

# **A MONTE CARLO METHOD TO DEVELOP RADIATIVE TRANSFER PARAMETERIZATIONS FOR TERRESTRIAL GCM**

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**Abstract** New accuracy requirements are described that make it necessary to reconsider the problem of parameterizing infrared radiative transfer in atmospheric global circulation models. On this basis a net-exchange Monte Carlo methodology is presented for exploration of the detailed physics of radiative exchanges in emitting, absorbing and scattering atmospheres, and first application examples are presented for Earth clear sky and cloudy atmospheres.

## **1 Introduction**

Global Circulation Models (GCM) are numerical models that include representations of all required physical processes and their interactions in order to predict the climate of a given planet. As far as terrestrial GCMs are concerned, up to half of the total computation time may be required for radiative transfer at all frequencies, among which infrared (IR) frequencies are by far the most time consuming. There are two major issues related to IR radiative transfer in this context : the need for a significant refining of the vertical discretisation of the atmosphere and the representation of radiation scattering by aerosols and clouds.

1. To date, 25 horizontal layers are typically used for the vertical discretisation of the atmospheric column at each point of an horizontal GCM grid (geographic coordinates). It is clear that in a near future accuracy requirements will lead to a significant extension of the top altitude of the considered atmosphere, as well as a finer meshing of the low atmosphere. Considering both tendencies, it is meaningful to plan radiative transfer modeling on the basis of an increase up to 50 layers. But for most of the radiative transfer parameterizations implemented in today's GCMs, because of spectral correlations due to the IR line structure of gaseous atmospheric components (although monochromatic trans-

mittances are exponential, spectral average transmittances are not), an increase by a factor of 2 of the number of layers induces a factor 4 increase of the computation times, which seems strictly unaffordable.

2. It was also recently shown that accurate long term climate modeling requires some level of representation of IR radiation scattering by water and ice clouds, as well as by mineral aerosols [7], [10]. Numerous numerical techniques may be used to simulate radiative transfer in absorbing and scattering media, but the associated computation costs are generally incompatible with GCM requirements. In practice, radiative transfer parameterizations implemented in today's GCMs are unable to accurately take multiple scattering into account.

These constraints motivate new research efforts concerning atmospheric radiative transfer parameterizations, with further detailed physical analysis (in terms of spatial decomposition, optical path distribution, spectral band decomposition, k-distribution, etc) of the most significant IR exchanges and of how these exchanges may be efficiently represented in the GCM context.

In the present paper, our starting point (following a recent similar work concerning Mars atmosphere [14]), is to suggest the use of net-exchange formulations, as introduced by Green in [12]. In this approach, the quantity under consideration is directly the net exchange between pair of meshes. For each pair  $(i, j)$ , this corresponds to the energy emitted from  $i$  and absorbed by  $j$ , minus the energy emitted from  $j$  and absorbed by  $i$ . For most atmospheric applications, because of the line structure of the spectrum of atmospheric gases, the net-exchange matrix (considering the atmospheric layers, ground and space) is dominated by net-exchanges with ground and space, on the one hand, and by short distance net-exchanges between adjacent atmospheric layers, on the other hand. This means that computation efforts may be concentrated on these dominating terms, allowing very significant reductions of radiative transfer computation requirements at each GCM time step.

In this context, there appears a need for radiative transfer simulation techniques to produce accurate atmospheric net-exchange matrixes and to comfortably analyze each net-exchange type in order to propose and test approximation assumptions that may lead to accurate enough GCM parameterizations. A Monte Carlo algorithm that was designed for this purpose is presented hereafter and simulation examples are provided for Earth atmosphere in presence of scattering by mineral aerosols as well as by water clouds. In section 2, this methodologic choice is argued in some detailed on the basis of a rapid review of recent Monte Carlo developments. In section 3, simulation examples are presented and physically discussed, attempting to point out some of the major issues of accurate net-exchange representation of Earth cloudy atmospheres.

