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SPIN PHYSICS AND THE THEORY OF STRONG INTERACTIONS

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Taking the Minkowski space as the scene of quantum field theory implies an implicit assumption: the spin plays no dynamical role. This assumption (already challenged by reggeism) should be re-examined in the light of recent advances of experimental spin physics. To make a dynamical role of spin possible, it is proposed to use as a scene for the theory of strong interactions the whole Poincaré group. On the other hand characteristic functions, defined on the Poincaré group, provide the only known way to describe the distinctive peculiarity of resonances, namely that a resonance is both one particle and several particles. Regge trajectories are interpreted as evidence for a mass-spin correlation among the virtual hadrons of vacuum. It is conjectured that the form of this correlation is deducible from a central limit theorem on the Poincaré group. And that a statistical mechanics of hadronic vacuum is important for strong interaction theory.

Key words: resonances, dynamical spin, strong interactions, Poincaré group, hadronic vacuum, virtual hadrons.

1. INTRODUCTION

Experimental spin physics provides important criteria to test particle theories, in particular with regard to strong interactions [1]. It is more sensitive to the structure of interactions than experiments which disregard spin (unpolarized beams and targets, no polarization mea-

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surements). But it is also more delicate. The advancement of our knowledge of spin effects has been slow, not only because of the practical difficulty of experiments, but also because of theoretical prejudices. The various theories of strong interactions were developed at times when a largely accepted idea was that spin is an "unessential complication". And the experimental evidence for important spin effects was generally received as a surprise [2].

The situation has changed now. One of the two aims which motivated the construction of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven is the study of spin effects. More precisely, it has been to resolve the "spin crisis" which had come from disagreements between theoretical predictions and experimental results [3]. As Leader put it a few years ago, with the increased kinematic range at RHIC "either the trend of the experimental results must begin to change or we must seriously begin to challenge the validity of QCD." [4]

Our starting point here is a conjecture: sooner or later, the disagreements between some experimental results of spin physics and QCD predictions will become irremediable, all escape routes being barred. If that turns out to be true, it will be necessary to reconsider the foundations of the theories of strong interactions. It will be assumed here that what should be questioned is not specifically QCD but, more generally, any theory which does not take into account the qualitative difference between strong and electroweak interactions.

Before the discovery and experimental study of resonances, it was natural to assume that the strong interactions differ only quantitatively from the electromagnetic and weak ones. Resonances, however, turned out to be something radically new. They are unstable hadrons, but their mode of instability is not the same as that of, e.g., a neutron or a neutral pion: they have a mass spectrum which cannot be neglected [5,6]. If a particle has a weak or electromagnetic decay, it is often a reasonable approximation to neglect the interaction which causes its instability and to consider it as a stable particle. On the other hand, for a strongly decaying particle such an approximation does not make sense, since it amounts to neglecting the strong interactions themselves.

Thus, the discovery of resonances has brought to light the specificity of the strong interactions. For the first time it has shown that the difference between them and the other interactions is not only quantitative, but also qualitative. The theory of strong interactions should look, therefore, for conceptual tools adapted to the study of phenomena of an original type. It is assumed here that neither quantum field theories, nor S-matrix type theories, if they are taken literally, can provide these tools. On the other hand, some of the physical concepts that they introduced may become, provided that they are suitably reinterpreted, basic elements of the theory to come.

We rely on two series of facts: the existence and elementary properties of resonances, and the existence and fundamental properties of Regge trajectories. These facts have been long known. However, due to the influence of a wished analogy between the strong interactions and the better known electromagnetic and weak interactions, their original character has been greatly underestimated.

In short, the following steps have already been taken towards a theory of strong interactions concerned with their specific properties. First, the notion of a dynamical role of spin has come from a reflection about reggeism [7]. Later a kinematical analysis of resonances has led to the central notion of characteristic function [5,6]. Then, in a general frame in which the specificity of strong interactions was again stressed, the notion of global characteristic function has been defined [6]. More recently [8,9], a conception of resonances as properties of hadronic vacuum has been developed; it requires the use of a new concept, that of characteristic distribution. In the following the notions used in this context will be reminded without dwelling on technical details. Then a research program will be sketched.

