Differential treatment of the PBL between inner and outer regions of cold pools; I: over ocean

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Key Points:

- cold pool strength is stongly reduced when the splitting of the PBL is taken into account.
- cold pool strength is closer to observations when the splitting of the PBL is taken into account.
- simulated precipitation variability is improved.

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Abstract

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1 Introduction

2 Surface atmosphere coupling

2.0.0.1 Notations : We consider fields function of position (x, y) and time t, where (x, y) belongs to a very large domain (large when compared to cold pool sizes and to grid cell size). Most often the t dependance will be omitted. For each field, say ϕ, ϕ^* designates the average values over the domain, ϕ^w the average value over the (w) region, and ϕ^x the average value over the (x) region. Fluxes are positive downward.

At the surface, the boundary layer model is coupled with the subsurface model. The subsurface model may represent surface water of an ocean, soil at the surface of continent, land ice, or sea ice. In all cases we assume that the subsurface model is constrained by mixed boundary conditions, that is by an affine relationship between the surface humidity q_s^* and the surface moisture flux $\hat{\phi}^*$:

$$\hat{\phi}^* = \hat{\mu} - \hat{\lambda} q_s^* \tag{1}$$

The coupling between the two models is implemented in the following way: the boundary layer model computes the coefficients $\hat{\lambda}$ and $\hat{\mu}$; from these boundary conditions the subsurface model determines the values of the variables $\hat{\phi}^*$ and q_s^*; from these surface values the boundary layer scheme computes the humidity values in the whole troposphere.

However it is not directly $\hat{\lambda}$ and $\hat{\mu}$ that are used as boundary conditions for the subsurface model but another set of equivalent parameters. An intermediate variable is introduced, namely the apparent atmosphere humidity q^a, related to the surface flux by:

1. a "surface exchange" like equation:

$$\hat{\phi}^* = \hat{K^a}(q^a - q_s^*) \tag{2}$$

2. the sensivity coefficients of the boundary layer model to the surface flux:

$$q^a = \hat{A}^a + \hat{B}^a \hat{\phi}^* \Delta t \tag{3}$$

Then the boundary conditions are given by the three coefficients $\hat{K^a}$, $\hat{A}^a$ and $\hat{B}^a$. The equivalence with the boundary condition (1) may be shown by eliminating q^a between the two equations (2) and (3), which yields a relation of the form (1) with:

$$\begin{aligned} \hat{\mu} &= \frac{\hat{K^a}\hat{A}^a}{1 - \hat{K^a}\hat{B}^a\Delta t} \\ \hat{\lambda} &= \frac{\hat{K^a}}{1 - \hat{K^a}\hat{B}^a\Delta t} \end{aligned} \tag{4}$$

For a given pair $(\hat{\mu},\ \hat{\lambda})$ there exists an infinity of triplets $(\hat{K^a},\ \hat{A}^a,\ \hat{B}^a)$ such that the relations (4) hold. $\hat{K^a}$ may be chosen arbitrarily different from zero and $\hat{A}^a$ and $\hat{B}^a$ are given by:

$$\begin{aligned} \hat{A}^a &= \frac{\hat{\mu}}{\hat{\lambda}} \\ \hat{B}^a &= \frac{1}{\Delta t}[\frac{1}{\hat{K^a}} - \frac{1}{\hat{\lambda}}] \end{aligned} \tag{5}$$

In the LMDZ GCM $\hat{K^a}$ is chosen equal to the exchange coefficient between the first layer of the atmospheric model and the surface in which case: (i) the surface flux reads $\hat{\phi}^* = \hat{K}(q_1^* - q_s^*)$ so that q^a is identical to q_1^*, and (ii) the coefficients $\hat{A}^a$ and $\hat{B}^a$ are directly given by the resolution of the vertical diffusion equation in the atmosphere.