## 2 Net-exchange matrix computation with Monte Carlo

Radiative net-exchanges between atmospheric layers (or between an atmospheric layer and ground or space) are multiple integrals over the altitudes across the inhomogeneous layers, over the space of all possible optical paths joining these two altitudes and over the relevant

frequency interval. Numerically speaking, a multiple integral computation method is therefore required and the Monte Carlo method is one of such methods with the well known advantage that an uncertainty interval is systematically associated with each numerical estimation, which is always useful for exploration purposes such as the present one.

However, the Monte Carlo method is seldom mentioned for IR atmospheric applications because of the strong convergence difficulties which standard algorithms encounter in optically thick conditions such as those at spectral line center frequencies. The reasons for such convergence difficulties are quite simple : if an atmospheric layer is optically thick, most emitted photons are absorbed by the layer itself and only the photons emitted close to the boundaries may participate to the radiative exchanges with the rest of the atmosphere. This practically means that a standard Monte Carlo approach will require that a very large number of photon bundles are emitted until statistical significance is reached in terms of radiative power exchanges.

One of the reasons why a Monte Carlo methodology is retained for the present study is that this difficulty has been recently overcome on the basis of reformulations and of sampling optimizations [4], which now allows to think of the Monte Carlo method for practical IR atmospheric applications. The underlying principles of such optimized Monte Carlo algorithms are the following :

1. Addressed quantities are not the radiative exchange rates from one geometrical element to another (the radiative power corresponding to photons emitted by a first element and absorbed by a second one) but instead directly the radiative net-exchange rates between two geometrical elements (the exchange rate from the first element to the second one minus the exchange rate from the second element to the first one). This simply means that when an optical path is followed from one point to the other, this path is not used to evaluate the radiative energy transported one way along this path, but instead directly to evaluate the contribution of the path in terms of radiative net-exchanges : thanks to the reciprocity principle, randomly generated optical paths are simultaneously considered both ways (which induces very little algorithmic changes to existing standard Monte Carlo algorithms [5]).

The first consequence (that was at the origin of such net-exchange Monte Carlo developments [2, 1, 18]) is that no specific difficulty is encountered for quasi-isothermal configurations. With a standard Monte Carlo algorithm, if a given element exchanges radiation with other elements at very similar temperatures, then total emitted and total absorbed radiative powers take similar values (the system is indeed close to radiative equilibrium). The radiative balance of the considered element is therefore approximated as the difference between two approximate quasi-identical quantities, which leads to unpractical numerical difficulties. With net-exchange Monte Carlo algorithms, these difficulties vanish because reciprocal exchange rates are computed simultaneously leading to numerical uncertainties on the net-exchange rates that are at the scale of the addressed radiative balance.

The second consequence is that numerical uncertainties are also much smaller, when

dealing with optically thick systems (in terms of absorption), than those of standard algorithms. The reason is similar : in an optically thick system, most radiative exchanges occur at short distances, that is to say between elements at similar temperatures (assuming a continuous temperature field). The above described numerical advantages are therefore at work for the same reason that the system is (locally) close to radiative equilibrium [2, 4, 18].

2. Another formulation change is made, compared to standard Monte Carlo algorithms, concerning volume and angular integrations. When defining a new optical path starting from a volume element (here an atmospheric layer), common approaches consist of a uniform statistical sampling of a position within the volume and of an isotropic statistical sampling of an initial optical path direction. When doing so, if the volume element is optically thick, since absorption free paths are very short compared to the volume size, most optical paths end within the volume and do not participate to the radiative exchanges with its environment. Only the optical paths starting in the vicinity of the volume boundary may significantly contribute to radiative exchanges. This means, that a very large number of sampled optical paths are required to reach statistical significance [8, 9].

