Theory and equations for
"Craters from Impacts and Explosions"

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1. Dimensionless Forms

As the primary example, the volume V of a crater formed by a given impact can be expected to depend on the impactor radius a, its velocity U, and its mass density δ. Note that those three variables also define the kinetic energy, momentum, and mass of the impactor. The target has some strength measure Y, a mass density ρ and the surface gravity is denoted as g. Then there is some functional relationship:

$$V = f\left[\{a, U, \delta\}, \{\rho, Y\}, g\right] \tag{1}$$

There are 7 variables in this relation, and three independent dimensions, so the standard tools of dimensional analysis can be used to obtain the reduced form using 4 dimensionless groups:

$$\frac{\rho V}{m} = \bar{f}\left[\frac{ga}{U^2}, \frac{Y}{\rho U^2}, \frac{\rho}{\delta}\right], \quad m = \frac{4\pi}{3}\delta a^3 \tag{2}$$

which is commonly written as

$$\pi_V = \bar{f}\left[\pi_2, \pi_3, \frac{\rho}{\delta}\right] \tag{2a}$$

Generally most of the effects of the impact are in a region very much larger than the impactor. In that case a further approximation is to suppose that the impactor acts as an instantaneous "point-source", which has neither length nor time measures. Then it can have only one single measure of its magnitude, not three independent ones. Holsapple 1983 coined the term "coupling parameter" for that measure. It must be some single power-law variable of a, U and δ of the form

$$C = aU^{\mu}\delta^{\nu} \tag{3}$$

where μ and ν are exponents to be determined. This reduces the number of independent variables by two, and now the most general form can be written in terms of only two dimensionless groups, either as

$$\frac{\rho V}{m}\left(\frac{Y}{\rho U^2}\right)^{\frac{3\mu}{2}}\left(\frac{\rho}{\delta}\right)^{3\nu-1} = F\left[\frac{ga}{U^2}\left(\frac{\rho U^2}{Y}\right)^{\frac{2+\mu}{2}}\left(\frac{\rho}{\delta}\right)^{-\nu}\right] \tag{4}$$

or

$$\frac{\rho V}{m}\left[\frac{ga}{U^2}\right]^{\frac{3\mu}{2+\mu}}\left(\frac{\rho}{\delta}\right)^{\frac{6\nu-2-\mu}{2+\mu}} = G\left\{\frac{Y}{\rho U^2}\left[\frac{ga}{U^2}\right]^{\frac{-2}{2+\mu}}\left(\frac{\rho}{\delta}\right)^{\frac{2\nu}{2+\mu}}\right\} \tag{5}$$

A careful examination of these forms will divulge that indeed the only dependence on the impactor is indeed the combination of Eq. (3) in all places. The first form is the more useful when the event is small and dominated by the target strength. Then the gravity can be ignored, and the rhs of (4) is just a constant, so that

$$V \propto \frac{m}{\rho}\left(\frac{\rho U^2}{Y}\right)^{\frac{3\mu}{2}}\left(\frac{\rho}{\delta}\right)^{1-3\nu} \tag{6}$$

defining what is called the "strength regime" of cratering.

In the other limit, the event is sufficiently large and gravitational stresses dominate the strength measure. Then, dropping the strength term, the form (5) gives for this "gravity regime"

$$V \propto \frac{m}{\rho}\left[\frac{ga}{U^2}\right]^{\frac{-3\mu}{2+\mu}}\left(\frac{\rho}{\delta}\right)^{\frac{2+\mu-6\nu}{2+\mu}}$$

A general form with those limits and that interpolates between these two regimes is taken as

$$\pi_V = K_1\left\{\pi_2\left(\frac{\rho}{\delta}\right)^{\frac{6\nu-2-\mu}{3\mu}} + K_2\left[\pi_3\left(\frac{\rho}{\delta}\right)^{\frac{6\nu-2}{3\mu}}\right]^{\frac{2+\mu}{2}}\right\}^{-\frac{3\mu}{2+\mu}} \tag{7}$$

$$\pi_V = \frac{\rho V}{m},\quad \pi_2 = \frac{ga}{U^2},\quad \pi_3 = \frac{Y}{\rho U^2}$$

which is the starting point for the fits here for impact cratering. (The last exponent of (7) is often denoted as α.)