It is worth emphasizing that the equality $\hat{K^a} = \hat{K}$ (and its consequence $q^a = q_1^*$) is chosen in order to make it possible to separate the part of the subsurface boundary conditions due to surface processes ($\hat{K^a}$) and the part due to boundary layer processes ($\hat{A}^a$ and $\hat{B}^a$). When dealing with a split boundary layer, even though the same consideration of physical significance is accounted for, q^a will be different from q_1^*.

3 Splitting

For the sake of brevity, we present the full computations only for humidity. We shall outline the differences concerning enthalpy afterward.

3.1 Basic equations

The main assumptions of the model are:

1. All fields are horizontally homogeneous within (w) and within (x).
2. Model equations are linear during each time step: exchange coefficients are supposed constant (they are computed with the field values at the end of the previous time step).
3. Surface fluxes are given by:

$$\begin{cases} \hat{\phi}^w &= \hat{K^w}(q_1^w - q_s^w) \\ \hat{\phi}^x &= \hat{K^x}(q_1^x - q_s^x) \end{cases} \tag{6}$$

4. Surface moistures q_s^w and q_s^x differ by a prescribed amound $\hat{\Delta}_s$:

$$\delta q_s = \hat{\Delta}_s \tag{7}$$

5. The boundary layer scheme provides, within each region (w) and (x), a linear relationship between humidity q_1 at the first level and the surface moisture flux $\hat{\phi}$:

$$\begin{cases} q_1^w &= (\hat{A^w} + \hat{B^w}\hat{\phi}^w \Delta t) \\ q_1^x &= (\hat{A}^x + \hat{B}^x\hat{\phi}^x \Delta t) \end{cases} \tag{8}$$

In addition to these five equations, (6), (7) and (8), there are six equations relating for each of the field $\hat{\phi}$, q_s and q_1 the average value, the values in the regions (w) and (x), and the difference between the values in the regions (w) and (x). For instance for the moisture flux:

$$\begin{cases} \hat{\phi}^* &= \sigma_w\hat{\phi}^w + (1-\sigma_w)\hat{\phi}^x \\ \delta\hat{\phi} &= \hat{\phi}^w - \hat{\phi}^x \end{cases} \tag{9}$$

and similarly for the surface moisture difference and for the humidity at the first level.

To sum up, there are eleven linear equations for the twelve variables $\hat{\phi}^*$, $\hat{\phi}^w$, $\hat{\phi}^x$, $\delta\hat{\phi}$, q_1^*, q_1^w, q_1^x, δq_1, q_s^*, q_s^w, q_s^x, δq_s. from which it is possible (by elimination of all variables but $\hat{\phi}^*$ and q_s^*) to extract an affine relashionship between the average flux $\hat{\phi}^*$ and the average surface humidity q_s^*:

$$\hat{\phi}^* = \hat{\mu} - \hat{\lambda} q_s^* \tag{10}$$

where $\hat{\mu}$ and $\hat{\lambda}$ are coefficients which may be expressed in terms of the coefficients of the eleven linear relations. It is this relation which constitutes the mixed boundary condition for the surface model.

The purpose of the following is to determine the coefficients $\hat{\mu}$ and $\hat{\lambda}$. Moreover, in order to facilitate the coupling with surface models, we shall rewrite the mixed boundary conditions in the form of equations similar to equations (6) and (8): we shall first determine $\hat{A}^a$ and $\hat{B}^a$ assumimg that $\hat{K}^a$ is known. Then we shall choose $\hat{K}^a$ in such a way that it is only function of surface conditions (and not of the boundary layer above).