2. THE DYNAMICAL ROLE OF SPIN

Quantum field theory (QFT), which satisfactorily describes the electroweak interactions, considers the relevant particles as quanta of certain fields. If the interactions are switched off, such a field satisfies a free wave equation, which implies the relation

$$\left(P^2 - m^2\right)\phi = 0\ . \tag{1}$$

(Here P^2 denotes the differential operator which corresponds to the Lorentz square of the energy-momentum vector, namely, minus the d'Alembertian). Accordingly, the energy-momentum vector p of the particle satisfies $p^2 = m^2$: the particle is on its mass shell. But as soon as there are interactions this property no longer holds. Instead of Eq. (1), the field satisfies an equation like (1), but with a non-zero right-hand side. Hence the relation $p^2 = m^2$ is no longer satisfied, and the particle can go for an excursion out of its mass shell. This "mass flexibility" comes from the fact that the scene of quantum field theory is the Minkowski space. The spacetime coordinates are primary, and the energy-momentum vector appears as a result of a Fourier transformation.

For the spin degrees of freedom, on the other hand, there is no such flexibility. With or without interactions, they are entirely determined by the nature of the field. They appear as purely kinematical labels, associated to the auxiliary vector space in which the field takes its values. For instance, a lepton is a quantum of a Dirac field, which implies that it has a spin 1/2. The case of gauge bosons is similar, apart from the fact that the representation of the little group is not the same for a virtual photon as for a (massless) free photon: the latter has only two possible polarizations, while the first has three [10]. The "spin rigidity" is a necessary consequence of the fact that the scene of usual quantum field theories is the Minkowski space. In short, besides their usual explicit assumptions these theories contain an implicit one: *The spin plays no dynamical role* [7]. This assumption agrees with all known properties of electroweak interactions.

In strong interactions, on the other hand, the role of spin is quite different. The experimental successes of reggeism, albeit limited, are enough to suggest that it cannot be considered as a purely kinematical label. The simplest interpretation of a Regge trajectory is that when a hadron goes out of its mass shell, at the same time it goes out of its spin shell. But the theories whose scene is Minkowski space cannot allow such a spin flexibility. The most natural way to regain it is to take as a scene for the theory, instead of the Minkowski space, the whole Poincaré group. This will allow us to treat on the same footing both Casimir operators of the group, namely, the above mentioned P^2 and the Lorentz square W^2 of the 4-vector W, whose components are the Pauli-Lubanski operators:

$$W_\mu = \varepsilon_{\mu\nu\rho\sigma} P^\nu M^{\rho\sigma} \ . \tag{2}$$

A function or a distribution ϕ related to a free particle satisfies, as an object defined on the group, two "wave equations":

$$\left(P^2 - m^2\right)\phi = 0 \ , \tag{3}$$

$$\left(W^2 + m^2 s(s+1)\right)\phi = 0 \ . \tag{4}$$

They imply the equation

$$\left(W^2 + s(s+1)P^2\right)\phi = 0 \ , \tag{5}$$

whose meaning is that the particle is on its spin shell. But as soon as there are interactions, we have no reason to assume that either of the Eqs. (3) and (5) is satisfied.

Let us take a closer look at the physical meaning of Regge trajectories. If we consider a resonance as a property of a scattering amplitude, it concerns a definite partial wave, characterized by precise values of the orbital angular momentum and of the spin. But if we consider it as an unstable hadron, it has both a mass spectrum and a spin spectrum. (The latter notion, although perhaps not familiar, is a quite natural one. As remarked long ago by Beltrametti [11]. for an unstable particle the spin value cannot be sharply defined.) Therefore we should not consider a Regge trajectory as a functional relation, but rather as a statistical correlation between mass and spin.