The two constants K_1 and K_2 and the two exponents μ and ν come from experiments and the database. It is fairly well established (see the references) that for relatively dissipative materials such as "dry" soils[1] the exponent μ is about 0.4, and for wet targets is about 0.55. The exponent ν is 1/3 if it is the mass and velocity of the impactor that determine its measure, but experiments give uncertain values, ranging from about 0.2 to 0.4.

2. Impact Cratering Volume

In Holsapple 1993, figures[2] are given for impacts into each of six target types, with definite values of the two exponents, in the figures 4, 5, 6 and 7 Those were obtained by the same data as now being considered, and the impact data is augmented by the fact that impacts give about the same volume as an explosive with the same specific energy and mass density if the explosive is buried about 1.5 impactor radii. (see Holsapple, 1979)

Here I take the same curves as those figures, and added two more: lunar regolith and cold ice. Cold ice was assumed to have the same gravity regime crater size as the other non-porous materials. Estimates for the strengths of the materials were made, and then the constant K_2 was chosen to give the strength regime asymptote shown in the figures. The results used in this web application are then (units are all cgs):

[1] That is, "nominally dry": even desert alluvium can have a few % moisture.

[2] . Note that there are errors in the Table 1 and; in the legends of the figures, the exponent on π_3 is incorrectly written as $(2+\mu)/\mu$, it should be $(2+\mu)/2$. However, the curves are correct.

Material	**K_1**	**K_2**	**μ**	**ν**	**Y (dynes/cm^2)**	**ρ (gm/cm^3)**
Water	0.98	0	0.55	.33	0	1
Dry Sand	0.132	0	0.41	.33	0	1.7
Dry Soil	0.132	0.26	0.41	.33	2E6	1.7
Wet Soil	0.095	0.35	0.55	.33	5E6	2.1
Soft Rock (Hard Soil)	0.095	0.215	0.55	.33	1E7	2.1
Hard Rock	0.095	0.257	0.55	.33	1E8	3.2
Lunar Regolith	0.132	0.26	0.41	.33	1E5	1.5
Cold Ice	0.095	0.351	0.55	.33	1.5E5	0.93

These strength and mass density values can be changed by the user if the choice "Other Soil" or "Other Rock" is made in the pull-down menu. So far, the other two properties shown, the porosity and friction angle are not explicitly used in the estimates; their effects are implicitly included by the division of the targets into the material types.

For the programmed impactors the mass density is needed. I use

Impactor type	**Mass density δ**
Aluminum	2.7
Plastic	0.95
Steel	7.8
C-Type	1.8
S-Type	3.0
Comet	0.8

Any changes are made by choosing "Other" in the pull-down menu for the impactor type. Gravity is pre-set for Terrestrial, Lunar or two asteroid diameters, the input is arbitrary if "Other" is selected. The velocity can be set to any value, but a warning ensues for a values below 1 km/sec, where the data is sketchy and the point source assumption becomes iffy. For non-vertical impacts, the vertical component U cos(θ) is used. The energy and mass of the impactor are calculated and presented, as well as the Pi-groups and the crater volume.

