EXOTOP December 4th and 5th at LATMOS, Guyancourt, France

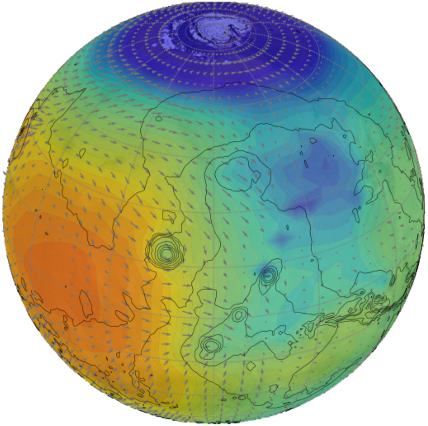
Turbulence and Drags from Non-orographic Gravity Waves in the Mars Planetary Climate Model

Jiandong Liu[1,2]; Ehouarn Millour[1]; Francois Forget[1]; Francois Lott[1], J.Y. Chaufray

[1]LMD/IPSL, Sorbonne Université, ENS, Université PSL, École Polytechnique, Institut Polytechnique de Paris, CNRS, Paris, 75005, France.
[2] LPC2E, CNRS/Université d'Orléans, UMR 7328, Orléans, 45071, France.

Context

- **Introduction**
- **Formalism (Drags+Turbulence)**
- **Simulations v.s. Observations**



Sciences:

small waves, big effects

Non-orographic GWs:

> Waves EP-flux

> > Divergences of EP-fluxes (Drags)

> > > Induced turbulence/mixing

> > > > Mixing fluxes induced drags

Planetary waves

> β effect, $\Im(m)$,fast decay

Thermal tides

> Forced planetary-scale GWs

Orographic GWs:(bugged)

> Drags from Blocked Flows

>Propagating Waves from Top of Blocked Flows

> >induced Mixing (not parameterized, see tomorrow's talk)

No inertial filtering is considered currently, thus these schemes do not apply to fast-rotated Planets such as Jupiter, Saturn, etc; See discussion demain

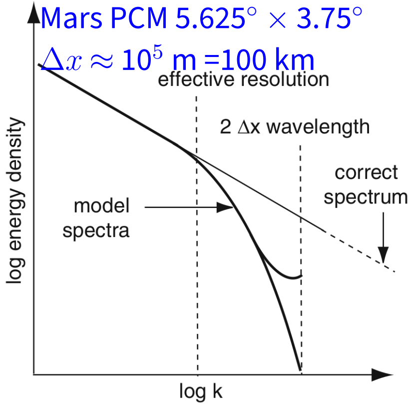
Gravity Waves in Mars PCM

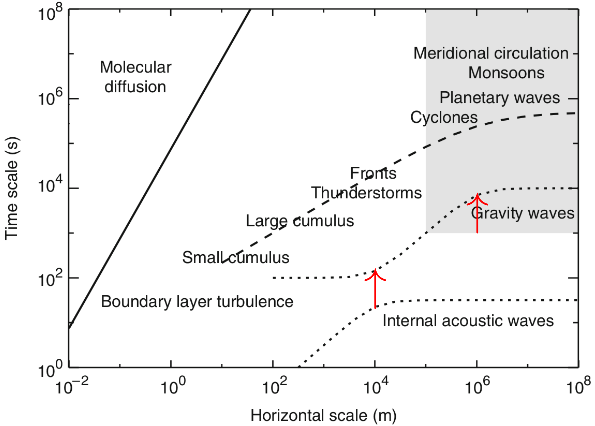
$k_{eff} \in [2\pi/10\Delta x, 2\pi/4\Delta x]$

Model physical $\delta t \approx$ 4.6$\times$ 10^2 s

Mars PCM 5.625° $\times$ 3.75°

$\Delta x \approx 10^5$ m =100 km





Lauritzen et al.(2011)

Modeled gravity waves: 10^4-10^6 m with life cycle of 88775 s

Setting Up

Variables	Ranges
wavenumber k, l [m^{-1}]	$2\pi/10\Delta x \leq k \leq \max(\pi/\sqrt{\Delta x \Delta y}, 2\pi/\Delta x/2)$
phase velocity c [m s^{-1}]	$c = \|\vec{u}_r\|, 2\|\vec{u}_r\|$
saturation factor S_c	$1 < S_c \leq 3$ (due to mixing)
viscosity-eddy $\mu*$	$\mu \bar{D}_{eddy}(z_b)$
source altitude z_r	ave(PBL), vPBL, other
max EP-flux E_{max}	$2\Omega \sin\phi \times \frac{\overline{v'\theta'}}{\bar{\theta}_z}$ (RMC by GWs at z_b)

wavelength: $\lambda = 2\pi/k$; frequency: $\omega = kc$

Liu et al.(2023),2025a,2025b

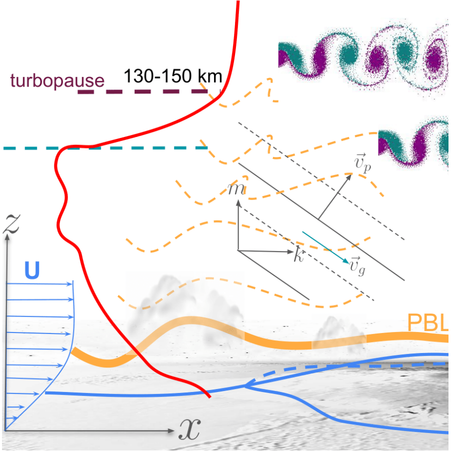
EP-Flux

Pseudo-Momentum:

$$\vec{E}_j^{z_r}(k_j, l_j, \omega_j) = \Re\left\{\rho_r \frac{\hat{u}\hat{w}^*}{2}\right\}$$

$$\hat{u} = -\frac{m}{|\vec{k}|}\hat{w}$$

$$\vec{E}_j^{z_r}(k_j, l_j, \omega_j) = -\rho_r \frac{\vec{k}}{2|\vec{k}|} m_j(z_r) |\hat{w}_j(z_r)|^2 \frac{\vec{k}\Omega}{|\vec{k}\Omega|}$$



$$\hat{w}_j(z_{ll+1}) = \underbrace{\Theta[\Omega(z_{ll+1}) \times \Omega(z_{ll})]}_{critical-layer} Min\left\{\hat{w}_j(z_{ll})\sqrt{\frac{m_r(z_{ll})}{m_r(z_{ll+1})}} e^{-\frac{\mu}{\rho}\left(\int_{z_{ll}}^{z_{ll+1}} m_i^{ave} d\zeta\right)}, \underbrace{\hat{w}_{j,s}}_{saturation}\right\}$$

turbulence (m_i^{ave})

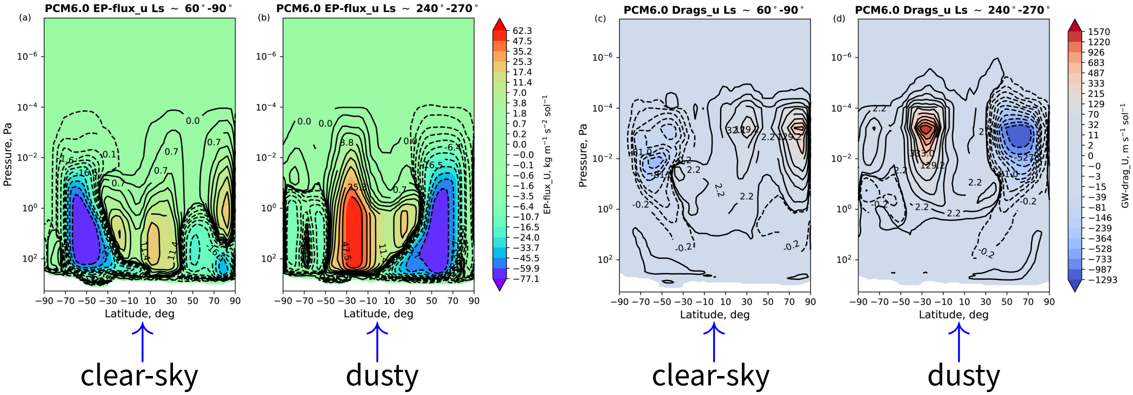
kinematic viscosity ($\frac{\mu}{\rho}$)

Lott et al.(2012),2013

How Do Non-orographic Gravity Waves Propagate ?

EP-flux [kg m^{-1} s^{-2} sol^{-1}]

Drags [m s^{-1} sol^{-1}]