If the mass of a resonance is close to its nominal value, the spin spectrum has its center at the nominal value of the spin. In such a state, the resonance can be said to be quasi-real. If the mass takes values more remote from the nominal one, the spin spectrum is shifted in the direction suggested by the usual notion of trajectory. This correlation extends also to the region of negative masses squared, as attested by the fact that in many cases, the exchange of a Regge pole gives a good fit to an experimental cross section. One of the most striking examples is provided by the trajectory of the ρ meson [12]: it contains several well attested resonances, and its continuation in the region of negative mass squared gives a good fit to the reaction $\pi^- p \to \pi^0 n$.

In the latter kinematical region, properly speaking it is no longer a matter of spin but of the representation of the little group. (By convenience, however, the word "spin" can be used also in such cases.) It has been shown by Hadjioannou [13] and Joos [14] that the Regge formalism in the crossed channel can indeed be interpreted in terms of the representations of the Poincaré group with negative mass squared.

Analogously, in both kinematical regions a complex spin can be considered as a description of a spin spectrum of a special form. This becomes straightforward if one casts a glance at the expression [15] of a Legendre function of complex index in terms of Legendre polynomials:

$$P_\lambda(\cos\theta) = (\sin\lambda\pi/\pi)\sum_{s=0}^{\infty}(-1)^s\left[(\lambda - s)^{-1} - (\lambda + s + 1)^{-1}\right]P_s(\cos\theta)\ . \tag{6}$$

The amplitude of the spin spectrum is the expression between square brackets. If $Re\ \lambda \gg 1$, the first term is predominant and we get a "Breit-Wigner" spin spectrum with average value $Re\ \lambda$ and width $Im\ \lambda$.

In this connection it is interesting to note that QCD does not succeed much to explain the partial successes of reggeism. There has been recent attempts to understand Regge poles (the pomeron and others) from the point of view of QCD [16,17]. But, as Kaidalov puts it, "The derivation of the Regge poles in QCD is a difficult problem closely related to nonperturbative effects in QCD and to the problem of confinement."

3. THE RELEVANCE OF EXISTING THEORIES

The brilliant successes of QFT in the realm of electroweak interactions have brought many theoreticians to consider it as an indispensable frame also for strong interaction theory. Up to now, however, the soundness of this opinion has not been convincingly proved. Our assumption here is that the very concept of quantum field is of limited relevance for the theory of strong interactions. Of course QFT is a convenient way of taking into account some postulates such as Poincaré invariance, the principles of quantum mechanics and cluster decomposition. These general postulates, however, are not concerned with the specific dynamical properties of strong interactions. QFT's main weak point, on the other hand, is that it tacitly introduces the above mentioned assumption that the spin plays no dynamical role.

There are also S-matrix type theories and models (dispersion relations, bootstrap, Regge poles, duality, etc.). It is regrettable that these investigations tend to sink into oblivion. Firstly, because an important feature common to all of them is still valid and important, namely, the fact that they recognize, explicitly or not, the specificity of strong interactions. Secondly, because several of their results could be reinterpreted in a style closer to the experimental facts, more remote also from the complex plane. Such is the case, as stated above, for reggeism.

In the following the most important notion will be that of virtual particle, borrowed both from QFT and from S-matrix type theories. It is often said that it is a controversial one, because it can be considered either as a (purely formal) property of perturbation theory, or as a description of (real) vacuum fluctuations. But the very definition of a virtual particle, as a particle absent from both the initial and the final state of a process, immediately implies that resonances are virtual particles. Therefore in strong interaction theory the notion of virtual hadron is a necessary and a central one.

4. WHAT IS A RESONANCE ?

4.1. One is Many

A resonance is usually characterized by giving its mass, width, spin, and decay modes with their branching ratios. Thus it appears as an unstable hadron. The connection of this picture with experiment, however, is rather indirect, via the Heisenberg-like relation between width of mass spectrum and lifetime. Let us instead pay more attention to how resonances are given to us by experimental physics. It will turn out then that a resonance appears (at least in the simplest cases, which we assume to be the most significant) as several hadrons. The $\Delta(1232)$ appears as πN; the $\rho(770)$, as a dipion; the $\omega(782)$, as a tripion, and so on. We shall consider this double nature of resonances as their essential distinctive feature. As almost all hadrons are resonances, we shall regard this dual nature as a general characteristic of hadrons. This holds independently of such dynamical assumptions about the structure of hadrons as those related to partons, quarks and gluons, etc.