2.1 Impact Crater Shapes: Simple Craters

The shapes of simple craters are calculated from

$$R=K_r V^{1/3}$$

$$D=\text{depth}=K_d V^{1/3}$$

The values indicated by the data and programmed are:

Material	K_r	K_d
Water	0.8	0.75
Dry Sand	1.4	0.35
Dry Soils (some cohesion)	1.1	0.6
Soft Rock	1.1	0.6
Cold Ice	1.1	0.6

The rim diameter is assumed to be 1.3 times the excavation diameter and the lip height 0.36 times the rim diameter, consistent with the data and measured lunar simple craters. The ejecta volume is assumed to be 80% of the excavation volume. The crater formation time is from Schmidt and Housen 1987, and the Figure 12 in Holsapple, 1993a as

$$T=0.8\sqrt{\frac{V^{1/3}}{g}} \tag{8}$$

2.2 Melt and Vapor Volumes

Melt and vaporization of target material occurs when the initial impact pressure is high enough. For melt, I assume that the velocity threshold is $U^2=10E_{melt}$. I take a generic value for the melt energy for silicates as 5E10. I use the "less than energy scaling" from Holsapple, 2003 matched to some of the results from Pierazzo et al 1997 and get

$$V_{melt}=0.5V_{projectile}\left[\frac{U^2}{5E10}-10\right]^{0.9} \tag{9}$$

Vapor production is in a volume much closer to the impactor, so I use strict energy scaling with a generic vapor energy of 1.5E11:

$$V_{vapor} = 0.4 V_{impactor} \left[\frac{U^2}{1.5\ 10^{11}} - 10 \right]^{1.0} \tag{10}$$

I have not yet added the melt and vapor for impacts into ice, there are significant questions abut its many phases at cold temperatures.

2.3 Complex Craters

For craters with a simple radius greater than some value R^*, the simple excavation crater with the radius R_e undergoes a late-time readjustment into a much broader and shallower "complex crater". The data for lunar craters by Pike 1977 gives a transition to complex shapes beginning at 10.6 km rim diameter. The transition in rim heights begins at the larger size, 22.8 km diameter. The onset of flat floors is gradual, but is fully developed at 20 km diameter.

Let R_r^f denote the final rim radius, and R_r^t the transient (simple) rim radius. Holsapple, 1993b presented an analysis of the transformation from simple to complex craters. It is based on an incompressible readjustment from the details of simple crater shapes measured in laboratory experiments and those observed for lunar craters, using primarily the data of Pike 1977. The approach is outlined in Holsapple, 1993a. The primary result is an expression for the ratio of the final to transient rim radius:

$$\frac{R_r^f}{R_r^t} = 1.02 \left(\frac{R_r^f}{R^*} \right)^{0.079} \tag{11}$$

which gives that, using the ratio 1.3 for the transient rim to excavation rim radii,

$$R_r^f = 1.33 R_e^{1.086} \left(R^* \right)^{-0.086} \tag{12}$$

For the transition radius I assume that $R^* \propto \dfrac{Y}{\rho g}$ and, for lunar craters $D^* = 2R^*$ is 10.6 km.

The Pike data for lunar craters gives for the depth of complex craters $d = 1.044 \left(D_r^f \right)^{0.301}$ in km units. This matches the simple crater result, $d = 0.2 D_r^f$ at the transition onset using the dimensionally consistent form

$$d = 0.2D^* \left(\frac{D_r^f}{D^*} \right)^{0.301} \quad (13)$$

For the rim height, Pike gives $h = 0.236 \left(D_r^f \right)^{0.399}$ for complex craters and $h = 0.036D$ for simple. With the transition at 22.8 km diameter, that gives the equation

$$h = 0.036D^* \left(\frac{D_r^f}{D^*} \right)^{0.399} \quad (14)$$

The flat floor diameter is given for lunar complex as $D_f = 0.187 \left(D_r^f \right)^{1.249}$ for diameters greater than 20 km. Assuming this begins at zero at the 10.6 km onset of complex craters, the fit used was

$$D_f = 0.292 \left(D^* \right)^{-0.249} \left(D_r^f - D^* \right)^{1.249} \quad (15)$$

Finally, the volume below the rim uses a profile with a flat floor, and a uniform slope from the floor diameter to the rim diameter with the rim height. It is given as

$$vol = \frac{\pi d}{4} \left[D_f^{\,2} + \frac{1}{3} \left(D_r^f - D_f \right) \left(D_r^f + 2D_f \right) \right] \quad (16)$$

Note that the section of the output for complex craters only appears when the crater sizes are larger than the transition diameter.