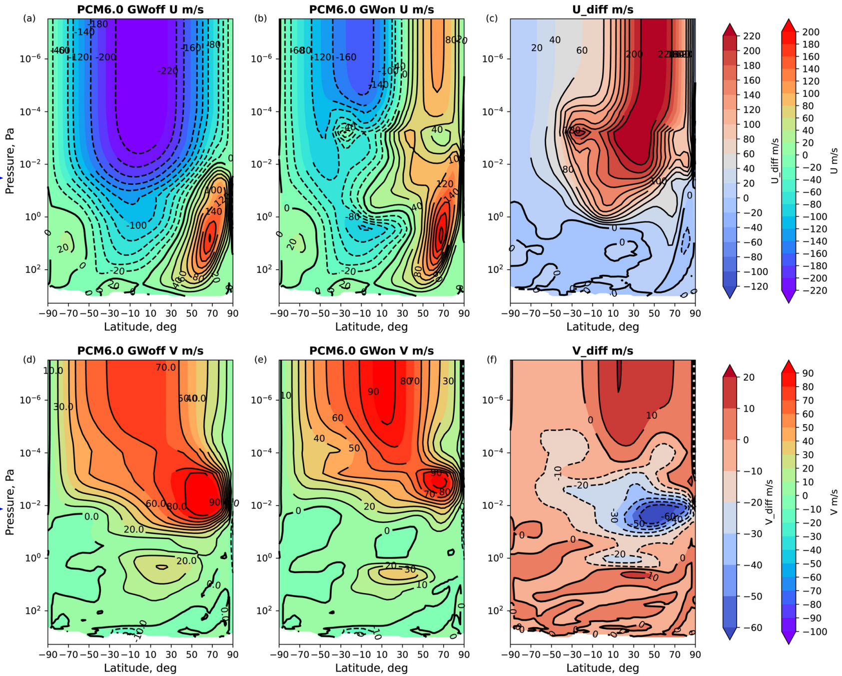


Liu et al.(2023),JGR-Planets:10.1029/2023JE007769

Effects on Flows

zonal winds

meridional winds



Drag on Circulation

$$\Psi = \frac{2\pi a \cos\phi}{g} \int_0^p \overline{v}\, dp'$$

clear-sky | dusty

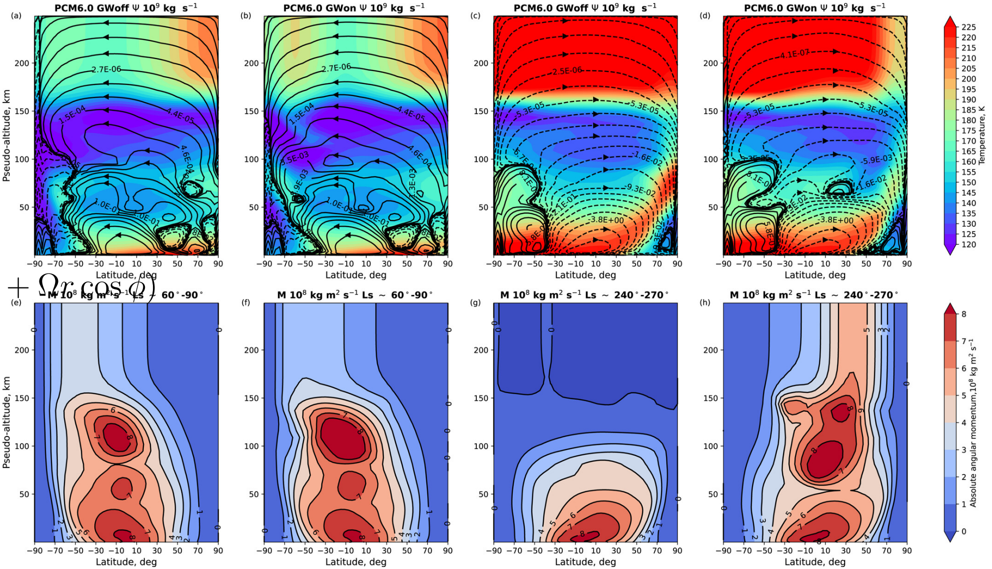
Φ

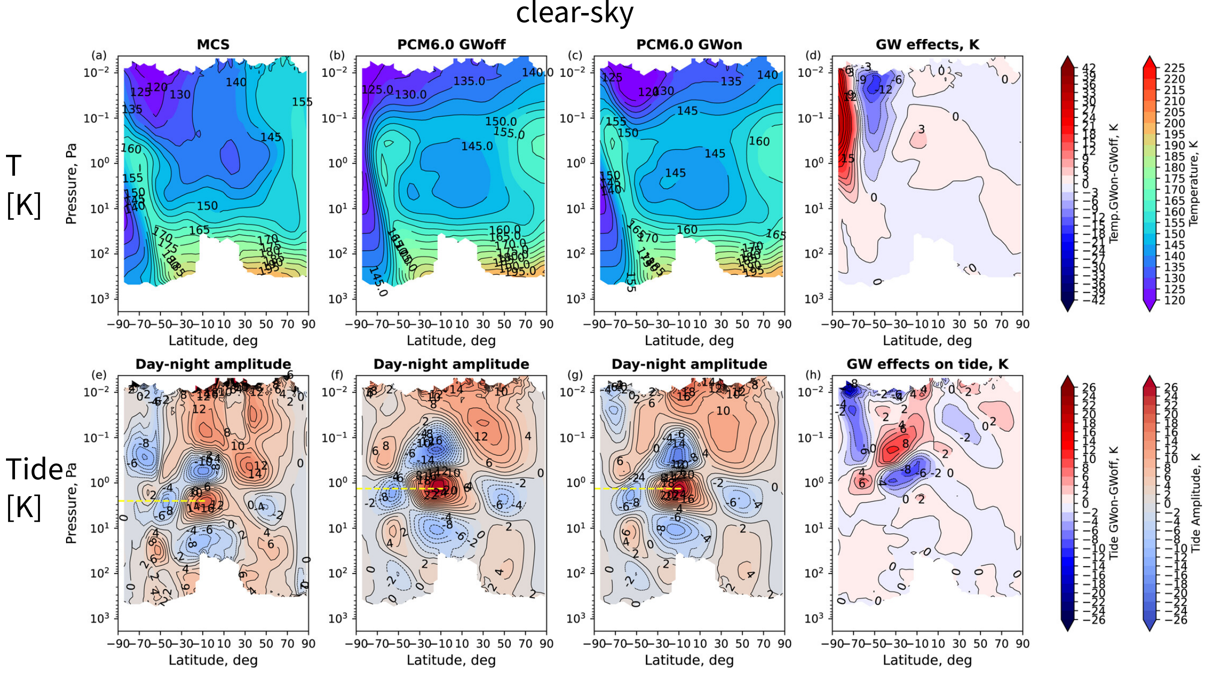
$[10^9 \text{ kg s}^{-1}]$

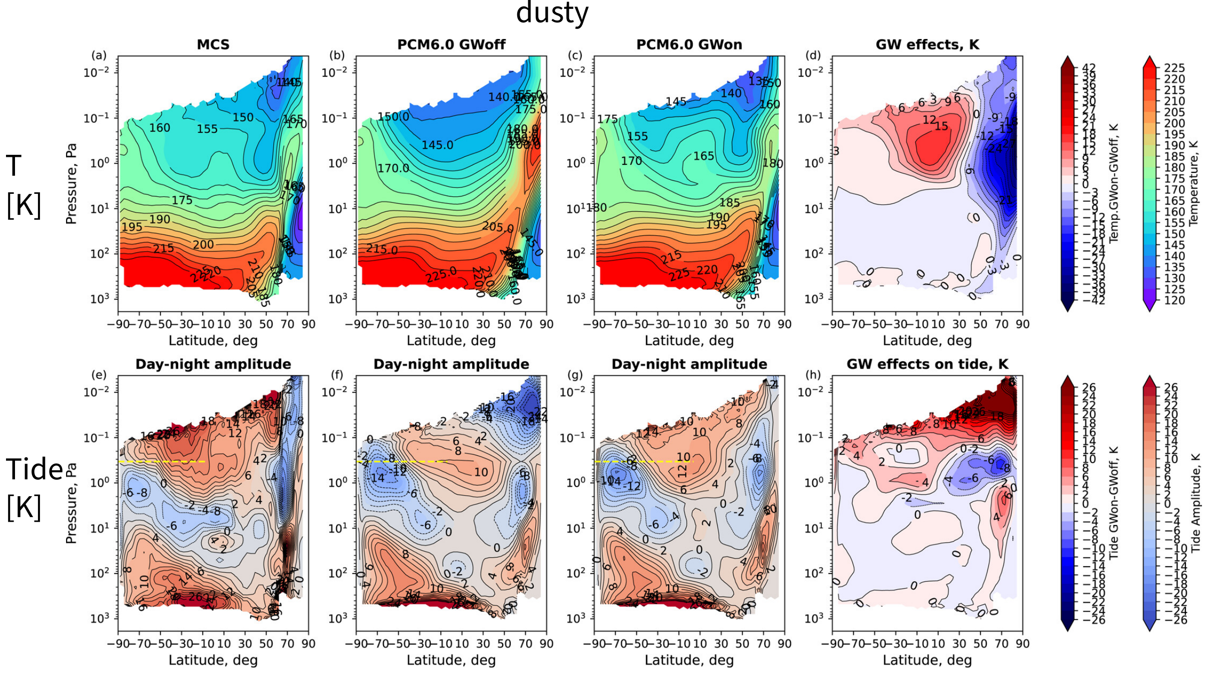