At the time when the discovery of the first resonances had first brought to light the dual nature of hadrons, there was no available mathematical formalism suited to describe it. One had to choose: there is either one particle, or several particles. Thus, if one wanted to take into account the specificity of strong interactions, he had to find a quantum formalism capable of describing a system which is both one and several particles. Such a formalism is provided by the concept of global characteristic function.

4.2. A Mathematical Formalism that Suits Resonances

Let us consider a stable particle, and let χ denote the signature of the irreducible representation of the Poincaré group asssociated to it: in the most familiar cases, $\chi = (m, s, \varepsilon)$, where ε is the sign of energy, most usually $+$. In the following the Poincaré group (or more exactly its universal covering group, the inhomogeneous $SL(2, C)$) will be denoted by G. The set of equivalence classes of irreducible representations of G, called the dual of G, will be denoted by $\widehat{G}$. (For more mathematical details, see Refs. 6 and 8). Let $\mathcal{H}_\chi$ be the Hilbert space of the representation χ. The state of the particle is most generally represented by a trace class operator W on $\mathcal{H}_\chi$. Let $U_{\chi,g}$ be the operator on $\mathcal{H}_\chi$ which represents the element g of the Poincaré group. Then the characteristic function of the state is the function on G defined by

$$\phi_g = Tr\left(WU_{\chi,g^{-1}}\right). \tag{7}$$

(For convenience reasons which appear in Ref. 9, the names of the variables are written as indices).

Similarly, for a system of n particles whose associated irreducible representations are $\chi_1, \cdots, \chi_n$, the state is defined by a trace class operator W on the space

$$\mathcal{H}_{\chi_1 \cdots \chi_n} = \mathcal{H}_{\chi_1} \otimes \cdots \mathcal{H}_{\chi_n}. \tag{8}$$

Let us define on this space the operators

$$U_{\chi_1 \cdots \chi_n, g_1 \cdots g_n} = U_{\chi_1, g_1} \otimes \cdots \otimes U_{\chi_n, g_n}. \tag{9}$$

Then the characteristic function is defined by

$$\phi_{g_1 \cdots g_n} = Tr\left(WU_{\chi_1 \cdots \chi_n, g_1^{-1} \cdots g_n^{-1}}\right). \tag{10}$$

Let us now define the *global characteristic function* ϕ_{glob} of the n-particle system: it is the characteristic function of that system, considered as a single particle. The latter will be called the *global particle* of the system. Mathematically, the global characteristic function is the restriction of the n-particle characteristic function to the diagonal subgroup of G^n:

$$\phi_{glob\ g} = \phi_{g, \cdots, g}. \tag{11}$$

In fact the concept of global particle is familiar, although most often in implicit form. Such notions as, for instance, center of mass motion, effective mass, total energy-momentum or angular momentum, require the use of global particles to be given a systematic formulation.

It can be shown that the function ϕ_{glob} is more regular than the characteristic function (7) of a single (stable) particle. In particular, it has a Fourier transform, which is the field of operators on the dual $\widehat{G}$ defined by

$$\rho_\chi = \int_G \phi_{glob\ g}\, U_{\chi,g}\, dg. \tag{12}$$

Suppose that for some reason we know that the signature χ of the global particle lies in a Borel part K of the dual $\widehat{G}$. (This will happen in particular if the mass of a resonance is cut into slices, which was done for instance in the experiment of Baton and Laurens [18] about the production of ρ-mesons. But more generally, the experimental device always puts some restrictions on the mass of a resonance.) Then the state of the global particle, conditioned by our knowledge that χ belongs to K, is represented by the *conditional statistical operator*

$$W(K) = \int_K^{\oplus} \rho_\chi\, d\chi \tag{13}$$

on the space

$$\mathcal{H}(K) = \int_K^{\oplus} \mathcal{H}_\chi\, d\chi. \tag{14}$$

(The definitions of the Haar measure dg and of the Plancherel measure $d\chi$ have been recalled in Ref. 6.)

