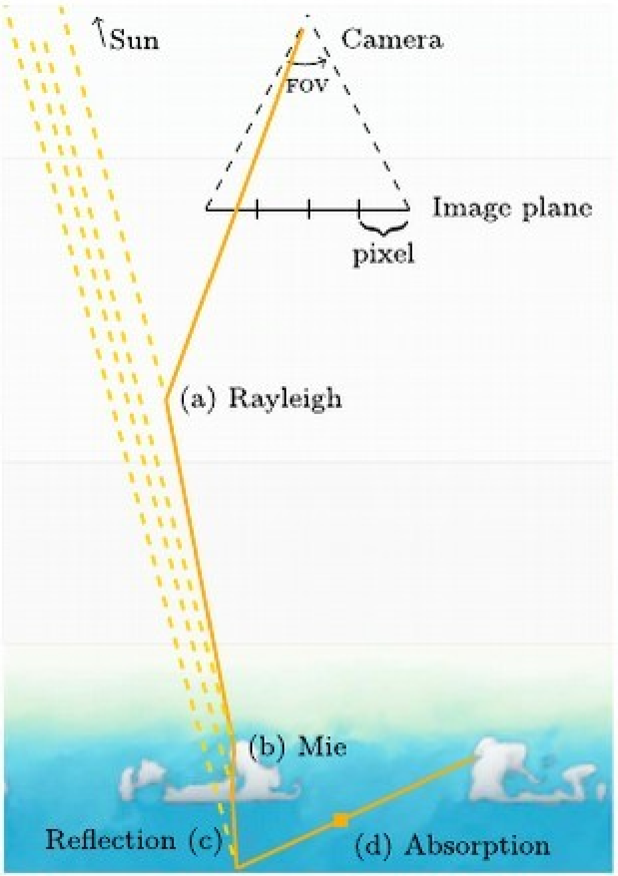
Ensemble of sub-columns with a high resolution vertical grid (Markov chain)