$$M = r\cos\phi\,(\bar{u} + \Omega r \cos\phi)$$

M

$[10^9 \text{ kg m}^2 \text{ s}^{-1}]$

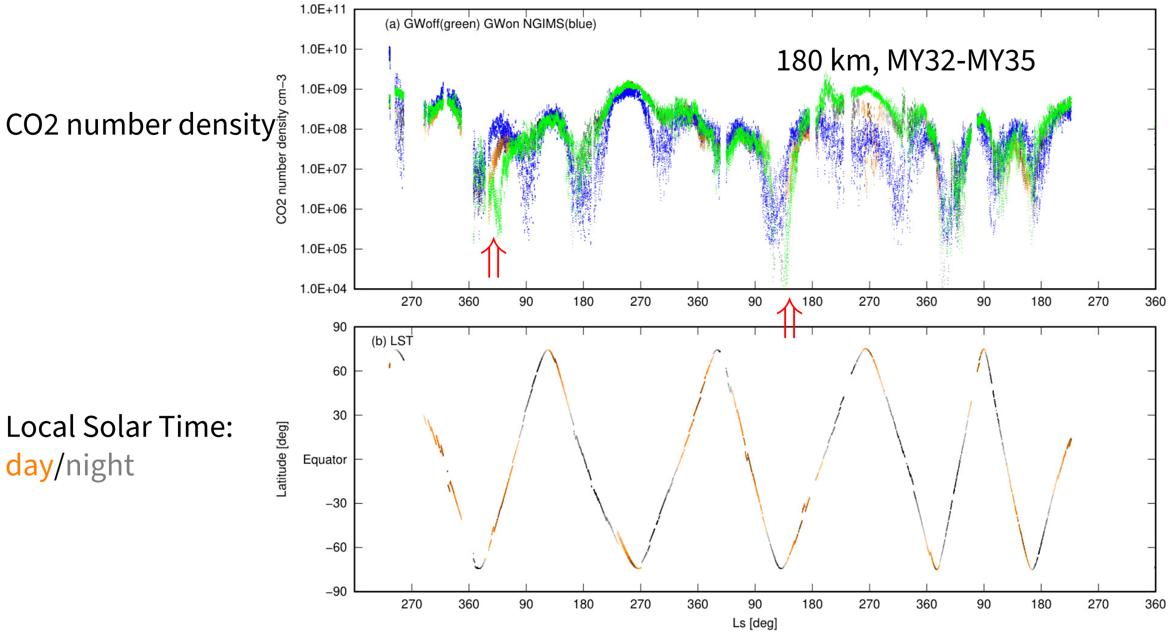



clear-sky
T
[K]
Tide
[K]
MCS
PCM6.0 GWoff
PCM6.0 GWon
GW effects, K
Day-night amplitude
Day-night amplitude
Day-night amplitude
GW effects on tide, K
Pressure, Pa
Latitude, deg
Temp.GWon-GWoff, K
Temperature, K
Tide GWon-GWoff, K
Tide Amplitude, K