A simple systematic solution to this convergence difficulty was proposed in [4] that consists in a reformulation of these two integrals (volume and angle) into an integral over the volume envelope, then over the outside hemisphere (defining the optical path initial direction), then over the segment intersecting the volume in the backward direction (defining the initial point). The advantage of doing so is that the abscissa along the segment (from the boundary in the inward direction) is a measure of the distance to the boundary, and this abscissa can be statistically sampled in a biased manner, as function of optical thickness, in order to ensure best statistical convergence.

3. In terms of pure numerical optimization (outside reformulation efforts), all sampling procedures are reconsidered in [4] in order to minimize the statistical variance and therefore reduce the number of required sampling events for a given accuracy. Angular sampling, narrow band sampling (when a narrow band spectral discretization is used), k-sampling (when a k-distribution formulation is used), etc, are performed according to probability density functions that do not correspond strictly to the statistical physics of photon emission and transport, but that are best adjusted to the effective contribution of each event to the addressed radiative net-exchange. Compensation terms are then introduced to ensure that the final solution is unchanged and the whole process of probability density function adjustment only modifies the numerical convergence speed toward the same theoretical limit <sup>1</sup> (the corresponding numerical benefits being particularly significant as far as gaseous spectral integration is concerned [6]).

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<sup>1</sup>Theoretically speaking, any non-zero probability density function can be used for statistical sampling in Monte Carlo algorithms. Probability density functions can be chosen to exactly match the physics of particle transport but this may lead to unrealistic computation requirements. In practice, probability density function adjustment always plays a key role in the process of designing efficient Monte Carlo algorithms.

Apart from these recent convergence enhancements, another reason why we retain the Monte Carlo method in the present context (where detailed physical analysis are required) is that it allows, with very little additional computational cost, to systematically evaluate all sensitivities of the addressed quantities to all physical parameters. It was indeed recently shown that sensitivities to temperatures, absorption coefficients, scattering coefficients, phase function asymmetry parameters, etc, could be computed simply on the basis of any existing Monte Carlo algorithm without the need of any additional random sampling [3]. This feature of Monte Carlo methods is easy to understand in the case of physical parameters that do not appear in the probability density functions used for statistical sampling. As far as present application is concerned, Monte Carlo computed net-exchange rates correspond practically to sums of partial net-exchanges along sampled optical paths ; the sensitivity of such a net-exchange to any gaseous concentration, for instance, will therefore directly be a sum of first order derivatives of partial net-exchange rates (that take simple analytical forms) with respect to the considered concentration. Things are less trivial when the considered parameter appears in the sampling probability density functions. This is in particular the case if one considers a sensitivity to any parameter that affects scattering : parameter changes affect the scattering probability density functions used for optical paths sampling (scattering free paths and scattering angles sampling) and correction terms must be added to the sum of partial net-exchange derivatives. These correction terms are given in [3] and are shown to introduce no specific implementation difficulty.

Finally, the Monte Carlo method is undoubtedly a comfortable tool for systematic quantitative analysis of the relative contributions of multiple transport processes. To make it short, referring to any Monte Carlo implementation experience, there is indeed no technical difficulty associated to the splitting of a given net-exchange rate, for physical interpretation purposes, into various contributions such as frequential contributions for which an optically thick assumptions is meaningful (leading to simple diffusive parameterizations), those for which optically thin assumptions are meaningful, etc, this even within a given narrow band if a k-distribution formulation is used. Similarly, there is no difficulty associated to the splitting of a net-exchange rate into the contributions of direct net-exchanges, versus those via one single diffusion, those via two diffusions, etc. For each such partial mechanism analysis, the algorithm is kept unchanged and only the averaging process is modified according to the considered physics.