Thèse de R. Lebrun

Monte-Carlo computation of 3D cloudy scenes

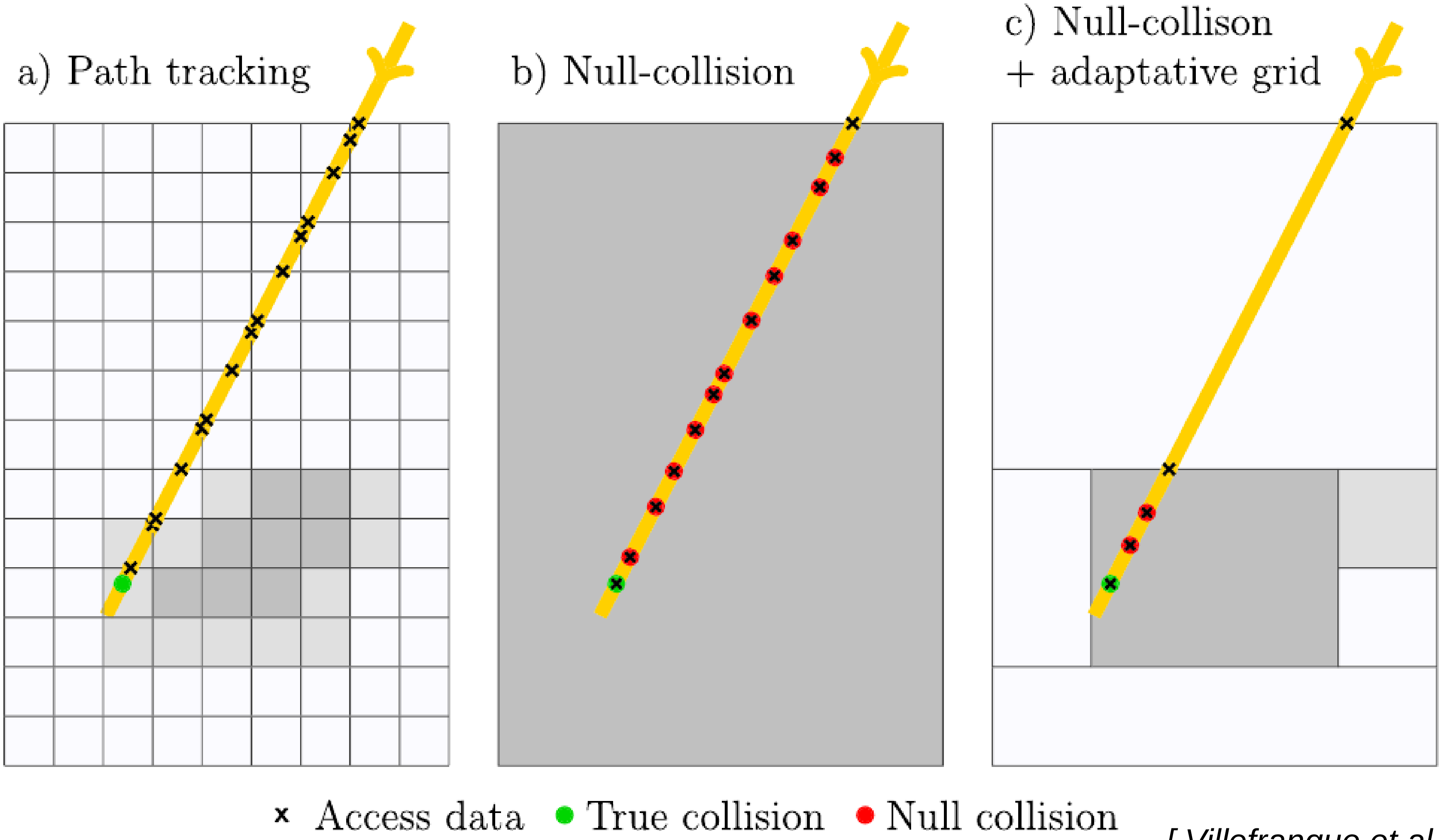


← a) Schematic illustrating the rendering algorithm

b) Example of a radiance field produced by the renderer

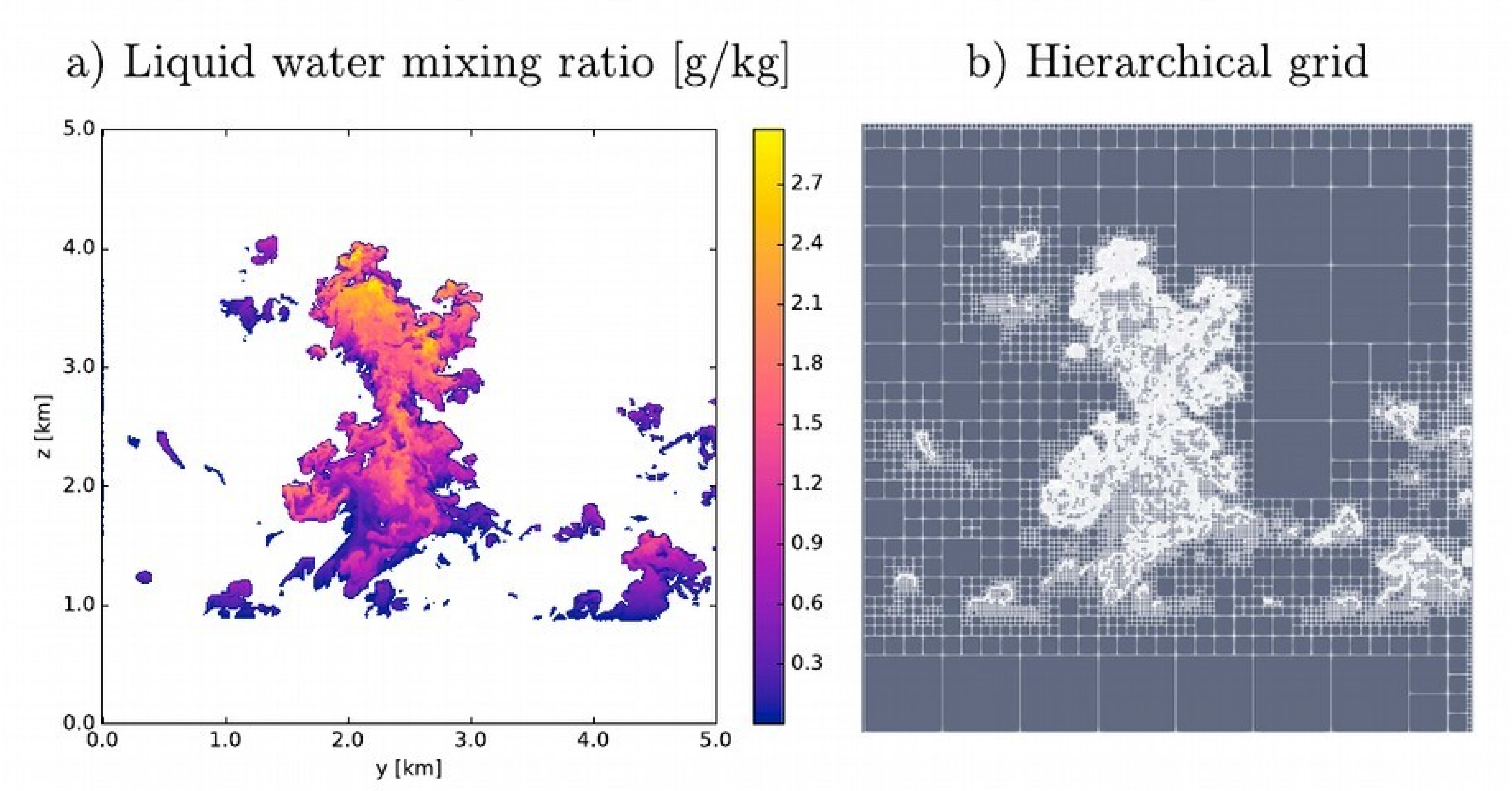
[Villefranque et al., 2019]