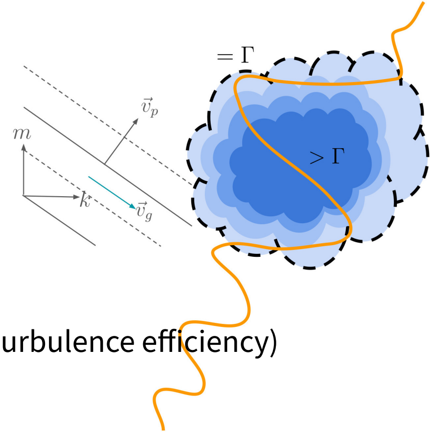
dusty
T
[K]
Tide
[K]
(a) MCS
(b) PCM6.0 GWoff
(c) PCM6.0 GWon
(d) GW effects, K
(e) Day-night amplitude
(f) Day-night amplitude
(g) Day-night amplitude
(h) GW effects on tide, K
Pressure, Pa
Latitude, deg
Temp.GWon-GWoff, K
Temperature, K
Tide GWon-GWoff, K
Tide Amplitude, K

NGIMS/GWoff/GWon



Turbulence from Waves

Assumption 1: Turbulence triggered by the release of wave momentum, viz.
$Turbulence \propto \frac{1}{\rho}\nabla(\vec{E})$



Assumption 2: Non-superadiabatic Principle
$|\frac{dT}{dz}|_{gw} = \Gamma$ at z_b (Lindzen 1981)

Assumption 3: Symmetric Principle

Assumption 4: Linear Damping (instability-to-turbulence efficiency)

Wave Thermodynamics: m_i at z_b

Assumption 2: Non-superadiabatic Principle

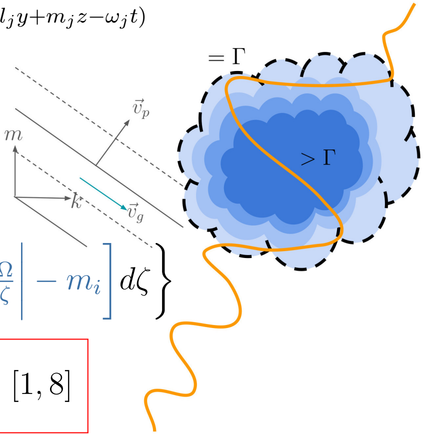
$$-i\Omega T' + w'\Gamma = 0 \qquad w'_j = \hat{w}_j(z)\underbrace{e^{z/2H}}_{\rho(z)} e^{i(k_j x + l_j y + m_j z - \omega_j t)}$$

$$i\vec{k}_j(c-\bar{u})\delta T = \hat{w}_j e^{z/2H}\Gamma$$

$$\Re\left\{\frac{d\delta T}{dz}\right\} = -\frac{\Gamma A |m_r|^{3/2} e^{\int_0^z \left(\frac{1}{2H} - m_i\right) d\zeta}}{N|\vec{k}|}$$

$$\Re\left\{\frac{d\delta T}{dz}\right\} = -\frac{\Gamma A}{N|\vec{k}|}\exp\left\{\int_0^z \left[\frac{1}{2H} + \frac{3}{2}\left|\frac{1}{N}\frac{\partial N}{\partial \zeta} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \zeta}\right| - m_i\right] d\zeta\right\}$$

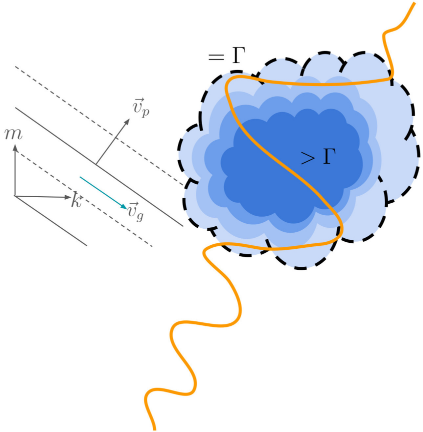
$$\boxed{\frac{1}{2H} + \frac{3}{2}\left|\frac{1}{N}\frac{\partial N}{\partial z} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial z}\right| = m_{i,s}, \qquad z = z_b, \forall j \in [1, 8]}$$



Wave Thermodynamics: NSP

Assumption 2: NSP $|\frac{dT}{dz}|^2_{gw} = \Gamma^2$

$$\Re\{\frac{d\delta T}{dz}\} = -\frac{\Gamma A|m_r|^{3/2}e^{\int_0^z(\frac{1}{2H}-m_i)d\zeta}}{N|\vec{k}|}$$



Saturation altitude z_b and A^2

$$A^2 m_r^{-1}\exp\left(-2\int_0^{z_b} m_i d\zeta\right) = m_r^{-4}N^2|\vec{k}|^2\exp\left(-\int_0^{z_b}\frac{1}{H}d\zeta\right), \quad z = z_b$$