3.2 Solving

3.2.1 Determining $\hat{\lambda}$ and $\hat{\mu}$

We introduce the coefficients $\hat{K'}$:

$$\begin{cases} \hat{K'}^x = \dfrac{\hat{K}^x}{1 - \hat{K}^x \hat{B}^x \Delta t} \\ \\ \hat{K'}^w = \dfrac{\hat{K}^w}{1 - \hat{K}^w \hat{B}^w \Delta t} \end{cases} \tag{11}$$

These coefficients are effective exchange coefficients taking into account the boundary layer feedbacks:

$$\begin{aligned} \hat{\phi}^w &= \hat{K'}^w (\hat{A}^w - q_s^w) \\ \hat{\phi}^x &= \hat{K'}^x (\hat{A}^x - q_s^x) \end{aligned} \tag{12}$$

Now applying the second product identity (A.4) to the fields $\hat{K'}$, $\hat{A} - q_s$ and their product $\hat{\phi}$ yields:

$$\hat{\phi}^* = \hat{K'}^* (\hat{A}^* - q_s^*) + \sigma_w \sigma_x \delta \hat{K'} (\delta \hat{A} - \delta q_s) \tag{13}$$

or, since δq_s is prescribed (Eq. 7):

$$\hat{\phi}^* = \hat{K'}^* \hat{A}^* + \sigma_w \sigma_x \delta \hat{K'} (\delta \hat{A} - \hat{\Delta}_s) - \hat{K'}^* q_s^* \tag{14}$$

We recognize in this equation the mixed boundary condition we are seeking. Identification with equation (1) yields:

$$\begin{aligned} \hat{\mu} &= \hat{K'}^* \hat{A}^* + \sigma_w \sigma_x \delta \hat{K'} (\delta \hat{A} - \hat{\Delta}_s) \\ \hat{\lambda} &= \hat{K'}^* \end{aligned} \tag{15}$$

Now we want to write the boundary conditions in terms of the triplet $(\hat{K}^a,\ \hat{A}^a,\ \hat{B}^a)$. As explained in section (2), since $\hat{K}^a$ is then a free parameter, we first determine $\hat{A}^a$ and $\hat{B}^a$ for a given $\hat{K}^a$.

3.2.2 Solving for $\hat{A}^a$ and $\hat{B}^a$ for a given $\hat{K}^a$

The coefficients $\hat{A}^a$ and $\hat{B}^a$ are given by equations (5) where $\hat{\lambda}$ and $\hat{\mu}$ are given by equations (15):

$$\begin{aligned} \hat{A}^a &= \hat{A}^* + \sigma_w \sigma_x \frac{\delta \hat{K'}}{\hat{K'}^*} (\delta \hat{A} - \hat{\Delta}_s) \\ \hat{B}^a &= \frac{1}{\Delta t} [\frac{1}{\hat{K}^a} - \frac{1}{\hat{K'}^*}] \end{aligned} \tag{16}$$

The problem of coupling a split boundary layer with a uniform subsurface model is now solved. Whatever the non-zero coefficient $\hat{K}^a$ the apparent atmosphere humidity reads

$$q^a = \hat{A}^a + \hat{B}^a \hat{\phi}^* \Delta t$$

A possible choice for $\hat{K}^a$ would be $\hat{K}^a = \hat{K'}^*$. Then $\hat{B}^a = 0$ and $q^a = \hat{A}^a$ so that the mixed boundary condition for the subsurface model reads:

$$\hat{\phi}^* = \hat{K}^a (\hat{A}^a - q_s^*)$$

which means that the subsurface model behaves as if coupled to a fixed moisture atmosphere. This is a particularly simple formulation. However it has a drawback: the coefficient $\hat{K}^a$ is dependent on both the surface exchange processes and on the boundary layer processes. Moreover, obviously q^a (which does not depend on $\hat{\phi}^*$) will be in general different from q_1^* (which varies with $\hat{\phi}^*$), even if there is no cold pool. We shall now seek a more satisfactory formulation.