Altogether, considering today's needs for further detailed analysis of IR atmospheric radiative exchanges, we estimated that it was meaningful to apply the above described Monte Carlo methodology to atmospheric configurations and we implemented the net-exchange algorithm of [4] together with a correlated-k spectral model [11]. The considered spectrum ranges from  $4 \mu m$  to  $100 \mu m$  and is divided into 121 narrow bands of width  $20 \text{ cm}^{-1}$ . For each atmospheric layer, given the average pressure, the average temperature and the  $H_2O$ ,  $CO_2$  and  $O_3$  molar fractions, the k-distribution parameters are derived from LOWTRAN7 database [16]. Aerosols and droplets are assumed spherical and their absorption and scattering properties are assumed independant of frequency within each narrow band ; these properties are modeled with Mie theory on the basis of given size distributions.

The original Monte Carlo algorithm of [4] was modified in such a way that multiple scat-

tering optical paths are randomly generated as in any classical diffusion particle transport algorithm : once an optical path is initiated at a given position, in a given direction, a pure scattering free path optical thickness is randomly generated according to an exponential distribution and the optical path is followed along a straight line until the corresponding scattering location is reached ; at this location, a new direction is randomly generated according to the local scattering phase function and a new scattering free path optical thickness is generated, etc. No specific optimization procedure is here applied as function of scattering characteristics.

Similarly to what was made for pure absorption configurations, optimizing the algorithm as function of scattering could be performed via a reformulation of the optical path domain integral and/or an optimization of the scattering sampling laws. As far as reformulation is concerned, the work by [13] on radiation shielding is undoubtedly worth a close attention and, as far as sampling laws are concerned, we performed a preliminary work on the adjustment of the mean free path probability density function and the phase angle probability density function on the basis of analytical results corresponding to the pure backscattering 1D limit case ; this work will be the objective of a forthcoming publication.

### 3 Test cases and conclusions

Two examples of net-exchange Monte Carlo simulation results are presented in this section in order to briefly comment on the main difficulties encountered when attempting to design efficient radiative transfer parameterization for GCM applications. The same vertical discretisation is used in both examples with 50 atmospheric layers up to 100 km. The first example corresponds to a clear sky configuration typical of Earth atmosphere (the temperature, pressure and gaseous concentration profiles are those of the reference tropical atmosphere of [15]). The second example corresponds to the same temperature, pressure and gaseous concentration profiles, on which both a dust cloud and a liquid water cloud are superimposed. The dust cloud extends from the surface up to 2.5 km and corresponds to the saharian dust observations reported in [17]. The water cloud is a simple homogeneous cloud of  $31\mu m$  water drops that extends from 7.5 km up to 8 km with a total water content of  $10g/m^2$  (integrated of the cloud thickness). Figure 1 displays the simulated heating rates <sup>2</sup> for both examples as function of altitude (the figure is repeated twice with different scales) and Fig. 2 displays the corresponding net-exchange matrixes.

The heating rate profile for clear sky conditions shows standard features of Earth atmosphere. Above 12 km, the atmosphere is close to IR radiative equilibrium, whereas at lower altitudes significant IR radiative cooling partially compensates other heat transfer exchanges such as convection and water phase changes. Also worth a mention is the rapid change of heating rates close to 4 km that corresponds to a decrease of water vapor concentration with altitude which, from a radiative point of view, induces around this altitude a transition from optically thick

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<sup>2</sup>The heating rate of a given atmospheric layer corresponds to the radiative budget of the layer and is either expressed in  $W/m^2$ , or in  $W/m^3$  (when divided by layer width), or in  $K/s$  (when divided by layer width and by the volumic heat capacitance). In the present text, the  $W/m^3$  convention is retained.