Wave Thermodynamics: Saturated EP-flux

Assumption 2: Non-superadiabatic Principle $|\frac{dT}{dz}|^2_{gw} = \Gamma^2$

$$A^2 m_r^{-1} \exp\left(-2\int_0^{z_b} m_i d\zeta \right)$$

$$= m_r^{-4} N^2 |\vec{k}|^2 \exp\left(-\int_0^{z_b} \frac{1}{H} d\zeta \right)$$

$$z \in [z_b, z_b + \delta z_{ll}]$$

$$\Longrightarrow \quad |\hat{w}_j(z_b)|^2 \approx A^2 m_r^{-1} \exp\left(-2\int_0^{z_b} m_i d\zeta \right)$$

$$= m_r^{-4} N^2 |\vec{k}|^2 \exp\left(-\int_0^{z_b} \frac{1}{H} d\zeta \right)$$

$$= \frac{\Omega^4}{N^2 |\vec{k}|^2} \exp\left(-\int_0^{z_b} \frac{1}{H} d\zeta \right)$$

$$\Downarrow$$

$$\hat{w}_{j,s} = \frac{\Omega^2}{|\vec{k}|N} e^{-z/2H} S_c \frac{k^*}{|\vec{k}|}$$

$$\Longleftarrow$$

$$\vec{E}_j^{z_{ll+1}} = \frac{\vec{k}\Omega}{|\vec{k}||\Omega|} \Theta[\Omega(z_{ll+1})\Omega(z_{ll})]$$

$$Min\left\{ |\vec{E}_j^{z_{ll}}| e^{-2\frac{\mu m_i}{\rho}\delta z}, \rho_r S_c^2 e^{-\frac{z_{ave}}{H}} \frac{|\Omega|^3 k^{*2}}{2N|\vec{k}|^4} \right\}$$

Wave Thermodynamics: Linear Damping

Assumption 4: Linear Damping

$$D^j_{eddy}\frac{\partial}{\partial z^2}\begin{Bmatrix}\bar{u}\\ \delta T\end{Bmatrix} = -m_r^2 D^j_{thermal}\begin{Bmatrix}\bar{u}\\ \delta T\end{Bmatrix}$$

$$Prandtl = D^j_{eddy}/D^j_{thermal} = 1$$

$$[i\vec{k}(c-\bar{u}) + m_r^2 D_{eddy}]\delta T = w'\Gamma$$

$m = m_r + im_i$, we rewrite $c = c_r + ic_i$,

$$\vec{k}c_i = m_r^2 D^j_{eddy}$$

Wave Thermodynamics: Linear Damping

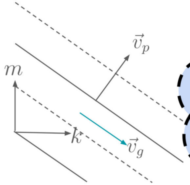
Assumption 4: Linear Damping: The turbulence m_i and c_i,

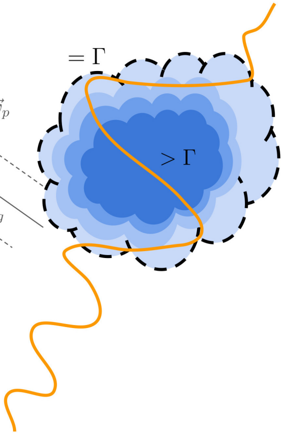
$$c = c_r + i c_i$$

$$m = \frac{N|\vec{k}|}{\vec{k}(\bar{u} - c)} \quad \Longleftarrow \quad \vec{k} c_i = m_r^2 D_{eddy}^j$$

$$= \frac{N|\vec{k}|}{\vec{k}(\bar{u} - c_r) - i m_r^2 D_{eddy}^j}$$

$$= -\frac{N|\vec{k}|\Omega}{\Omega^2 + \cancel{m_r^4 D_{eddy}^2}} + i \frac{N|\vec{k}| m_r^2 D_{eddy}^j}{\Omega^2 + \cancel{m_r^4 D_{eddy}^2}}$$





$$m_i = \frac{N|\vec{k}| m_r^2 D_{eddy}}{\Omega^2} = \frac{m_r^3 D_{eddy}^j}{\Omega}, \quad z \in [surface, top]$$

Eddy Diffusion Coefficient $z = z_b$

Since

$$m_i = \frac{N|\vec{k}|m_r^2 D_{eddy}^j}{\Omega^2}, z \in [surf, top]$$

Saturated vertical wavenumber of turbulence,

$$\frac{1}{2H} + \frac{3}{2}\left|\frac{1}{N}\frac{\partial N}{\partial z} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial z}\right| = m_{i,s}, \qquad z = z_b$$

$$D_{eddy}^j = \frac{\Omega^4}{N^3|\vec{k}|^3}\left(\frac{3}{2}\left|\frac{1}{N}\frac{dN}{dz} - \frac{1}{\Omega}\frac{d\Omega}{dz}\right| + \frac{1}{2H}\right)$$

$$D_{eddy}^j = S_{mix}\frac{\Omega^4}{N^3|\vec{k}|^3}\left(\frac{3}{2}\left|\frac{1}{N}\frac{dN}{dz} - \frac{1}{\Omega}\frac{d\Omega}{dz}\right| + \frac{1}{2H}\right), \qquad z = z_b$$