The physical meaning of Eq. (13) is the following: to define the state of a resonance we must consider it as an incoherent superposition of states with sharp masses, and also of states with sharp spin.

As a particular case, one might of course take for K the whole dual $\widehat{G}$; one would get then the *unconditional statistical operator*

$$W = \int_{\widehat{G}}^{\oplus} \rho_\chi\, d\chi. \tag{15}$$

The function ϕ_{glob} can be expressed in terms of the field ρ_χ by the Fourier inversion formula

$$\phi_{glob\ g} = \int_{\widehat{G}} Tr\,(\rho_\chi\, U_{\chi,g^{-1}})\, d\chi. \tag{16}$$

4.3. Characteristic Functions in Probability Theory

Let us now briefly review the usual notion of characteristic function, familiar in probability theory. If X is a real-valued random variable, it defines a probability measure μ on the real line. The characteristic function of X is the function ϕ of a real variable t defined by

$$\phi_t = \int \exp(itx)\ d\mu(x)\ . \tag{17}$$

Now if $\mathcal{X}$ is a vector-valued random variable, its components $X_1, \cdots, X_n$ are n real-valued random variables. It defines similarly a probability measure μ on R^n; the characteristic function of $\mathcal{X}$ is the function ϕ of n real variables defined by

$$\phi_{t_1 \cdots t_n} = \int \exp i\,(t_1 + \cdots + t_n)\, d\mu\,(x_1, \cdots, x_n)\ . \tag{18}$$

Let us now consider the random variable X defined as the sum of the n random variables $X_1, \cdots X_n$:

$$X = X_1 + \cdots + X_n\ . \tag{19}$$

It turns out that its characteristic function Φ is the diagonal restriction of the function ϕ defined by Eq. (18) :

$$\Phi_t = \phi_{t, \cdots, t}\ . \tag{20}$$

If X denotes the coordinate of a point, then t has the dimension of a spatial frequency. The coordinate x can be considered as an element of the additive group of real numbers, R. Then the spatial frequency t belongs to the dual of that group, which is isomorphic to R.

Now the definition (7) says that the value at g of a one-particle characteristic function is the expectation value of $U_{\chi, g^{-1}}$; clearly this is a generalization of the definition (17), according to which the value at t of the characteristic function of a random variable is the expectation value of $\exp(itX)$. Similarly, the definition (10) of an n-particle characteristic function generalizes the definition (18) of the characteristic function of a vector-valued random variable; and the definition (11) of the global characteristic function of a system of n particles generalizes the definition (20) of the characteristic function of the sum of n random variables. Otherwise stated, the notion of global particle is a generalization of the notion of sum of independent random variables. Mathematically, these generalizations amount to replacing the additive group of real numbers by some group G; for the problems of interest here, G is the Poincaré group.

5. LOOKING FOR RESONANCES IN VACUUM

Granted that a resonance is both one and several particles, a question remains: what does it mean exactly to call such an object a resonance?

In other branches of physics there are several types of resonance phenomena, among which we shall consider two.

A. *Resonance of a preexisting system.* In classical phenomena such as the excitation of a mechanical or electrical oscillator, resonance is the anomalously high response of a system to a driving force (mechanical force, electrical voltage) whose frequency is close to some proper frequency of the system.

B. *Resonance of a compound system.* In many nuclear reactions, the incident and the target objects may form a more or less long-lived compound system. (See, for instance, Wigner's review [19]). The cross section for the formation of that system is small unless the total energy of the colliding pair coincides closely with one of the energy levels of the compound system. In the latter case it shows a sharp maximum, and this phenomenon is called resonance.

The problematic nature of the concept of a resonance as a particle was early recognized by Feynman, who told his students around 1962: "When there is a very *sharp* resonance, it corresponds to a very definite energy, just as though there were a particle of that energy present in nature. When the resonance gets wider, then we do not know whether to say there is a particle which does not last very long, or simply a resonance in the reaction probability." [20]

One should ponder over the remark that there may be "a particle of that energy present in nature." Present in nature, but where?