3. Explosive Cratering

Explosive cratering is much like impact cratering, but there is one additional independent variable: the depth of burial of the explosive. The specific energy of the impactor, which is $\frac{1}{2}U^2$ is replaced by the specific energy Q of the explosive material.

Also, it is more common to use the explosive mass (weight) W rather than its radius. Thus, the dimensional form for the crater volume becomes

$$\frac{\rho V}{m} = \bar{f}\left[\left(\frac{g}{Q}\right)\left(\frac{W}{\delta}\right)^{\frac{1}{3}}, \frac{Y}{\rho Q}, \frac{\rho}{\delta}, \frac{d}{a}\right] \tag{17}$$

Note that the relation between the explosive gravity π_2 group and that of the explosive is

$$\pi_2 = \left(\frac{g}{Q}\right)\left(\frac{W}{\delta}\right)^{\frac{1}{3}} = 3.22\frac{ga}{U^2} \tag{18}$$

If the point-source assumption is added, then again the explosive radius a, its specific energy Q and its mass density d can only occur in a power-law group. Specifically, This equation (17) must then have the restricted form

If I told you anything more, I would have to shoot you…..(To be completed)

4. References:

(Note: many of the Holsapple references can be downloaded from http://keith.aa.washington.edu/papers.html)

Holsapple, K. A., 2003 "Does melt volume give the signature of the impactor?" *Proceedings, Workshop on Bridging the Gap between Modeling and Observations*. Houston TX, February, 2003.

Holsapple, K. A., 1993a "The Scaling of Impact Processes in Planetary Sciences", *Annual Reviews of Earth and Planetary Sciences* **21** pp333-373.

Holsapple, K.A. 1993b. "The Size of Complex Craters", *Lunar and Planetary Science XXIV.*

Holsapple, K.A., 1987 "The Scaling of Impact Phenomenon", *Intl. J. Impact Engr., 5,* 343-355.

Holsapple, K.A., 1980. "The Equivalent Depth of Burst for Impact Cratering", *Lunar and Planetary Science XI.*

Holsapple, K.A. and S. Peyton, 1987. "The Scaling of Nuclear Weapons Effects for Near Surface Bursts", *Rpt. DNA 6543F, Defense Nuclear Agency*, Washington, D.C.

Holsapple, K.A. and R.M. Schmidt, 1982, "On the Scaling of Crater Dimensions 2. Impact Processes", *J. Geophy. Res., 87, No. B3,* 1849-1870.

Holsapple, K.A. and R.M. Schmidt, 1980. "On the Scaling of Crater Dimensions 1. Explosive Processes" *J. Geophys. Res. 85, No. B12*, 7247-7256.

Housen, K.R., and K.A. Holsapple, 2003. "Impact Cratering on Porous Asteroids", *Icarus…*

Housen, K.R., R.M. Schmidt, and K.A. Holsapple, 1983. "Crater Ejection Scaling Laws: Fundamental Forms Based on Dimensional Analysis", *J. Geophys. Res. 88, No. B3*, 2485-2499.

Schmidt R.M., Housen K.R. and K.A. Holsapple, 1988. "Gravity Effects in Cratering", DNA-TR-86-182, *Defense Nuclear Agency,* Washington D.C.

Schmidt, R.M. and K.A. Holsapple, 1982. "Estimates of Crater Size for Large-Body Impact: Gravity-Scaling Results", *Geo. Soc. Am., Special Paper No. 190.*

Schmidt, R.M. and K.A. Holsapple, 1980. "Theory and Experiments on Centrifuge Cratering", *J. Geophys. Res. 85,* 235-252.