Monte-Carlo computation of 3D cloudy scenes



[Villefranque et al., 2019]

Monte-Carlo computation of 3D cloudy scenes



[Villefranque et al., 2019]

Monte-Carlo computation of 3D cloudy scenes

a) BOMEX b) ARMCu 1 c) ARMCu 2 d) FIRE

[Villefranque et al., 2019]

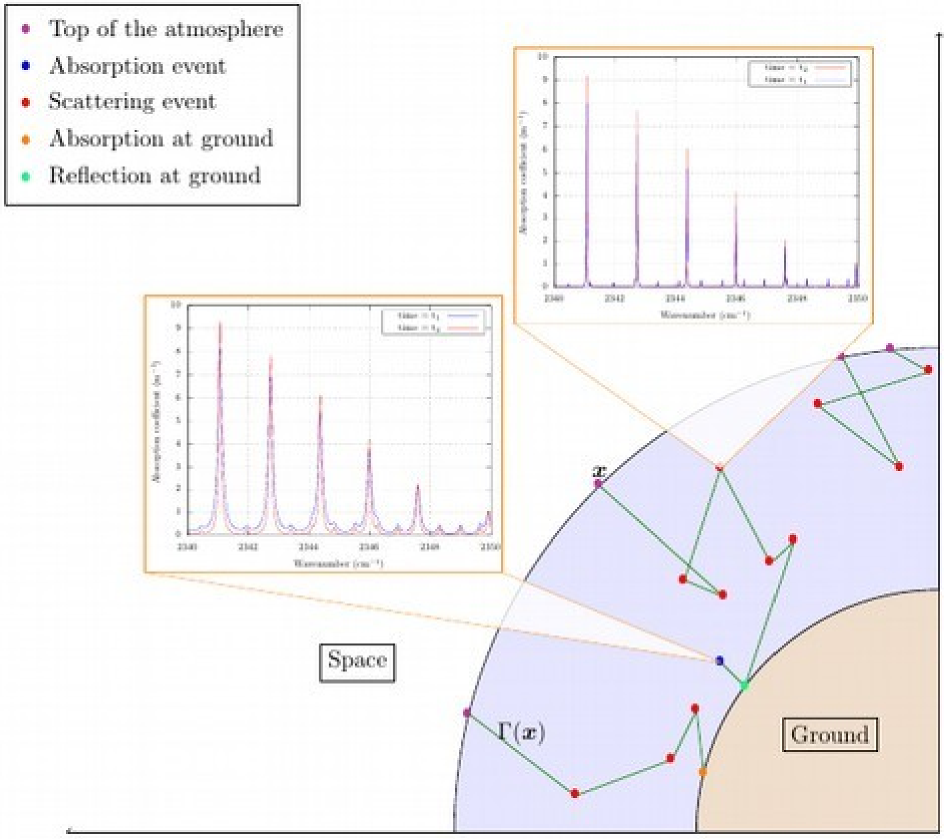
Monte-Carlo computation of 3D atmosphere with sampling of molecular transition (line-by-line)

$$\bar{\phi} = \int_{\Delta t} p_T(t)dt \int_{TOA} p_S(\vec{x})dS(\vec{x}) \int_0^{+\infty} p_N(\nu)d\nu$$

$$\int_{2\pi} p_U(\vec{u})d\vec{u} \int_0^{+\infty} \hat{p}_{\mathcal{L}}(l)dl \Bigg\{$$