Eddy Diffusion Coefficient $z > z_b$

Assumption 1: Turbulence triggered by momentum releasing, viz,
$Turbulence \propto \nabla(\vec{E})$

$$
\begin{aligned}
-\frac{1}{\rho}\frac{\partial \vec{E}_j^z}{\partial z} &= \alpha_{eff}\frac{\Omega^3}{N|\vec{k}|^2} m_i \frac{\vec{k}}{|\vec{k}|} \\
&= \alpha_{eff}\frac{\Omega^3}{N|\vec{k}|^2}\left(\frac{3}{2}\left|\frac{1}{N}\frac{dN}{dz} - \frac{1}{\Omega}\frac{d\Omega}{dz}\right| + \frac{1}{2H}\right)\frac{\vec{k}}{|\vec{k}|}, \qquad z > z_b
\end{aligned}
$$

$$
\boxed{D^j_{eddy} = \alpha_{eff}\frac{\Omega}{N^2|\vec{k}|}\left\{-\frac{1}{\rho}\frac{\partial \vec{E}_j^z}{\partial z}\right\}, \qquad z > z_b}
$$

Eddy Diffusion Coefficient $z < z_b$

Assumption 3: Sysmetric Principle: empirical fit

$$D^j_{eddy} = \mathcal{B} \times n_q^{-1/2}$$

Since we know $n \propto \exp -\frac{z}{H}$,

$$D^j_{eddy} = D^j_{eddy}(z_b) e^{\beta_{diff}(z-z_b)/H}, \qquad z < z_b$$

General Equation

Eddy diffusion coefficient $\forall j$th monochromatic wave:

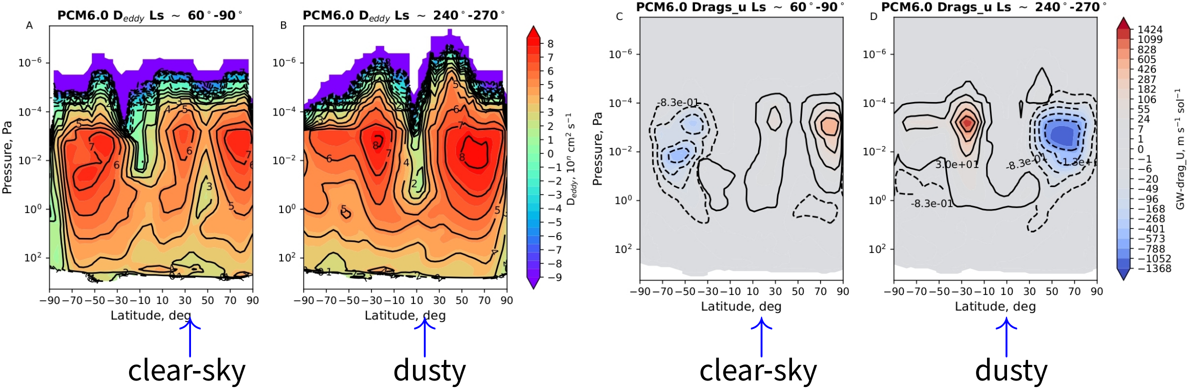
$$D_{eddy}^{j} = \begin{cases} Min\left[\alpha_{eff}\frac{\Omega}{N^2|\vec{k}|}\left(-\frac{1}{\rho}\frac{\partial \vec{E}}{\partial z}\right), S_{mix}\frac{\Omega^4}{N^3|\vec{k}|^3}\left(\frac{3}{2}\left|\frac{1}{N}\frac{dN}{dz}-\frac{1}{\Omega}\frac{d\Omega}{dz}\right|+\frac{1}{2H}\right)\right] & , z \geq z_b \\ D_{eddy}(z_b)\exp\left[\beta_{diff}\frac{(z-z_b)}{H}\right] & , z < z_b \end{cases}$$

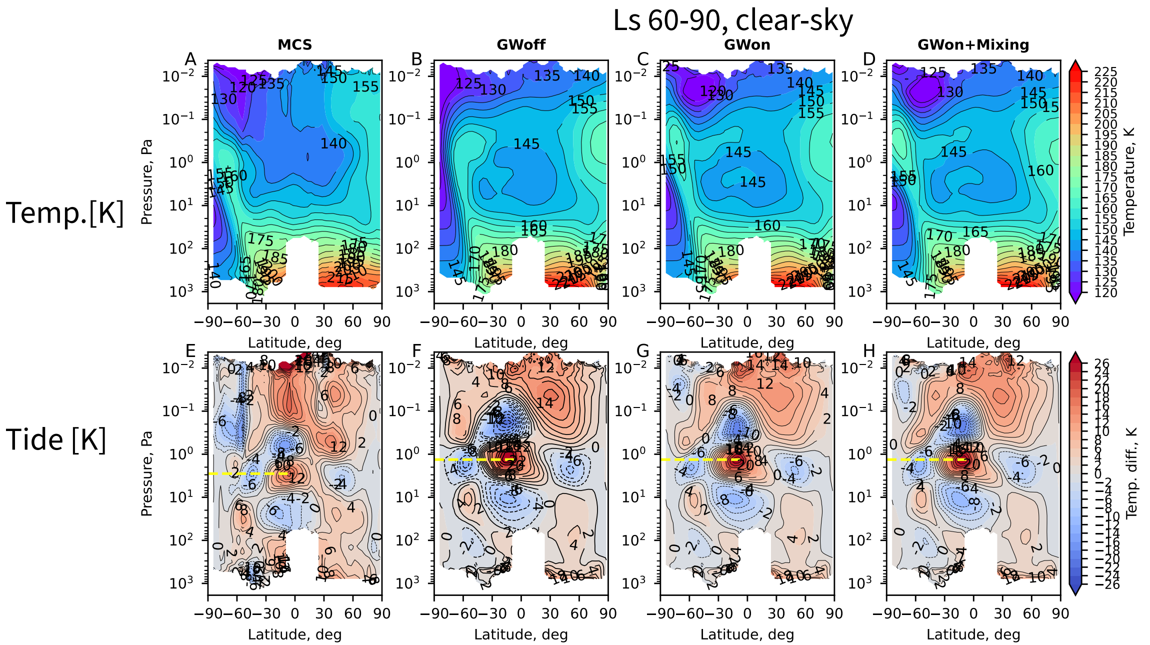
$\alpha_{eff} = S_{mix} = 0.1$ and $\beta_{diff} = \frac{3}{2}$