I can see no plausible answer to this question, except: present in vacuum. It has been indeed for some time a familiar idea that vacuum is the medium which contains virtually all possible processes. Now if we take it seriously, we are led to think that vacuum is the natural location of resonances, and that resonance phenomena in particle physics, while they belong of course to type B (for instance the Δ, as said above, is also a πN system), also belong to type A. The preexisting system is the hadronic vacuum, and the driving force which excites its motion is the pair target particle-incident particle.

6. PROSPECTS FOR A DYNAMICS

Can the above sketched general principles become the kernel of a theory of strong interactions? One might first try to build a quantum field theory on the Poincaré group [21]. For the time being, however, this appears as a purely formal possibility: the known phenomena give us no hint about a possible way of describing the interactions in such a frame. The most obvious evidence for the nature of interactions is the existence of resonances. Rather than looking for fields, it seems therefore more appropriate to interpret the functions or distributions on the Poincaré group as characteristic functions or distributions.

6.1. General Description

Assuming that hadronic vacuum is the basic object, we may try to describe it as a gas of virtual hadrons. The problem will be then to build a statistical mechanics of this gas. It is a system with zero total energy-momentum, angular momentum, etc. More precisely, we shall assume that any particle present in vacuum belongs to a system whose energy-momentum, as well as the other quantum numbers, are zero. Such a system is called a *keneme*; it can transform into nothing, and vice versa. This assumption implies that some of the particles present in vacuum have a negative energy.

In order to describe the state of a system, quantum mechanics uses a statistical operator. It is a positive trace class operator W on the Hilbert space $\mathcal{H}$ of the pure states of the system.

In order to describe the state of a system of N particles (especially in the case where N is infinite), quantum statistical mechanics uses a sequence of statistical operators W_n $(n = 1, 2, \cdots, N)$ relative respectively to the n-particle aspects of the system. (See, for instance, Bogoliubov [22].)

As has been shown in Ref. 9, relativistic invariance does not allow to describe the virtual hadrons of vacuum by trace class statistical operators. We must rather use the notion, which goes back to von Neumann, of relative statistical operators, which are allowed to have an infinite trace. Accordingly, the characteristic functions are replaced by characteristic distributions.

These distributions are a convenient substitute for the statistical operators. Furthermore, the distribution $F(n)$ is related to the processes in which p particles give rise to q particles $(p + q = n)$. This is due to the fact that a process (p particles $\rightarrow q$ particles) is related by crossing to the processes ($\emptyset \rightarrow p + q$ particles $\rightarrow \emptyset$), where $\emptyset$ stands for "nothing." In Ref. 9 this has been explicitly written, and shown in detail for the case $n = 2$.

6.2. The Case n = 2

Let us consider now the particular case of the two-particle kenemes. The one-particle state of vacuum is described by a statistical operator of the form (13). An element g of the invariance group G transforms the operator (13) into the operator

$$W' = \int_K^{\oplus} \rho'_\chi \, d\chi, \tag{21}$$

with

$$\rho'_\chi = U_{\chi,g} \, \rho_\chi \, U_{\chi,g^{-1}}. \tag{22}$$

Now the invariance of vacuum by the elements of G implies that for almost any χ one has

$$\rho'_\chi = \rho_\chi. \tag{23}$$

Hence for almost any $\chi \varepsilon \widehat{G}$ and for all $g \varepsilon G$, ρ_χ commutes with $U_{\chi,g}$; by Schur's lemma, this implies that ρ_χ is almost everywhere a multiple of the identity operator:

$$\rho_\chi = \sigma_\chi I_\chi, \tag{24}$$

where I_χ is the identity operator on $\mathcal{H}_\chi$ and σ_χ is a positive real number. As $\mathcal{H}_\chi$ is almost everywhere infinite-dimensional, the operator (21) is not trace class; hence we cannot define a one-particle vacuum characteristic function, corresponding to the one-particle statistical operator (13)-(14). Indeed, this function would be defined by

$$f_g = \int_{\widehat{G}} \sigma_\chi Tr\left(U_{\chi,g^{-1}}\right) d\chi, \tag{25}$$

but the trace does not exist. It turns out, however, that a distribution over the group G can be defined, which plays the role of the trace of $U_{\chi,g}$. By analogy with the case of compact groups, it is called the character of the representation χ. We shall write formally

$$\Delta_{\chi g} = Tr\left(U_{\chi,g}\right). \tag{26}$$

The characters of the representations of the Poincaré group have been computed explicitly [23-25].