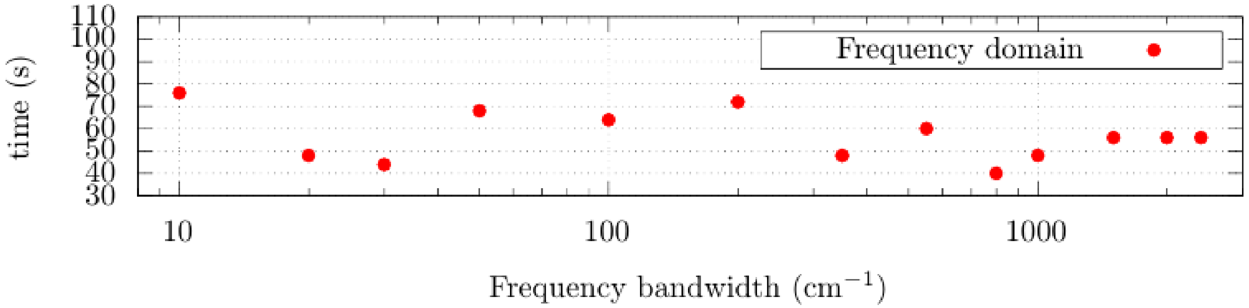
$$\sum_{j=1}^{Nt(s)} P_{J,\nu}(j)\Bigg(P_{a,\nu,j}(\vec{x}',t) \frac{\pi L_\nu^{eq}(T(\vec{x}'))}{p_N(\nu)}$$

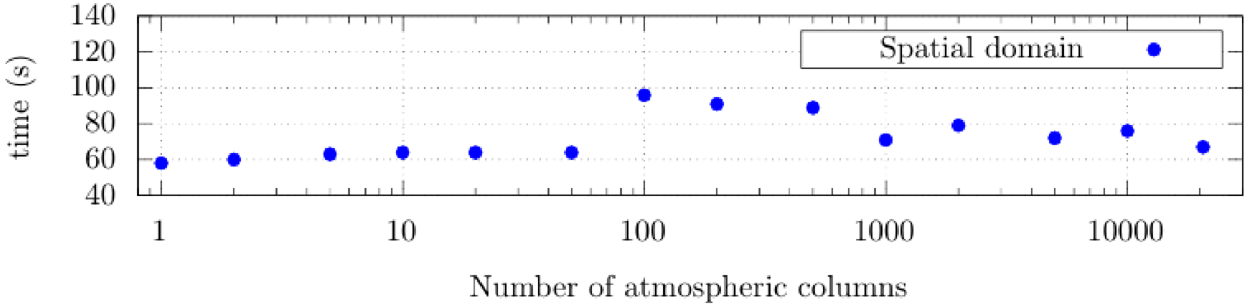
$$+ (P_{n,\nu,j}(\vec{x}',t)) \frac{\pi L_\nu(\vec{x}',\vec{u},t)}{p_N(\nu)} \Bigg) \Bigg\}$$

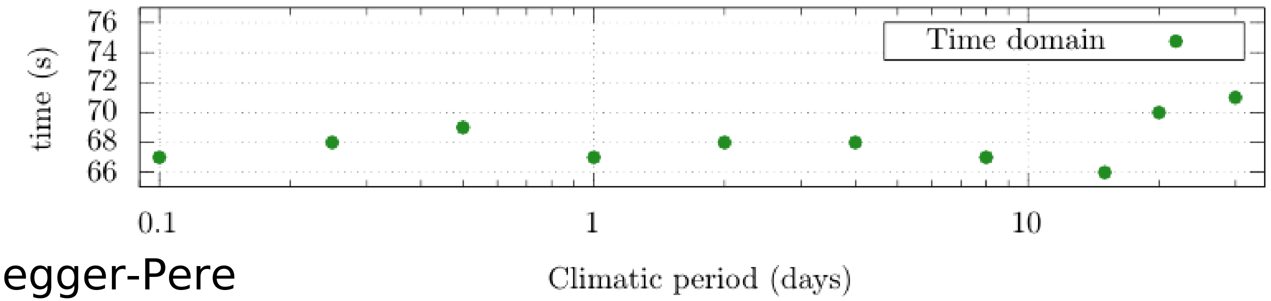


PhD thesis of Yaniss Nyffenegger-Pere

Monte-Carlo 3D line-by-line model: insensitivity to the integration domain







PhD Yaniss Nyffenegger-Pere


FIN