Eddy Diffusion Coefficient

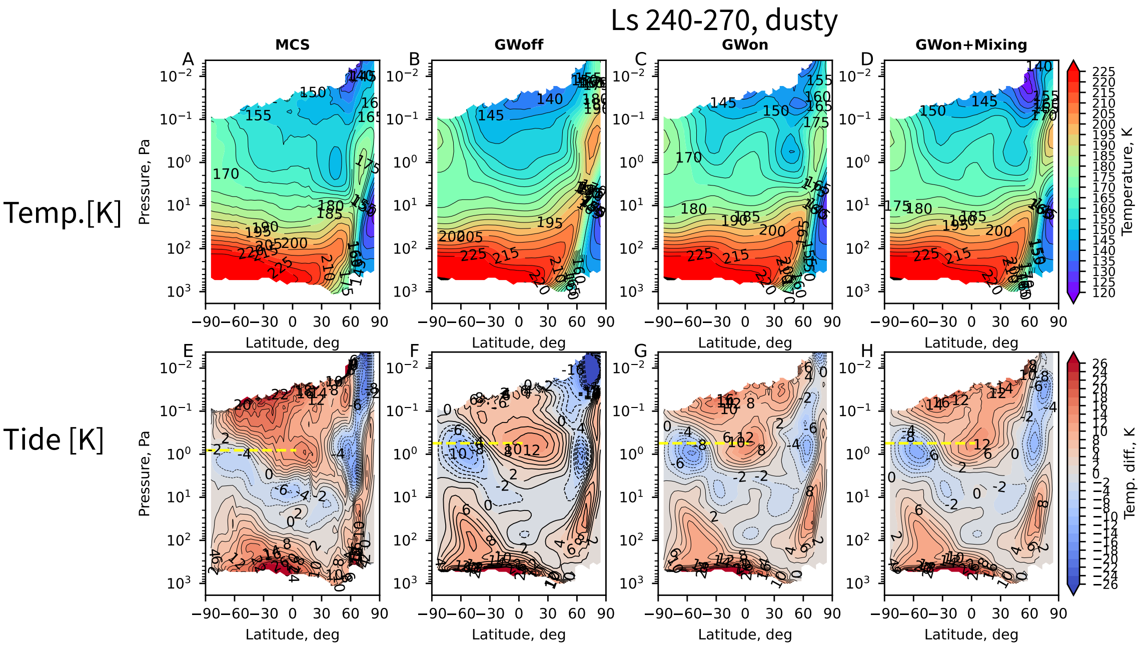
$$D_{eddy} = \sum_{j=1}^{M} D_{Eddy}^{j}, M = 8$$

Drags (GW drags not mixing drags)





Ls 60-90, clear-sky
MCS
GWoff
GWon
GWon+Mixing
A
B
C
D
E
F
G
H
Temp.[K]
Tide [K]
Pressure, Pa
Latitude, deg
Temperature, K
Temp. diff., K



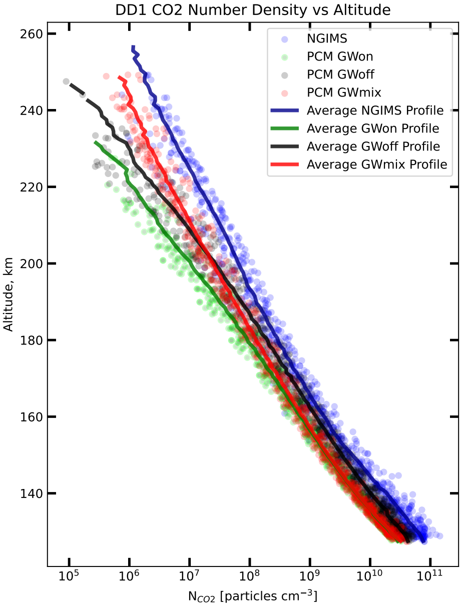
Ls 240-270, dusty
MCS
GWoff
GWon
GWon+Mixing
Temp.[K]
Tide [K]
Pressure, Pa
Latitude, deg
Temperature, K
Temp. diff., K
A
B
C
D
E
F
G
H

MAVEN/NGIMS

GWoff

GWon

GWon+mixing



Thank you for your time and interest in this topic. Please feel free to ask any questions.

Je vous remercie pour votre temps et votre intérêt. N’hésitez pas à poser vos questions.

承蒙聆听，不胜荣幸，有疑但问.

jiandong.liu@lmd.ipsl.fr
jiandong.liu@cnrs-orleans.fr

LMD