3.2.3 Determining $\hat{K}^a$

Similarly to the non-split case we might choose $\hat{K}^a = \hat{K}^*$. Although this will be our final choice, it needs some more justification, since the weighting between $\hat{K}^w$ and $\hat{K}^x$ could be very different from the one given by $\hat{K}^*$. In order to guide our choice we shall look at the variable q_1^* and compare it to q^a. The moisture field q_1 is related to the surface fluxes by equations (6):

$$\begin{cases} \hat{\phi}^w &= \hat{K}^w(q_1^w - q_s^w) \\ \hat{\phi}^x &= \hat{K}^x(q_1^x - q_s^x) \end{cases}$$

Using the same technique as in section (3.2.1) we apply the second product identity (A.4) to the fields $\hat{K}$, $q_1 - q_s$ and their product $\hat{\phi}$. It yields:

$$\hat{\phi}^* = \hat{K}^*(q_1^* - q_s^*) + \sigma_w\sigma_x\delta\hat{K}(\delta q_1 - \hat{\Delta}_s) \tag{17}$$

(It should be noted that this equation is not a mixed boundary condition for the subsurface model since it involves four unknown variables: $\hat{\phi}^*$, q_1^*, q_s^*, and δq_1).

Elimination of $\hat{\phi}^*$ between equations (2) and (17) yields an expression for the difference $q^a - q_1^*$:

$$q^a - q_1^* = (1 - \frac{\hat{K}^a}{\hat{K}^*})(q^a - q_s^*) + \sigma_w\sigma_x\frac{\delta\hat{K}}{\hat{K}^*}(\delta q_1 - \hat{\Delta}_s) \tag{18}$$

This equation shows that when there are no cold pools ($\sigma_w = 0$ or $\sigma_x = 0$) then:

$$q^a - q_1^* = (1 - \frac{\hat{K}^a}{\hat{K}^*})(q^a - q_s^*)$$

which implies that $q^a = q_1^*$ if and only if $\hat{K}^a = \hat{K}^*$. Consequently, in order to guarantee consistency with the no-cold pool case we choose $\hat{K}^a = \hat{K}^*$.

On the other hand, the equation for $\hat{B}^a$ may be rewritten in the form of a sum instead of a difference. First $\hat{B}^a$ reads also $\hat{B}^a = (1/\Delta t)(\hat{K'}^* - \hat{K}^*)/(\hat{K}^*\hat{K'}^*)$. Second it is easy to show that:

$$\hat{K'}^* - \hat{K}^* = \Delta t\,[\sigma_w\hat{K}^w\hat{K'}^w\hat{B}^w + \sigma_x\hat{K}^x\hat{K'}^x\hat{B}^x]$$

Hence $\hat{B}^a$ reads also:

$$\hat{B}^a = \frac{\sigma_w\hat{K}^w\hat{K'}^w\hat{B}^w + \sigma_x\hat{K}^x\hat{K'}^x\hat{B}^x}{\hat{K}^*\hat{K'}^*} \tag{19}$$

Hence the final formulas:

$$\begin{cases} \hat{K}^a &= \hat{K}^* \\ \hat{A}^a &= \hat{A}^* + \sigma_w\sigma_x\frac{\delta\hat{K'}}{\hat{K'}^*}(\delta\hat{A} - \hat{\Delta}_s) \\ \hat{B}^a &= \frac{\sigma_w\hat{K}^w\hat{K'}^w\hat{B}^w + \sigma_x\hat{K}^x\hat{K'}^x\hat{B}^x}{\hat{K}^*\hat{K'}^*} \end{cases} \tag{20}$$

Equations (20) express the boundary conditions for the subsurface model coupled to an equivalent horizontaly homogeneous atmosphere. It enables the subsurface model to determine the average surface flux $\hat{\phi}^*$. There remain to determine the repartition of this moisture flux among the two regions (w) and (x); otherwise stated, there remain to determine the difference $\hat{\delta\phi}$.

3.3 Back to the atmosphere

In order to compute the flux difference $\hat{\delta\phi}$ we apply the third product identity (A.6) to the fields $\hat{K'}$, $\hat{A} - q_s$ and their product $\hat{\phi}$:

$$\hat{K'}^* \hat{\delta\phi} \; - \; \delta\hat{K'}\hat{\phi}^* \; = \; \hat{K'^x}\hat{K'^w}(\delta\hat{A} \; - \; \delta q_s) \tag{21}$$