As a result the distribution $F(2)$, representative of the two-particle kenemes, admits the following spectral representation:

$$F(2) = \int_{\widehat{G}} \widehat{\Delta}_\chi^{g_1 g_2^{-1}} \ \sigma_\chi \ d\chi. \tag{27}$$

Equivalently, the one-particle vacuum open distribution has the form

$$H(1) = \int_{\widehat{G}} \widehat{\Delta}_\chi \ \sigma_\chi \ d\chi. \tag{28}$$

6.3. The Mass-Spin Correlation: a Conjecture

The mass-spin correlation of vacuum virtual hadrons is contained in the spectral function σ. Is it possible to find the general form of this correlation?

As said above, it appears experimentally as a Regge trajectory. It is known empirically that the trajectories have approximately universal properties: roughly speaking, they have all the same form and the same slope. This leads us to conjecture that the form of the mass-spin correlation can appear as a result of a universal property of hadrons.

As said above, a resonance is both one and several particles. We may assume, therefore, that a virtual hadron of vacuum is the global particle of several virtual hadrons. As a result, the distribution $H(1)$ is the global distribution of several virtual hadrons.

What about their interactions? Let us draw our inspiration from the mode of experimental manifestation of resonances. What do experimental physicists do, when they study a phenomenon in which the resonances obviously play a dominant role? Consider for instance the case of resonances such as the ρ or the ω mesons. One measures effective mass spectra of dipions or tripions, without bothering about interactions between pions. Such an experimental procedure suggests that *the study of virtual particles (as a particular case, of resonances) is enough to disclose the essential nature of interactions*.

Therefore we can say that the many-particle aspect of a hadron consists of independent hadrons. This holds in particular for the virtual hadrons, constitutive of hadronic vacuum. Their state is the global state of several independent virtual hadrons.

Of how many hadrons? Their number has no reason to be sharp, but it is natural to assume that its average value increases with the mass. Now the angular momentum of a cluster of n (real) particles tends to increase with n. If we assume this law to be valid also for a cluster of virtual hadrons, we get the intuitive result that the angular momentum of the cluster (i.e., the spin of the hadron) tends to increase with its mass. We have thus a simple qualitative picture of a Regge trajectory, at least in the region of positive mass squared. How could we transform it into a quantitative one?

Here the principles of statistical physics are of interest, especially in the form given to them by Khinchin [26]. For a system made of a large number n of components, probability theory can provide laws, the form of which does not depend of the special nature of the laws governing the separate components.

Khinchin also shows that when n tends to infinity, the probability law for a sum of n independent random variables tends to a simple standard form (in many cases, but not always, a Gaussian form). (See also Rényi's textbook [27].) Such a property is the content of what probabilists call a central limit theorem. In many cases, even for small values of n the probability law of the sum is very close to the limit. The practical interest of central limit theorems comes from that fact, which allows the exact law to be replaced by its limit.

Thus we get finally the following assumption: the form of $H(1)$ is given by the limit for infinite n of of the global characteristic distribution of n independent hadrons.

It is a likely conjecture that a central limit theorem exists for our situation, i.e., a theorem which gives the limiting form of the product of n characteristic distributions on the Poincaré group. An analog of this expected theorem effectively exists in a simple model situation: the approximate evaluation of relativistic phase space integrals by use of the usual central limit theorem [28]. A phase space integral is the convolution product of n distributions in momentum space, each of which describes a uniform probability on a mass shell. The main differences between this model situation and that which is of interest for us here are the following: 1°) Spin is disregarded, and 2°). The particles are on their mass shell.

It is hoped that the mathematical difficulties will be overcome, allowing the general form of the mass-spin correlation to be found and, more generally, a statistical mechanics of the hadronic vacuum to be built.

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