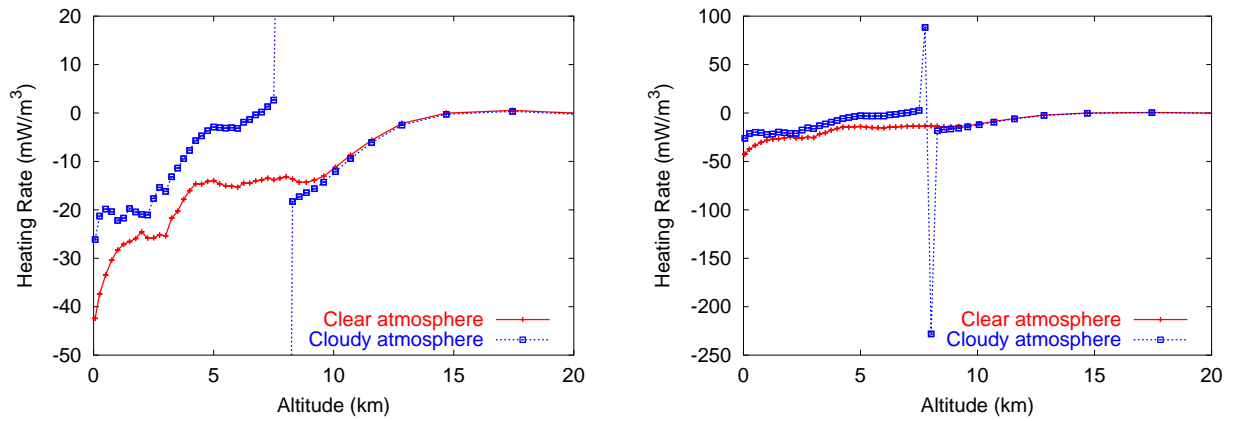


Figure 1: Heating Rates expressed in  $mW/m^3$  for clear sky conditions and cloudy conditions.

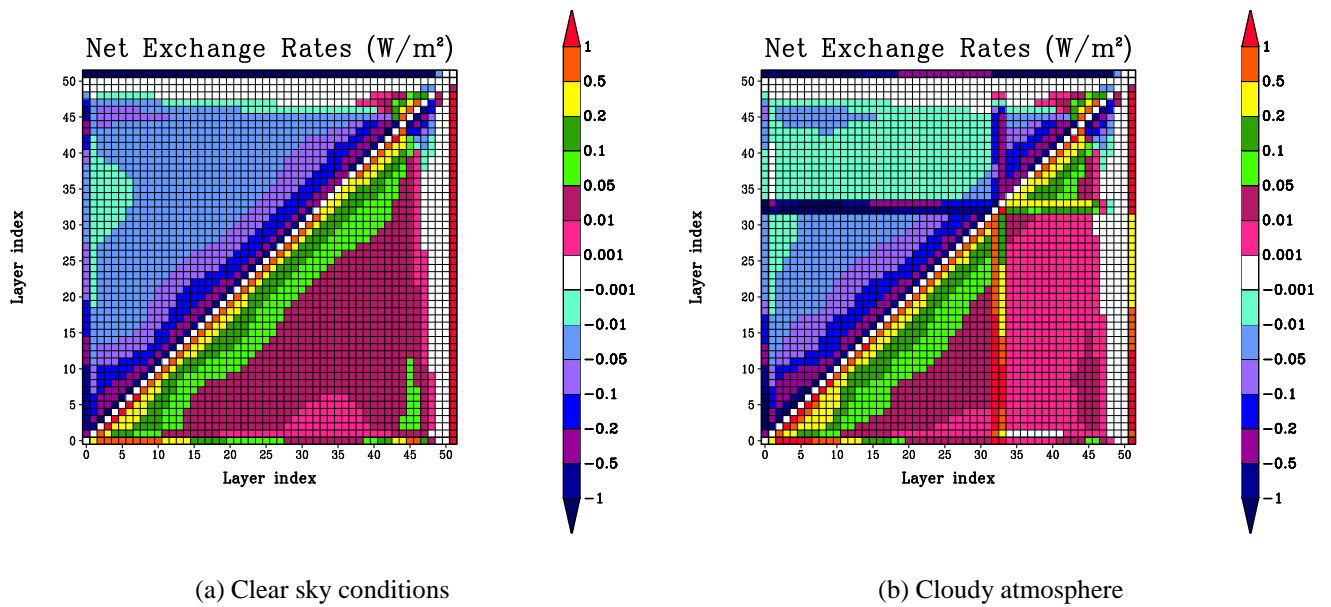


Figure 2: Net Exchange Rates matrixes for (a) clear sky conditions (no aerosols, no clouds) and (b) cloudy conditions : presence of aerosols and water clouds. Each element displays the Net Exchange Rate (in  $W/m^2$ ) between each pair of layers. Index 0 stands for ground, index 51 stands for space, and all indexes inbetween stand for the atmospheric layers.

conditions to intermediate conditions in the water vapor spectral bands.