Hence the expression for $\hat{\delta\phi}$:

$$\hat{\delta\phi} \; = \; \frac{\delta\hat{K'}}{\hat{K'}^*}\hat{\phi}^* \; + \; (\delta\hat{A} \; - \; \hat{\Delta}_s)\frac{\hat{K'^x}\hat{K'^w}}{\hat{K'}^*} \tag{22}$$

3.4 Formulas for static energy

The equations for the dry static energy $h \; = \; C_pT + gz$ are almost identical to those for humidity except for the surface variable: it is q_s in the case of humidity while it is $T_s \; = \; h_s/C_p$ in the case of dry static energy. Hence the equations equivalent to (15) read:

$$\begin{aligned} \mu &= K'^*A^* + \sigma_w\sigma_x\delta K'(\delta A - C_p\Delta_s) \\ \lambda &= K'^* \end{aligned} \tag{23}$$

, the equations equivalent to (2) and (3) read:

$$\left\{ \begin{aligned} \phi^* &= K^a(h^a - C_pT_s^*) \\ h^a &= A^a \; + \; B^a\phi^*\Delta t \end{aligned} \right. \tag{24}$$

, and the equations equivalent to (20) read:

$$\left\{ \begin{aligned} K^a &= K^* \\ A^a &= A^* + \sigma_w\sigma_x\frac{\delta K'}{K'^*}(\delta A - C_p\Delta_s) \\ B^a &= \frac{\sigma_w K^w {K'}^w B^w \; + \; \sigma_x K^x {K'}^x B^x}{K^*K'^*} \end{aligned} \right. \tag{25}$$

Finally the "return to atmosphere" formula reads:

$$\delta\phi \; = \; \frac{\delta K'}{K'^*}\phi^* \; + \; (\delta A \; - \; C_p\Delta_s)\frac{{K'}^x{K'}^w}{K'^*} \tag{26}$$

4 1D simulations

5 3D simulations

6 Conclusion

A: Elementary product identities

Let a, b and p be three fields such that $p^w \; = \; a^wb^w$ and $p^x \; = \; a^xb^x$. This appendix proves three identities relating the field a, b and their product p.

Since $a^w = a^* + \sigma_x \delta a$ and $a^x = a^* - \sigma_w \delta a$ (and similarly for b), the products p^w and p^x read:

$$\begin{aligned} p^w &= (a^* + \sigma_x \delta a)(b^* + \sigma_x \delta b) \\ p^x &= (a^* - \sigma_w \delta a)(b^* - \sigma_w \delta b) \end{aligned} \tag{A.1}$$

Hence:

$$\begin{aligned} p^w &= a^* b^* + \sigma_x^2 \delta a\, \delta b + \sigma_x (a^*\, \delta b + b^*\, \delta a) \\ p^x &= a^* b^* + \sigma_w^2 \delta a\, \delta b - \sigma_w (a^*\, \delta b + b^*\, \delta a) \end{aligned} \tag{A.2}$$

From which it is possible to derive expressions for δp $(= p^w - p^x)$ and p^* $(= \sigma_w p^w + \sigma_x p^x)$ yielding the first product identity:

$$\delta p = a^*\, \delta b + b^*\, \delta a + (\sigma_x - \sigma_w) \delta a\, \delta b \tag{A.3}$$

and the second product identity:

$$p^* = a^* b^* + \sigma_w \sigma_x \delta a\, \delta b \tag{A.4}$$

Another usefull identity is obtained by forming $a^*\, \delta p \ - \ p^*\, \delta a$:

$$\begin{aligned} a^*\, \delta p \ - \ p^*\, \delta a &= \delta b[(a^*)^2 \ + \ (\sigma_x - \sigma_w) a^*\, \delta a \ - \ \sigma_x \sigma_w (\delta a)^2] \\ &= \delta b (a^* + \sigma_x \delta a)(a^* - \sigma_w \delta a) \end{aligned} \tag{A.5}$$

Finally the third product identity reads:

$$a^*\, \delta p \ - \ p^*\, \delta a \ = \ a^x a^w\, \delta b \tag{A.6}$$