If one keeps in mind that such heating rates are dominated by net-exchanges with space (“cooling to space”), then it is quite easy to understand the modifications induced by the presence of clouds. Clouds introduce an additional optical thickness between low altitudes and space and the cooling rates decrease therefore at all altitudes below the cloud. For altitudes above the clouds, on the contrary, very little change is observed because cooling to space is unaffected, the only change in heating rates being that the atmosphere is heated by net-exchanges with the cloud top that is here colder than the bottom atmosphere and the surface. The same physical image may be easily used to interpret the fact that for optically thick clouds (such as the present water drop cloud) a strong heating of the bottom of the cloud is observed, together with a strong cooling of the cloud top.

All such first level physical pictures are clearly illustrated and quantified in Fig. 2 with the net-exchange matrixes. The first and last columns and lines of these matrixes correspond to ground and space respectively, intermediate columns and lines corresponding to the successive 50 atmospheric layers used for spatial discretization of the atmosphere. For clear sky conditions, the orders of magnitude of the last horizontal line (or of the last vertical column, the matrix being anti-symmetric), compared to the rest of the matrix, clearly indicate that IR radiative net-exchanges are dominated by net-exchanges with space. When similarly analyzing the cloudy atmosphere results, it appears (again because the water cloud is here optically thick) that, for the atmosphere below the cloud, no significant net-exchange with space is possible and the corresponding sub-matrix becomes dominated by net-exchanges with the cloud bottom (leading to lower cooling rates simply because the cloud is hotter than space). The sub-matrix corresponding to the atmosphere above the water cloud is mainly unaffected by the cloud, except that net-exchanges with the ground are replaced by less intense net-exchanges with cloud top.

The other point that appears quite clearly on such matrixes is that radiative net-exchanges within the atmosphere itself (net-exchanges between uncloudy atmospheric layers) are dominated by net-exchanges with adjacent layers. We already mentioned this point in introduction as a consequence of spectral correlations associated to gaseous line spectra. This illustrates the point that net-exchange formulation allows to significantly reduce the complexity of atmospheric IR radiation parameterization in the sense that there are obviously only a few numbers of net-exchanges that are worth detailed attention (and intensive computation efforts), even up to quite high accuracy levels. However, a rapid look at the two matrixes of Fig. 2 indicates that the presence of clouds may significantly modify the matrix structure which is source of practical difficulties in terms of rapid parameterization design. It is clear that for Earth, as was already observed for Mars, net-exchange matrixes are structured in such a way that strong computation amounts may be saved by a priori selection of the dominating terms, but unlike for Mars this “a priori” selection<sup>3</sup> is not simply and systematically the near diagonal terms plus net-exchanges

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<sup>3</sup>For Mars, very significant computation times could be saved in the GCM context by neglecting the temporal variations of the net-exchanges between distant layers, while only systematically recomputing the net-exchanges between adjacent layers and the net-exchanges between each layer and the boundary surfaces (ground and space).



with space and ground. Optically thick clouds require at least a specific treatment and, more generally speaking, detailed analysis of standard atmospheric configurations will be required if similar benefits are expected for Earth GCMs as was obtained for uncloudy Mars atmospheres.

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The extension of such an approach to Earth cloudy atmospheres will at least require a systematic recomputation of the net-exchanges between each clear atmosphere layer and the nearest cloud layers.

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